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Assessment of empirical formulae for determining the hydraulic conductivity of glass beads

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Abstract: Empirical formulae are often used in practice to quickly and cheaply determine the hydraulic conductivity of soil. Numerous relations based on dimensional analysis and experimental measurements have been published for the determination of hydraulic conductivity since the end of 19th century. In this paper, 20 available empirical formulae are listed, converted and re-arranged into SI units. Experimental research was carried out concerning hydraulic conductivity for three glass bead size (diameters 0.2 mm, 0.5 mm and 1.0 mm) and variable porosity. The series of experiments consisted of 177 separate tests conducted in order to obtain relevant statistical sets. The validity of various published porosity functions and empirical formulae was verified with the use of the experimental data obtained from the glass beads. The best fit was provided by the porosity function $n^3/(1–n)^2$. In the case of the estimation of the hydraulic conductivity of uniform glass beads, the best fit was exhibited by formulae published by Terzaghi, Kozeny, Carman, Zunker and Chapuis et al.

Keywords: Hydraulic conductivity; Empirical formulae; Porosity; Porosity function configuration.

INTRODUCTION

The determination of hydraulic conductivity via field pumping tests may be very costly and time-consuming. At the same time, laboratory testing using permeameters may not be a feasible solution in many cases due to time and cost restrictions. For this reason, in many practical studies, namely in preliminary aquifer assessment (EPG, 2009; Šoltész and Baroková, 2014, etc.), empirical relations appear to be a suitable alternative. However, empirical relations have been derived for specific conditions and have their applicability limits.

The typical form of empirical equations for the determination of hydraulic conductivity comes from dimensional analysis based on the Darcy-Weissbach equation (Kasenow, 2002; Vuković and Soro, 1992). The general problems with the proposed formulae lie in determining the characteristic pore diameter and expressing the effect of soil non-uniformity and the form of the appropriate porosity function which reflects the soil compaction rate.

Probably the first relation was proposed by Hazen (1892). It expresses the simple linear dependence between hydraulic conductivity and soil porosity. In his formula, Hazen did not consider the effect of soil non-uniformity. This is also the case with formulae proposed by Slichter (1899) and Terzaghi (1925). Kozeny (1927) proposed a formula that was modified by Carman (1937, 1939) to become the Kozeny-Carman equation.

Pavchich (VNIIG, 1991), Sauerbrey (1932), Krüger (1918), Kozeny (1953), Zunker (1932), Zamarin (1928), Koenders and Williams (1992), and Chapuis et al. (2005) derived the characteristic pore diameter from the effective grain size d_e and porosity function $\chi(n)$ based on the analysis of typical sphere configurations (VNIIG, 1991). Most authors (Hazen, Slichter, Terzaghi, Beyer, Harleman et al., Chapuis et al., and others) considered d_{10} to be an effective grain diameter, though Sauerbrey and Pavchich preferred d_{17}. Authors like Krüger, Kozeny, Zunker and others calculated the effective grain diameter from the grain size distribution curve. Mallet and Pacquant (1951) published frequently used tables expressing hydraulic conductivity as a function of d_{20}. This dependence was expressed by the United States Bureau of Reclamation (USBR) engineers via functional dependence.

Other generally less used formulae were proposed by Fair and Hatch (1933), Harleman et al. (1963), Alyamani and Sen (1993) and Chesnaux et al. (2011). The use of these formulae is restricted by their applicability limits.

The aim of the authors was to compare and assess the applicability of selected formulae. Vuković and Soro (1992) and Kasenow (2002) summarized and analysed the most important formulae with the conclusion that even when applying suitable empirical relations to the same soil sample, different resulting hydraulic conductivity values may be obtained. Further research conducted by Odong (2007) was focused on the evaluation and comparison of empirical relations with measured values. Cabalar and Akbulut (2016) measured the hydraulic conductivities of sands of different grain size and shape and compared them with some empirical formulae. Naeej et al. (2017) developed a M5 model tree used to predict hydraulic conductivity based on grain size distribution. An analysis of unconsolidated aquifer materials was performed by Hussain and Nabi (2016). Their aim was to compare seven empirical formulae with experimental data. Rosas et al. (2014) determined hydraulic conductivity from grain size distribution for 400 samples of sediments.

Some of the empirical formulae developed by different authors vaguely define applicability limits via the simple description of material type without any grain size distribution curves or quantification. This often leads to improper use of these equations. In many cases the input parameters (namely the effective grain size) in the empirical formulae need to be expressed in units other than those defined by the SI (e.g. mm, cm), such as hydraulic conductivity in cm/day, m/day, etc.

The objective of this paper is to summarize the most commonly used empirical formulae, convert them strictly into SI units and evaluate their applicability and reliability for glass beads of three different diameters. The assessment of porosity functions is also included in this paper. This analytical approach enables the influence of soil non-uniformity and grain shape to be excluded from the analysis. Moreover, the grain size is relatively well-defined which is suggested to provide lower uncertainty in resulting hydraulic conductivities when compared with more complex soils.

First, a dimensional analysis was performed, and the dependence between porosity and pore size was established, after which the relation between hydraulic conductivity and porosity was analysed. Second, the determined empirical relations were summarized and converted into SI units. Via laboratory experiments the hydraulic conductivities of glass beads of three different diameters were determined for variable porosity. Finally, the empirical formulae were verified using the results of experimental research.

DIMENSIONAL ANALYSIS

Traditionally, the system of pores was described as the system of parallel tubes oriented in the flow direction, a conception sometimes referred to as the "Hydraulic radius model" (Bear, 1972).

The head loss Δh is defined by the Darcy-Weissbach equation (Vuković and Soro, 1992):

$$\Delta h = \lambda \frac{L}{D} \frac{v^2}{2g}, \tag{1}$$

where L is the tube length, D is the diameter of the tube, v is the cross sectional velocity in the tube, g is the gravitational acceleration, and λ is the coefficient of friction loss, which in the case of laminar flow can be calculated as follows:

$$\lambda = \frac{64}{\mathrm{Re}}, \tag{2}$$

with the Reynolds number Re:

$$\mathrm{Re} = \frac{vD}{\upsilon}, \tag{3}$$

where υ is the kinematic viscosity.

