J. Hydrol. Hydromech., 61, 2013, 3, 241–249
DOI: 10.2478/johh-2013-0031

Bedload equation analysis using bed load-material grain size

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Abstract: Twelve predictive bedload sediment transport equations are rated against 14 sets of gravel-bed river field data collected by handheld bedload sampler in Narmab River, northeastern Iran. To evaluate these formulas two types of grain size namely bedload and bed material were used. The results show that the equations of Engelund and Hansen, Van Rijn and Einstein perform well with bed material grain size, while Shocklitsch, Meyer-Peter and Mueller, and Frijlink yield the best results using the bedload grain size.

Keywords: Sediment transport; Grain size; Gravel-bed river; Evaluation.

INTRODUCTION

One of the most important problems in river engineering is to predict bed load transport rates in torrential floods flowing from mountainous streams. Such flows have been known to set huge boulders in motion, some weighing several tons. These boulders may roll and bounce down alluvial fans causing severe damage to trees, fences, buildings and other structures in their stream paths. Damage by impingement is not limited to the large rocks but may also result from the transport of smaller particles. For instance, to turbines and pumps where sediment-laden flow causes excessive wear on runners, vanes and other appurtenant parts. Such problems encourage engineers to develop their knowledge of sediment transport.

Three modes of transport namely; rolling, sliding and saltation may occur simultaneously in bed load transport. The different modes of transportation are closely related and it is difficult, if not impossible, to separate them completely. Few bed load discharge equations were available when the systematic regulation of rivers began, but since the middle of the 20th century a variety of bed load discharge formulas has been developed. They were primarily based on laboratory investigations with controlled boundary conditions, equilibrium transport, and bed level stability.

Bed load transport relationships can be divided into two categories. First, there are those that assume the hydraulic roughness is known. These relationships can be used to estimate the transport rate in streams where the composition of the bed material and the flow conditions are known. Examples of such sediment transport relations include the Meyer-Peter and Mueller (1948) and Ackers and White (1973) formulas. The second category includes formulas that are a combination of a hydraulic resistance predictor and a transport predictor, for example, the Engelund and Hansen (1967), Karim and Kennedy (1983) and Van Rijn (1984a) formulas.

MATERIAL AND METHODS

Derivation of selected bed load formulas

Among the bed load formulas reported so far, engineers are required to choose a formula that has the most efficient answer to their problems. This is not easy since different formulas predict drastically different results. In this study, however, we did not analyse all bed load discharge formulas. Instead, we tested formulas based on their applicability to gravel bed rivers. Most of the sediment load formulas were developed on a limited database, untested model assumptions, and a general lack of field data. Consequently, the application of many formulas is limited to the special cases of their development; only a few are generally accepted for practical use. All formulas presented in our study predict bed load sediment discharge under uniform steady flow and do not include the wash load. Many formulas have appeared in the literature since Du Boys presented his tractive force relation in 1879. Twelve bed load discharge formulas were applied for the comparison of measured and calculated values, including Meyer-Peter and Mueller (1948), Schoklitsch (1950), Einstein-Brown (Brown 1950; Einstein 1950), Frijlink (1952), Yalin (1963), Engelund and Hansen (1967), Bijker (1971), Ackers and White (1973), Bagnold (1980), Van Rijn (1984a), Van Rijn – Stochastic (1987) and Cheng (2002) (Table 1). In Table 1, representative diameter particle size and type of sediment data to derive formulas are given.

Field data

The bed load discharge data analysed in this study were collected at the Narmab River, a sub-catchment of a large mountainous Catchment (Chelchay catchment) in north eastern of Iran. The Narmab River that originates from the connection of three tributaries is a mild stream and semi-armoured gravel bed river with a basin area of about 197 km^2. Both banks of gaging-station section, at which both stream flow measurements and sediment samples are taken, are somewhat stabilized and is immediately downstream of a water-stage recorder (Fig. 1). The measuring reach had a longitudinal bed slope of 0.03%.