The hydraulic gradient i along the tube:

$$i = \frac{\Delta h}{L}. \tag{4}$$

After substituting Eqs. (2 to 4) into the Darcy-Weisbach equation (1) and some manipulation, one obtains:

$$i = \frac{32 \upsilon v}{D^2 g}. \tag{5}$$

The average velocity in pores may be expressed using the Darcy law:

$$v = \frac{ki}{n_a}, \tag{6}$$

where n_a is the areal porosity and k is the hydraulic conductivity. Assuming areal porosity n_a is equal to volumetric porosity n (Bear, 1972), and joining Eq. (5) and (6), the hydraulic conductivity may be expressed as:

$$k = \frac{1}{32} \frac{g}{\upsilon} D^2 n. \tag{7}$$

The tube diameter D has to be substituted by the representative minimum pore diameter $d_0 = D$ (Vuković and Soro, 1992):

$$d_0 = \alpha f(n) d_e, \tag{8}$$

where α is a dimensionless coefficient that depends on the characteristics of the porous medium (structure, grain shape, uniformity, petrographic composition, tortuosity, etc.), $f(n)$ is the porosity function and d_e is the effective grain diameter of the porous medium. Eq. (7) then transforms into:

$$k = \frac{g}{32\upsilon} \alpha^2 n f^2(n) d_e^2. \tag{9}$$

By introducing $\chi(n) = nf^2(n)$ for the porosity function, Eq. (9) holds:

$$k = \frac{g}{\upsilon} \beta \chi(n) d_e^2, \tag{10}$$

where β characterizes the properties of the porous medium and includes the constant from Eq. (8). For materials with relatively uniform grain size, such as beads, two theoretical limits to porosity may be identified (Fig. 1) according to the configuration of the grains (Indraratna and Vafai, 1997). The minimum packing corresponds to the void ratio $e_{max} = 0.908$ and maximal achievable porosity $n_{max} = 0.476$ corresponds to the ratios:

$$\frac{d_0}{d_{grain}} = 0.414, \qquad \frac{d_{\max}}{d_{grain}} = 0.732, \tag{11}$$

where d_{grain} is the diameter of the uniform grain and d_{max} is the maximal pore diameter.

The maximal packing gives $e_{min} = 0.351$, $n_{min} = 0.260$ and the ratios:

$$\frac{d_0}{d_{grain}} = 0.155, \qquad \frac{d_{\max}}{d_{grain}} = 0.224. \tag{12}$$

Other grain configurations are random and the resulting porosity ranges from 0.260 to 0.476.

For non-uniform materials, Pavchich (VNIIG, 1991) proposed the following relation:

$$d_0 = 0.455 \sqrt[6]{C_U} \frac{n}{1-n} d_{17} \tag{13}$$

where C_U is the coefficient of uniformity, n is porosity and d_{17} is the grain diameter for 17% finer by weight. In the case of a spherical grain material (like glass beads) with $C_U \approx 1$, Eq. (13) can be written as follows:

$$\frac{d_0}{d_{17}} = 0.455 \frac{n}{1-n}. \tag{14}$$

EMPIRICAL FORMULAE

Empirical formulae for the hydraulic conductivity estimate k stem from Eq. (10), while the porosity function is frequently determined from Eq. (13). The following list was assembled via the comparison and critical analysis of the available literature sources. All formulae have been rewritten into dimensional

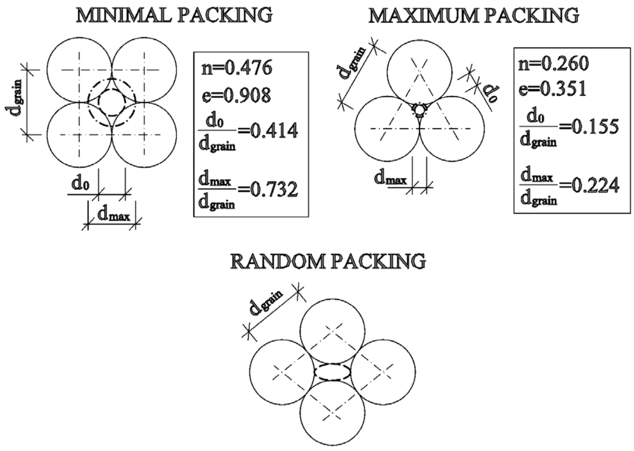

Fig. 1. Schematization of the packing of spherical grains, and possible pore size.

form (10), while the empirical coefficients have been recalculated in order to ensure SI units are used, i.e. grain diameters are expressed in [m] and hydraulic conductivity in [m/s]. The influence of temperature is included in the kinematic viscosity [m^2/s].

Hazen (1892)

$$k = \frac{g}{\upsilon} C_{H,2} [1 + 10(n - 0.26)] d_{10}{}^2 , \tag{15}$$

where d_{10} is the grain diameter for 10% finer by weight and coefficient $C_{H,2} = 6\text{x}10^{-4}$. Eq. (15) may be used to estimate the hydraulic conductivity of sand with d_e from 0.1 to 3 mm with the coefficient of uniformity $C_U < 5$.

Slichter's (1899) formula can be used to estimate the hydraulic conductivity of soil with d_e from 0.01 to 5.0 mm:

$$k = \frac{g}{\upsilon} 0.01 n^{3.287} d_{10}^2 . \tag{16}$$

Terzaghi (1925)

$$k = \frac{g}{\upsilon} C_T \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2 , \tag{17}$$

where C_T depends on the grain shape ($C_T = 10.7\text{x}10^{-3}$ for smooth grains and $C_T = 6.1\text{x}10^{-3}$ for coarse grains). Eq. (17) may be used for large-grained sands.

Beyer (1964)

$$k = \frac{g}{\upsilon} C_B d_{10}{}^2 , \tag{18}$$

where C_B is:

$$C_B = 0.0006 \log \frac{500}{C_U} . \tag{19}$$

This formula can be used for soils with $0.06 \le d_e \le 0.6$ mm, and with C_U ranging from 1 to 20.

Sauerbrey (1932)

$$k = \frac{g}{\upsilon} C_Z \frac{n^3}{(1-n)^2} d_{17}{}^2 , \tag{20}$$

where $C_Z = 3.75\text{x}10^{-3}$. Eq. (20) can be used for soils with d_e up to 5.0 mm.

Krüger (1918), Densch et al. (1930), Kasenow (2002)

The Krüger (1918) formula is mentioned in several publications (Densch et al., 1930; Kasenow, 2002; Vuković and Soro, 1992) in a different form. Vuković and Soro (1992) and also Kasenow (2002) mention the following dimensional form:

$$k = \frac{g}{\upsilon} C_K \frac{n}{(1-n)^2} d_e{}^2 , \tag{21}$$

where $C_K = 4.35\text{x}10^{-3}$, n is porosity and d_e is effective grain defined as follows:

$$\frac{1}{d_e} = \sum_{i=1}^{N} \frac{2\Delta g_i}{d_i^g + d_i^d} , \tag{22}$$

where Δg_i is the fraction of mass that passes between sieves i and i+1 where i is the smaller sieve, and d_i^g and d_i^d are the maximum and minimum grain diameter corresponding to the i-th fraction. Eqs. (21) and (22) can be used for sands of medium grain size with $C_U > 5$, N is the number of fractions.

However, Kasenow (2002) also mentions the form of Eq. (21) with a geometrically justified porosity function corresponding to Eqs. (20), (23), (27) and others. This form exhibits much better agreement with measured values than Eq. (21); see the Discussion section.

Kozeny (1927, 1953)

$$k = \frac{g}{\upsilon} C_{KO} \frac{n^3}{(1-n)^2} d_e{}^2 , \tag{23}$$

where $C_{KO} = 8.3\text{x}10^{-3}$ and d_e is effective grain size determined as follows:

$$\frac{1}{d_e} = \frac{3\Delta g_1}{2d_1} + \sum_{i=2}^{N} \Delta g_i \frac{d_i^g + d_i^d}{2 d_i^g d_i^d} , \tag{24}$$

with the same notation as in Eq. (22). This formula can be used for coarse-grained sands.

Zunker (1932)

$$k = \frac{g}{\upsilon} C_{ZU} \left(\frac{n}{1-n} \right)^2 d_e{}^2 , \tag{25}$$

where C_{ZU} is an empirical coefficient that depends on the porous medium (Table 1), d_e is given by the formula:

$$\frac{1}{d_e} = \sum_{i=1}^{N} \Delta g_i \frac{d_i^g - d_i^d}{d_i^g d_i^d \ln \frac{d_i^g}{d_i^d}} , \tag{26}$$

Table 1. Empirical coefficient for the Zunker formula (Kasenow, 2002).

Characteristics of the porous medium	C_{ZU} [–]
Uniform sand with smooth, rounded grains	2.4x10^{-3}
Uniform composition with coarse grains	1.4x10^{-3}
Nonuniform composition	1.2x10^{-3}
Nonuniform composition, clayey, with grains of irregular shape	0.7x10^{-3}

with the notation from Eq. (22). Eq. (26) can be applied for fine and medium-grained sands.

Zamarin (1928)

$$k = \frac{g}{\upsilon} C_{ZA} C_n \frac{n^3}{(1-n)^2} d_e^{\ 2} , \qquad (27)$$

where C_{ZA} = 8.64x10^{-3} is the empirical coefficient and C_n is a factor that depends on the porosity:

$$C_n = \left(1.275 - 1.5n\right)^2 . \qquad (28)$$

Effective grain size d_e is given for materials containing grains finer than 0.0025 mm as follows:

$$\frac{1}{d_e} = \frac{3\Delta g_1}{2d_1} + \sum_{i=2}^{N} \Delta g_i \frac{\ln \dfrac{d_i^g}{d_i^d}}{d_i^g - d_i^d} , \qquad (29)$$

where d_1 is the largest diameter of the finest fraction and Δg_1 is the weight of the finest fraction. For materials that do not contain fractions finer than 0.0025 mm, the effective grain size can be obtained as follows:

$$\frac{1}{d_e} = \sum_{i=1}^{N} \Delta g_i \frac{\ln \dfrac{d_i^g}{d_i^d}}{d_i^g - d_i^d} . \qquad (30)$$

Eq. (27) can be used for fine and medium-grained sands.

USBR (Mallet and Pacquant, 1951)

$$k = \frac{g}{\upsilon} C_{US} d_{20}^{\ 2} , \qquad (31)$$

where d_{20} is the diameter of the 20 percentile grain size of the material and C_{US} is:

$$C_{US} = 0.00048(1000 d_{20})^{0.3} . \qquad (32)$$

The USBR formula, also tabulated by Mallet and Pacquant (1951), is recommended for medium-grained sands with $C_U < 5$.

Pavchich (VNIIG, 1991)

$$k = \frac{0.04}{\upsilon} \varphi_1 \sqrt[3]{C_U} \frac{n^3}{(1-n)^2} d_{17}^{\ 2} , \qquad (33)$$

where φ_1 is the coefficient depending on the grain size ($\varphi_1 = 1$ for gravel sands, φ_1 = 0.35–0.40 for gravel), Eq. (33) can be used for grain sizes ranging from 0.06 mm to 1.5 mm.

Seelheim (1880)

$$k = 3570 d_{50}^{\ 2} , \qquad (34)$$

where d_{50} is the diameter of the 50 percentile grain size. The formula was tested on sands, clay and elutriated chalk.

Kozeny-Carman (Carrier, 2003)

The following equation, which depends on the specific surface area of grains, was derived by Kozeny and Carman:

$$k = \frac{g}{\nu} C_{KC} \frac{1}{S_0^{\ 2}} \frac{n^3}{(1-n)^2} , \qquad (35)$$

where C_{KC} = 480 ± 30 is the empirical coefficient, and S_0 is the specific surface of particles (1/m). For uniform spherical grains Eq. (35) can be written as follows:

$$k = \frac{g}{\nu} C_{KC} \frac{6}{d_e} d_e^{\ 2} \frac{n^3}{(1-n)^2} , \qquad (36)$$

where d_e is the uniform grain diameter (d_{grain}). The formula is not appropriate for clayey soils, but it is applicable for silts, sands and gravel sands.