We used BLSH bed load sampler in this study (Envcoglobal, Australia). BLSH is a handheld sampler for wading measurements in natural streams. This sampler is of Helley-Smith sampler type with a long handle to carry it in water. The unit is supplied with wading rods, for lowering sampler in the stream or river. Pilot experiments showed that in combination with the pressure difference effect caused by the sampler, a 0.2 mm mesh gives a hydraulic efficiency of between 0.9 and 1.1 as long as the extent of sampler fill is less than 30%; this requirement was satisfied for Narmab bed load measuring.

Table 1. Description of bed load equations used in this study.

Formula name	Publishing year	Representative diameter	Particle size range (mm)	Type of data and descriptions
Meyer-Peter and Mueller	1948–1954	D_a	0.4–28.65	Uniform sediments with varying specific gravity from 1.25 to 4
Schoklitsch	1950	D_{40}	0.3–5	Well sorted and graded sediment measured in small flumes. Not applicable for hyper-concentrated sand bed rivers
Einstein-Brown	1942–1950	D_{50}	0.785–28.65	Uniform sediment and light weight materials based on flume data
Frijlink	1952	D_a		Simple fit of Meyer-Peter and Mueller and Einstein
Yalin	1963	D_{50}	0.315–28.65	Uniform sediment and light weight materials
Engelund and Hansen	1967	D_{50}	0.19–0.93	Based on large flume data
Bijker	1971	D_{50}		Proposed based on the concept of Einstein formula
Ackers and White	1973	D_{35}	0.4–4.94	Uniform sediment and light weight materials
Bagnold	1980			Based on flume data and US streams
Van Rijn	1984	D_{90}	0.2–2	Based on 130 flume experiments
Van Rijn – Stochastic	1987	D_{50}		
Cheng	2002	D_{50}	0.7–22.5	Derived for computing bed load for the conditions of low to high shear stresses.

According to the water level changes, three to six sub-sections were considered for the measuring of bedload discharge in a reach without an obvious aggradation or degradation. The duration of sampling was decided on the bedload discharge to ensure that only 30% of the sampler was filled (Emmett 1980).

In addition to the bedload sampling, the flow depth, water surface width, river bed slope and flow velocity were measured. An OTT velocity meter was used to measure the time averaged velocity in sampling section. Water discharge was then calculated by means of multiplying area by velocity. A typical flow velocity profile which was obtained in 150–200 cm intervals across the river is shown in Fig. 2b. Lateral distributions of bed load concentration and depth for gauging-station section are shown in Fig. 2a and Fig. 2c, respectively, for three different times r. In Table 2 some hydraulic data such as; flow width, hydraulic radius, specific discharge, and temperature that are used for evaluation of formulas are presented. The width of river changes from less than 7 m during low flow events to ~19 m in maximum measured discharge. The range of changes in measured flow velocity was between 0.28 m s^{-1} and 1.32 m s^{-1}. These values prove that the river flow is highly variable and bed load discharges can change remarkably among wet and dry seasons (see Fig. 2a).

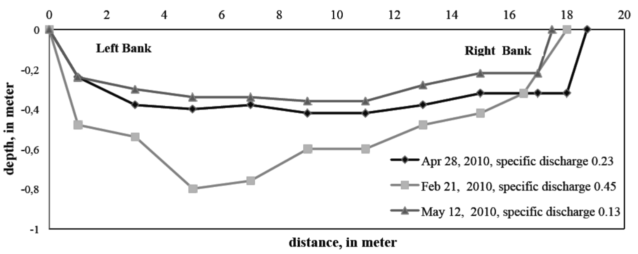

Fig. 1. Channel cross section at the gauging-station section for stream flow measurements and sediment sampling.

Properties of sediment

Transportation of sediment depends not only on the characteristics of the flow involved, but also on the properties of the sediment itself. Those properties of most importance in the sedimentation processes can be divided into properties of the particles and of the sediment as a whole. The most important property of the sediment particle or grain is its size. In most of the river sediment studies average size alone has been used to describe the sediment as a whole. To obtain more accurate results, a more precise description of the sediment is required.

Table 2. Maximum and minimum of hydraulic data used in calculation of the formulas.