Harleman et al. (1963)

$$k = 6.54\text{x}10^{-4} \frac{g}{\upsilon} d_{10}^{\ 2} . \qquad (37)$$

Koenders and Williams (1992)

This formula was derived from the Kozeny-Carman equation:

$$k = \frac{1}{\upsilon} \chi n \left(\frac{n}{1-n}\right)^2 d_{50}^{\ 2} , \qquad (38)$$

where χ is the proportionality coefficient (χ = 0.0035 ± 0.0005) and d_{50} is the median grain diameter. It is then applicable for silts, sands and gravelly sands.

The following authors used formulae that are rather different in form compared to Eq. (10).

Alyamani and Sen (1993)

$$k = 15046\left[I_o + 0.025\left(d_{50} - d_{10}\right)\right]^2 \qquad (39)$$

where I_o is the intercept point [m] of the line formed by points d_{50} and d_{10} with the grain size axis. The formula can be used for well-distributed samples only.

Chapuis et al. (2005)

$$k = 1219.9 \frac{n^{2.3475}}{(1-n)^{1.565}} d_{10}^{\ 1.565} \qquad (40)$$

The formula is applicable for soils with d_{10} ranging from 0.03 to 3 mm.

Fair and Hatch (1933)

$$k = \frac{g}{\upsilon} \frac{n^3}{(1-n)^2} \frac{1}{m\left(\frac{\theta}{100}\sum_{i=1}^{N}\frac{P_i}{d_{mi}}\right)}, \qquad (41)$$

where $m = 5$ is the empirically obtained packing factor, θ is the shape factor ranging from 6 to 7.7 (spherical to angular grains), d_{mi} is the geometric mean of the grain fraction, and P_i is the percentage of sand between adjacent sieves determined by the following equation:

$$P_i = 100 w f_i , \qquad (42)$$

where w_{fi} is the weight of the fraction retained on sieve i. For the geometric mean d_{mi} it holds that:

$$d_{mi} = \sqrt{ds_i ds_{i+1}} , \qquad (43)$$

where d_{si} is the size of the sieve openings for sieve i. This formula is applicable for sands.

NAVFAC DM7 (Chesnaux et al., 2011)

$$k = 0.2272(1.772189\text{x}10^{11})^{\frac{n}{1-n}}[d_{10}{}^{3.31917}]^{\frac{n}{1-n}} . \qquad (44)$$

The formula was derived for sands with n ranging from 0.23 to 0.41, C_U ranging from 2 to 12, $d_{10}/d_5 > 1.4$ and d_{10} ranging from 0.1 mm to 2 m

EXPERIMENTAL RESEARCH

Experiments were carried out in order to verify the porosity function and empirical formulae for uniform material laboratory experiments on glass beads of three different diameters. This experimental research aimed to obtain a sufficient number (at least 50) of hydraulic conductivity measurements for individual glass beads of different diameters with various porosities. The numbers of performed experiments are mentioned in Table 4.

Equipment

The laboratory experiments were performed using a permeameter (plastic cylinder) with upward vertical seepage flow. A permeameter consists of a cylinder containing the sample mounted on a frame. The lower part is connected to a water supply and the upper part is connected to an outlet pipe. Piezometers are located below and above the sample. The seepage flow is generated by a vertically movable water tank that can be adjusted to provide different hydraulic gradients. The movable tank is equipped with a pump that draws water from a storage tank. Water flowing through the permeameter outlet is collected and conveyed back to the storage tank. A schematic diagram of the experimental apparatus is shown in Fig. 2.

Preliminary measurements

A detailed investigation using an electron microscope showed that the sizes of the glass beads did not exactly match the interval declared by the manufacturer. Therefore, bead diameter measurements were conducted using a digital Vernier calliper for each declared (commercial) grain size in order to set up the grain size distribution curves (Fig. 3). The curves provided good fits with the electron microscope and the grain size characteristics were set (Table 3).

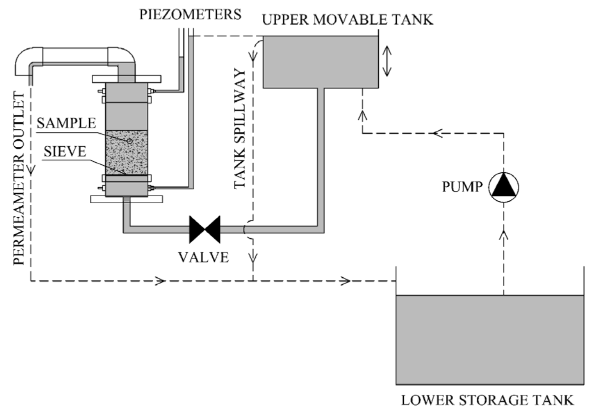

Fig. 2. Schematic diagram of the experimental apparatus.

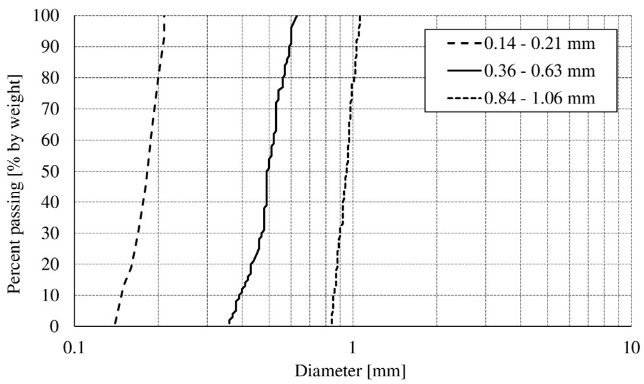

Fig. 3. Grain size distribution curves of the tested glass beads.

Experimental procedure and results

In order to obtain glass bead samples with randomly different porosity, the samples were added to the permeameter via methods involving free fall and compaction by vibration for variable durations. The porosity of each sample was determined by weighing it and then measuring its volume in a Darcy cylinder.

The various piezometric heads (and thus hydraulic gradients) were achieved by gradually raising the upper movable tank (Fig. 2). The seepage discharge was measured each time the tank was raised. In total, 177 laboratory experiments were performed on the glass beads (Table 4).

The dependence of the hydraulic conductivity k on the porosity n was evaluated separately for each bead diameter (Fig. 4). The porosity and hydraulic conductivity ranges are summarized in Table 4. The expected measurement accuracy for individual variables is summarised in Table 5.

COMPARISON OF EXPERIMENTAL RESULTS WITH EMPIRICAL FORMULAE

Comparison of measured porosity with empirical results

First, the porosity functions $\chi(n)$ used in the above-described empirical formulae (Table 2) were analysed. In Fig. 5 the correlation between porosity functions $\chi(n)$ and the ratio between measured hydraulic conductivity and effective grain size (k/d_e^2) is plotted. Only results with a fairly good fit are presented in Fig. 5, those being obtained by Terzaghi (1925), Sauerbrey (1932), Pavchich (VNIIG, 1991) and Chapuis et al. (2005).

Table 2. Summary of empirical formulae.

Number of formula	Author	β	$\chi(n)$	d_e	Use
1	Hazen (1892)	$6x10^{-4}$	$[1+10(n-0.26)]$	d_{10}	sands, 0.1 mm ≤ d_{10} ≤ 3 mm $C_U < 5$
2	Slichter (1899)	0.01	$n^{3.287}$	d_{10}	0.01 mm ≤ d_{10} ≤ 5 mm
3	Terzaghi (1925)	$10.7x10^{-3}$ – smooth grains $6.1x10^{-3}$ – coarse grains	$\left(\frac{n-0.13}{\sqrt[3]{1-n}}\right)^2$	d_{10}	large-grained sands
4	Beyer (1964)	$0.0006\log\frac{500}{C_U}$		d_{10}	0.06 mm ≤ d_{10} ≤ 0.6 mm 1 ≤ C_U ≤ 20
5	Sauebrej (1932)	$3.75x10^{-3}$	$\frac{n^3}{(1-n)^2}$	d_{17}	d_{17} ≤ 5 mm
6	Krüger (Kasenow, 2002)	$4.35x10^{-3}$	$\frac{n}{(1-n)^2}$	Eq. (22)	sands of medium grain size $C_U > 5$
7	Kozeny (1953)	$8.3x10^{-3}$	$\frac{n^3}{(1-n)^2}$	Eq. (24)	coarse-grained sands
8	Zunker (1932)	Table 1	$\left(\frac{n}{1-n}\right)^2$	Eq. (26)	fine and medium-grained sands
9	Zamarin (1928)	$8.64x10^{-3}$	$\frac{n^3}{(1-n)^2}(1.275-1.5n)^2$	Eq. (29) Eq. (30)	fine and medium-grained sands
10	USBR (Mallet and Pacquant, 1951)	$0.00048(1000d_{20})^{0.3}$		d_{20}	medium-grained sands $C_U < 5$; T = 15 °C
11	Pavchich (VNIIG, 1991)	$0.04\,\varphi_1\sqrt[3]{C_U}$	$\frac{n^3}{(1-n)^2}$	d_{17}	0.06 mm ≤ d_{17} ≤ 1.5 mm
12	Seelheim (1880)	3570		d_{50}	sands, clay and elutriated chalk
13	Kozeny-Carman (Carrier, 2003)	480 ± 30	$\frac{n^3}{(1-n)^2}$	d_{grain}	uniform spherical grains
14	Harleman et al. (1963)	$6.54x10^{-4}$		d_{10}	
15	Koenders and Williams (1992)	0.0035 ± 0.0005	$n\left(\frac{n}{1-n}\right)^2$	d_{50}	silts, sands and gravelly sands
16	Alyamani and Sen (1993)	15046		I_O, d_{50}, d_{10}	well-distributed sample
17	Chapuis et al. (2005)	1219.9	$\frac{n^{2.3475}}{(1-n)^{1.565}}$	d_{10}	0.03 mm ≤ d_{10} ≤ 3 mm
18	Fair and Hatch (1933)	1	$\frac{n^3}{(1-n)^2}$	$m\left(\frac{\theta}{100}\sum_{i=1}^{N}\frac{P_i}{d_{mi}}\right)$	sands $m = 5$ $6 \le \theta \le 7.7$
19	NAVFAC DM7 (Chesnaux et al., 2011)	0.2272	$(1.772189\cdot10^{11})^{\frac{n}{1-n}}$	d_{10}	0.23 ≤ n ≤ 0.41 2 ≤ C_U ≤ 12 $d_{10}/d_5 > 1.4$ 0.1 mm ≤ d_{10} ≤ 2 mm

Table 3. Grain size characteristics.

Diameter d_{min}–d_{max}	d_{10}	d_{17}	d_{20}	d_{60}	$d_{e,Krüger, Zamarin, Zunker}$	$d_{e,Kozeny}$	C_U
	mm	mm	mm	mm	mm	mm	–
0.14–0.21	0.16	0.17	0.17	0.19	0.18	0.18	1.19
0.36–0.63	0.39	0.43	0.43	0.52	0.49	0.48	1.33
0.84–1.06	0.85	0.87	0.88	0.97	0.94	0.92	1.14

For each data set a linear relation between the hydraulic conductivity and the porosity function was assumed depending on Eq. (10). Determination coefficients were evaluated in order to assess the best fit.

Table 4. Summary of performed experiments and the minimum and maximum values of porosity and hydraulic conductivity.