Available data series	Used data series		Slope	
17	14		0.0003	
Velocity (m s^{-1})	Hydraulic radius (m)	Specific discharge (m^2 s)	Width (m)	Tempe-rature (ºC)
Maximum				
1.32	0.52	0.49	18.7	20
Minimum				
0.28	0.16	0.016	6.7	6.5

In this study, two types of grain size namely; bedload grain size and bed material grain size are used for evaluating the bedload discharge formulas. Bed material grain sizes are the particle sizes which are found in appreciable quantities in that part of the river bed where is affected by transport. Bedload grain size is defined as size of the transported bed material which is almost in continuous contact with the bed, carried forward by rolling, sliding or hopping. In Fig. 3, cumulative frequency curves of bedload grain size for all 14 series of data and size distribution of bed material are shown. The results show that the corresponding particle size of sediment in transport is finer than those because of selective transport due to bed material.

Grain size distributions of bed load are more uniform than bed material (Fig. 3). By means of size distribution curves, median size, d_{50}, geometric mean size, d_g, geometric standard deviation, δ_g, arithmetic mean size, d_m, particle fall velocity, dimensionless particle diameter, D_* and sorting coefficient, S_o are obtained and shown in Table 3.

On the other hand, these sediment features can be written for all 14 bed load grain size. In Table 4 the maximum, minimum,