Grain diameter	Number of experiments	Porosity		Hydraulic conductivity	
		Minimum	Maximum	Minimum	Maximum
[mm]	[–]	[–]	[–]	[m/s]	[m/s]
0.14–0.21	52	0.377	0.446	0.00416	0.00922
0.36–0.63	53	0.368	0.437	0.00110	0.00220
0.84–1.06	72	0.353	0.416	0.00013	0.00036

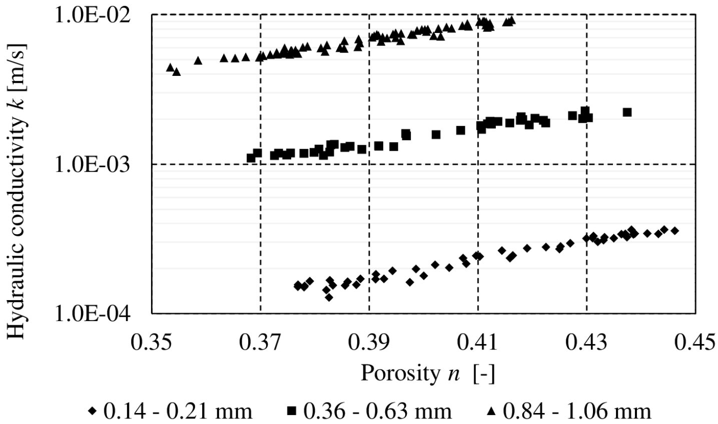

Fig. 4. Dependence of hydraulic conductivity on porosity.

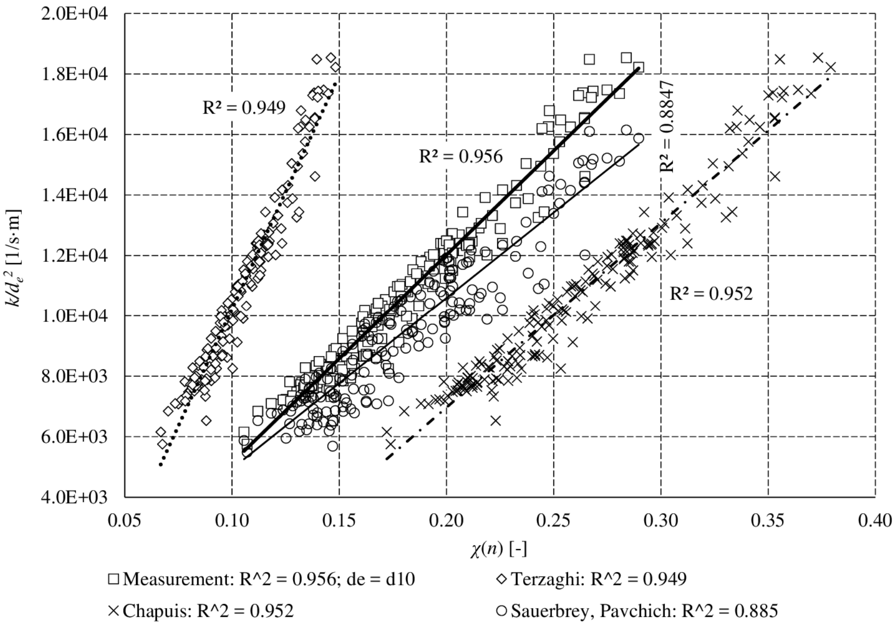

Fig. 5. Correlation between $\chi(n)$ and (k/d_e^2).

Table 5. Summary of measurement accuracy.

Accuracy of directly measured variables	
Variable	Accuracy
Glass bead diameter	0.01 mm
Weight	0.0001 kg
Permeameter diameter	0.25 mm
Length of sample	025 mm
Piezometric heights	0.25 mm
Temperature	0.25 °C
Time	0.05 s

The dependence between the most frequently used porosity function from Eq. (10) and the measured hydraulic conductivities and for $d_e = d_{10}$ was added to the graph. This provided the best fit with $R^2 \approx 0.956$. As regards the empirical formulae, the closest values to the measured data were obtained from the Chapuis (2005) and empirical Terzaghi (1925) porosity functions. A relatively good fit was also provided by the geometrically based relation (14) derived by Pavchich (VNIIG, 1991) and Sauerbrey (1932) with $R^2 \approx 0.885$.

Comparison of the measured hydraulic conductivity with the empirical formulae

Figs. 6 to 10 show comparisons of the measured conductivity values with the calculated values gained from the empirical formulas. The ratio k/d_e^2 used in the plots enables the joint comparison of results for all tested bead diameters. This comparison was not performed for formulae that do not meet applicability limits, such as Alyamani and Sen (1993), Fair and Hatch (1933) and Chesnaux et al. (2011).

To quantify the rate of agreement numerically, the sums of the standardised squares of the residuals $\Sigma\varepsilon$ were expressed in Table 6:

$$\Sigma\varepsilon = \sum_{i=1}^{N} \frac{\left(k_{i-calculated} - k_{i-measured}\right)^2}{k_{i-measured}^{\;2}} \qquad (45)$$

where $k_{i\text{-}calculated}$ and $k_{i\text{-}measured}$ are hydraulic conductivities obtained from empirical formulae and from measurements, respectively, and $N = 177$ is the number of measurements.

DISCUSSION

The analysis of the porosity functions shows that the best fit is provided by a commonly used porosity function based on Eq. (13), or on (14) with the effective grain d_{10}. A quite good fit is also achieved by the dependence proposed by Terzaghi (1925). This is especially true for the measured hydraulic conductivities when $d_e = d_{17}$.

In Figs. 6 to 10 it can be seen that there are considerable differences between the empirical formulae listed above. This is because the individual formulae were derived for specific conditions via different methods. Some are geometrically and physically justified, while others are pure regression dependencies which are not supported by dimensional analysis.

A visual check of Figs. 6 to 10 indicates that for uniform glass beads the best fit with the measured hydraulic conductivities is provided by the formulae published by Terzaghi (Fig. 6), Kozeny-Carman, Zunker (Fig. 8) and Chapuis et al. (Fig. 9). A still reasonable degree of agreement is given by the formulae by Hazen, Zamarin, Sauerbrey and Pavchich.

Table 6. Sums of standardized squared deviations for empirical formulae (ascending order).

Author	Formula number in Table 2	Eq. number	Sum of standardized squared deviations $\Sigma\varepsilon$
Kozeny-Carman (Carrier, 2003)	13	(35)	1.25
Zunker (1932)	8	(25)	1.38
Terzaghi (1925)	3	(17)	1.80
Zamarin (1928)	9	(27)	6.07
Pavchich (VNIIG, 1991)	11	(33)	7.45
Sauerbrey (1932)	5	(20)	7.63
USBR (Mallet and Pacquant, 1951)	10	(31)	16.36
Chapuis et al. (2005)	17	(40)	18.80
Kozeny (1953)	7	(23)	26.47
Harleman et al. (1963)	14	(37)	28.25
Beyer (1964)	4	(18)	30.91
Hazen (1892)	1	(15)	31.60
Seelheim (1880)	12	(34)	46.34
Slichter (1899)	2	(16)	51.29
Koenders and Williams (1992)	15	(38)	145.47
Krüger (Kasenow, 2002)	6	(21)	5284.86

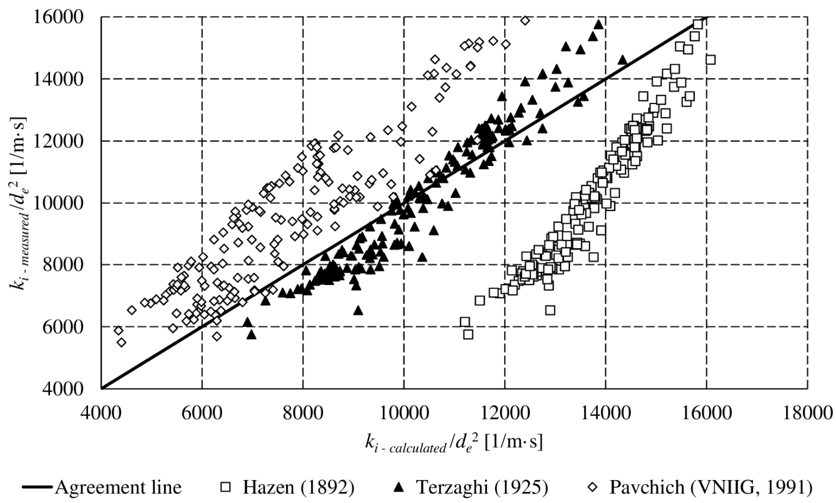

Fig. 6. Comparison of calculated and measured hydraulic conductivity – part 1.

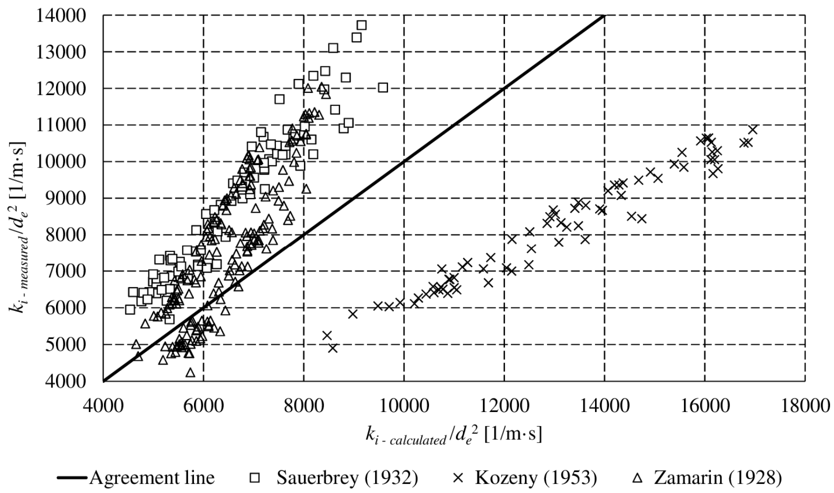

Fig. 7. Comparison of calculated and measured hydraulic conductivity – part 2.

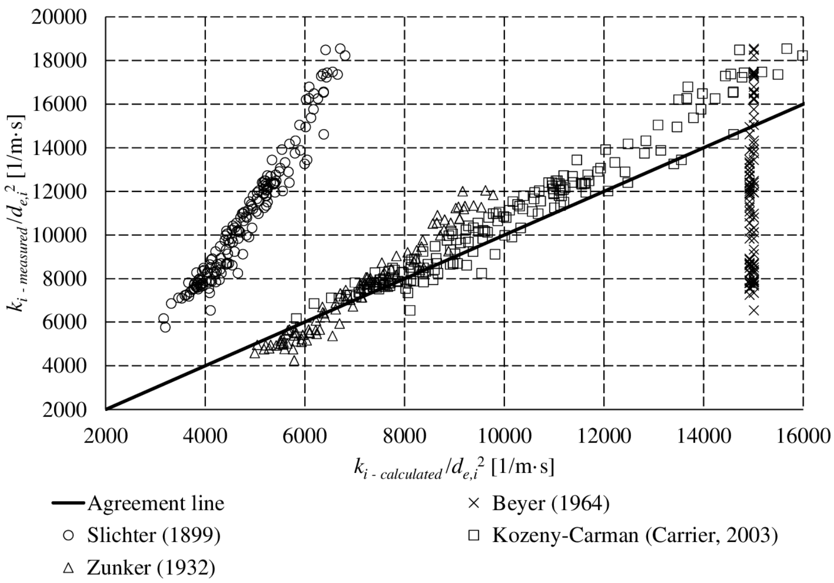

Fig. 8. Comparison of calculated and measured hydraulic conductivity – part 3.

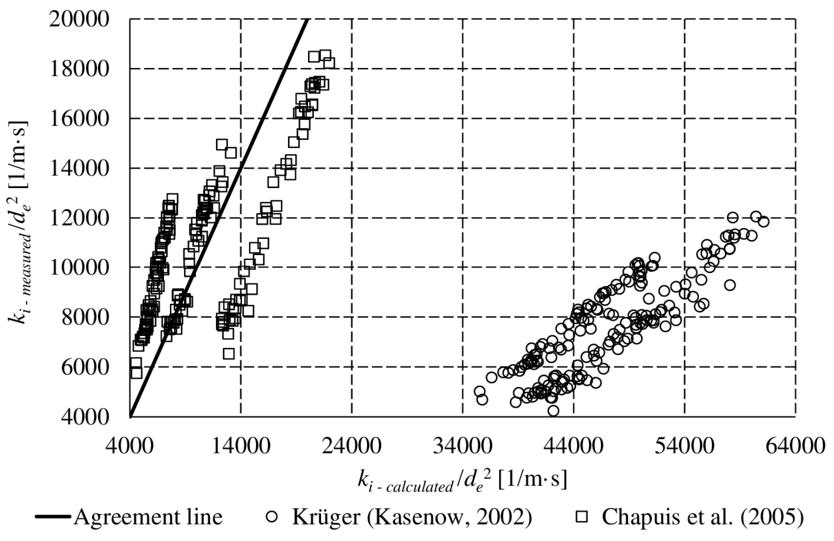

Fig. 9. Comparison of calculated and measured hydraulic conductivity – part 4.