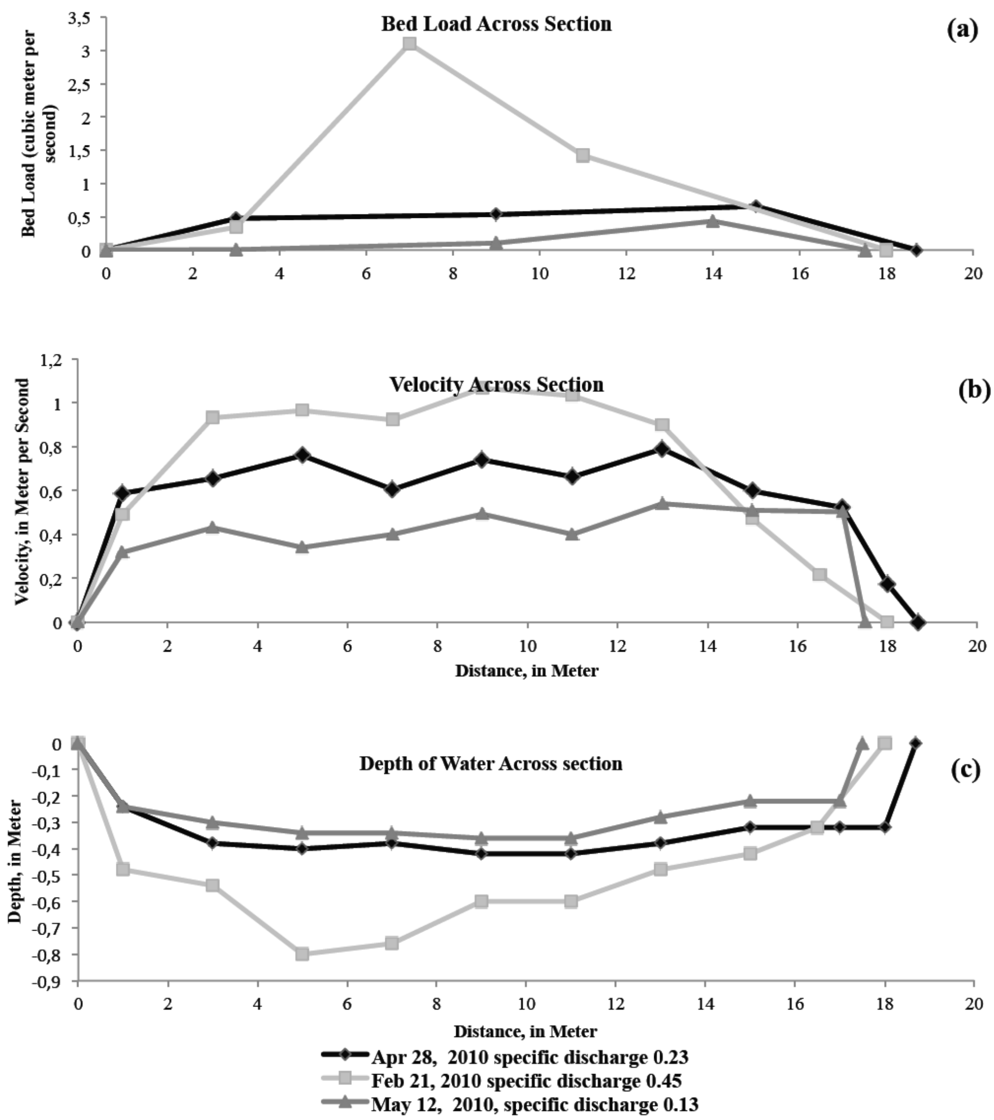

Fig. 2. Lateral distribution of bed load (A), velocity (B), depth (C) gauging-station section, Narmab.

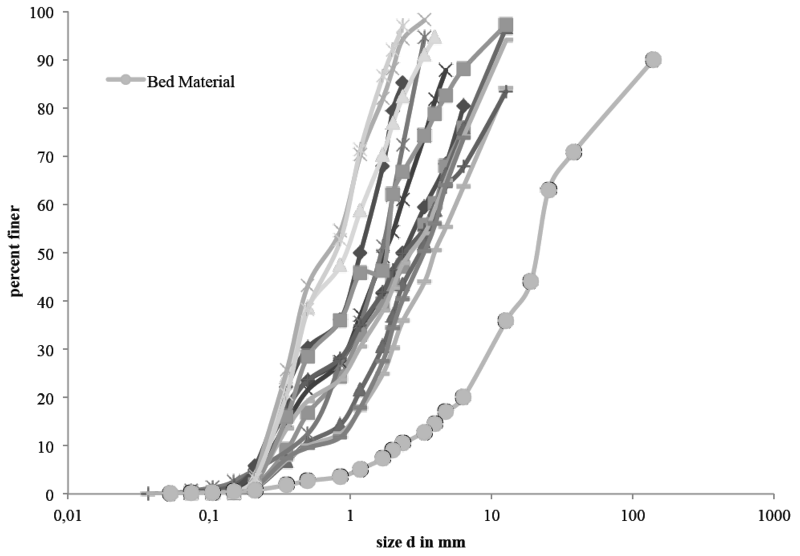

Fig. 3. Grain size distributions of bed load and bed material load. Bed material is shown with circle. Other curves are related to 14 bed load grain size.

Table 3. Sediment properties for bed material.

Median size (d_{50}), (mm)	Geometric mean size (d_g)	Geometric standard deviation (δ_g)	Arithmetic Mean size (d_m), (mm)	Fall velocity (W_S), (m s^{-1})	Particle diameter (D_*)	Sorting coefficient (S_0)
18	16.4	3.81	19.2	0.6	360	2.74

Table 4. Sediment properties for bed load.

	Median size (d_{50}), (mm)	Arithmetic mean size (d_m), (mm)	Fall velocity (W_S), (m s^{-1})	Geometric standard deviation (δ_g), (mm)
Maximum	3.35	3.4	0.28	6.1
Minimum	0.7	0.78	0.09	2.2
Mean	2.11	2.43	0.192	3.5

and mean of median size, mean size, fall velocity and geometric standard deviation are presented.

RESULTS AND DISCUSSION

Evaluation of formulas

Fourteen datasets were used to evaluate formulas. All these data are in the range of base flow to peak flood from October 2009 to May 2010.

A straightforward way to evaluate a formula is to compare the results of the predictions with the bed sediment discharge measured in natural streams. In making the calculations one needs to know hydraulic quantities, e.g., mean velocity, depth, and hydraulic radius.