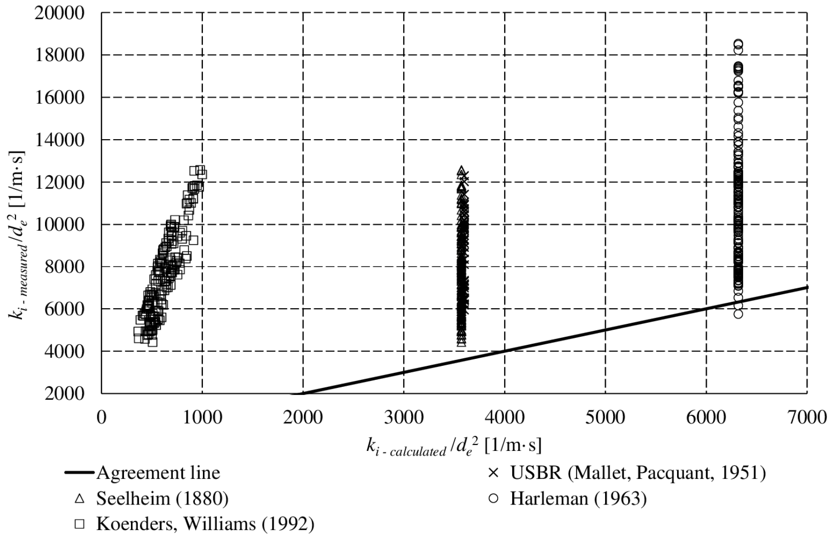

Fig. 10. Comparison of calculated and measured hydraulic conductivity – part 5.

Table 7. – Ratios of calculated and measured values.

Formula number in Table 2	Equation number	Agreement ratio	
		Min. (A_{min})	Max. (A_{max})
1	(15)	0.87	1.98
2	(16)	0.35	0.63
3	(17)	0.78	1.39
4	(18)	0.81	2.30
5	(20)	0.60	0.94
6	(21)	4.85	9.93
7	(23)	1.49	1.75
8	(25)	0.76	1.36
9	(27)	0.67	1.35
10	(31)	0.29	0.60
11	(33)	0.68	1.11
12	(34)	0.28	0.81
13	(35)	0.80	1.24
14	(37)	0.34	1.10
15	(38)	0.07	0.11
17	(40)	0.60	1.98

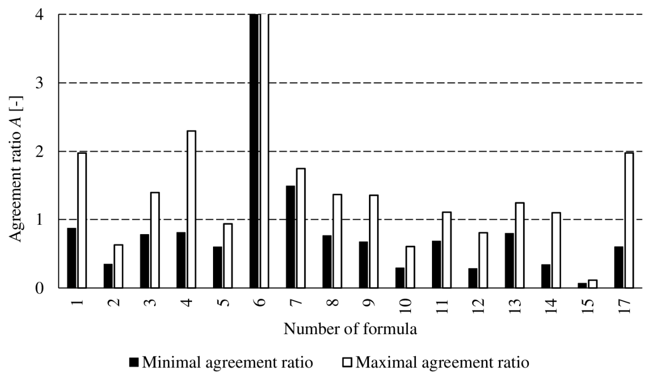

Fig. 11. Minimal and maximal agreement ratios.

In contrast, the worst agreement is provided by the formula derived by Krüger and also formulae which do not take the porosity effect into account, specifically those by Seelheim, USBR, Harleman et al. and Beyer, which show practically no agreement. Interesting results were achieved when the porosity function $n^3/(1–n)^2$ was implemented into the Krüger formula.

The originally poor fit shown in Fig. 9 was significantly improved by this alteration. Other empirical formulae tend to overestimate or underestimate the hydraulic conductivity more significantly in comparison to conducted measurements. To gain an idea about the rate of agreement, the ratios between the calculated and experimentally measured hydraulic conductivity values were computed:

$$A = \frac{k_{emp}}{k_{mea}}, \; A_{\min} = \min(A), \; A_{\max} = \max(A) \tag{46}$$

where A is the agreement ratio, k_{emp} is the hydraulic conductivity obtained from an empirical formula, and k_{mea} is the hydraulic conductivity obtained from measurements. The ratio A was enumerated for all measurements and formulae and its minimum and maximum values were identified for each formula (Table 7, Fig. 11). The best fit is represented by $A = 1.0$.

From the graphs in Figs. 6, 7, 9 two or three slightly different clusters in the term k/d_e^2 may be identified for individual formulae, namely e.g. Hazen (Fig. 6), Zamarin (Fig. 7), Krüger and Chapuis (Fig. 9). This fact may be attributed to the increasing effect of surface tension with decreasing grain size, a factor which is not included in the porosity function. Here, the "effective porosity" (Bear, 1972) should be used instead of "dry" porosity in empirical equations.

CONCLUSION

In the study, empirical formulae for determining hydraulic conductivity were presented and transformed into dimensional form using SI units (m, m/s, etc.). The advisability of using porosity functions in empirical formulae was examined along with their applicability for uniform spherical grains using the results of 177 laboratory tests on glass beads of three different diameters.

The best fit was provided by the geometrically derived porosity function $n^3/(1–n)^2$ based on Eq. (13) when $d_e = d_{10}$ was used. For uniform glass beads the best fit was exhibited by formulae published by Terzaghi (Eq. (17)), Kozeny-Carman (Eq. (36)), Zunker (Eqs. (25, 26)) and Chapuis et al. (Eq. (40)). If applied porosity function $n^3/(1–n)^2$ into the Krüger formula Eq. (21) very good fit with measured values is also achieved.

The comparison shows the increasing effect of surface tension and capillary forces with decreasing grain size. Further research should be focused on this effect and the use of "effective porosity" instead of standard porosity in empirical formulae.

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