Van Rijn (1984b) used 486 sets of river data to verify the methods of Engelund and Hansen (1967), Ackers and White (1973), Yang (1973), and Van Rijn (1984a). Bed material sizes were in the range of 0.1 to 0.4 mm. The results have been expressed in terms of a discrepancy ratio. The method of Van Rijn (1984) yields the best results for field data with 76% of the predicted transport rates within a factor 2 ($0.5 \leq r \leq 2$) of the measured values. Engelund and Hansen (1967), and Ackers and White (1973) yield good results with 63% of discrepancy ratio in factor 2. The method of Yang yields excellent results for small-scale river data, but very poor results for large scale river data.

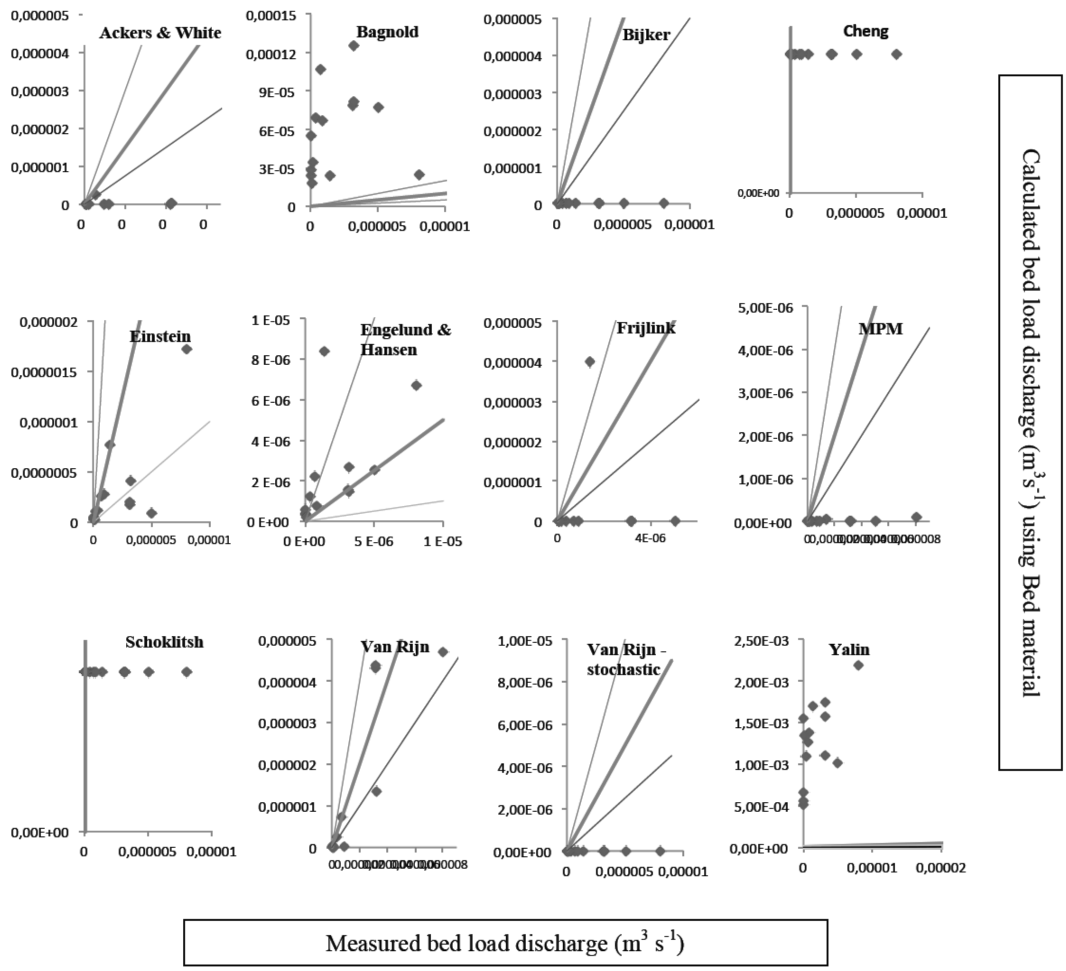

Fig. 4. Comparisons between calculated and measured bed load discharge (bed material grain size).

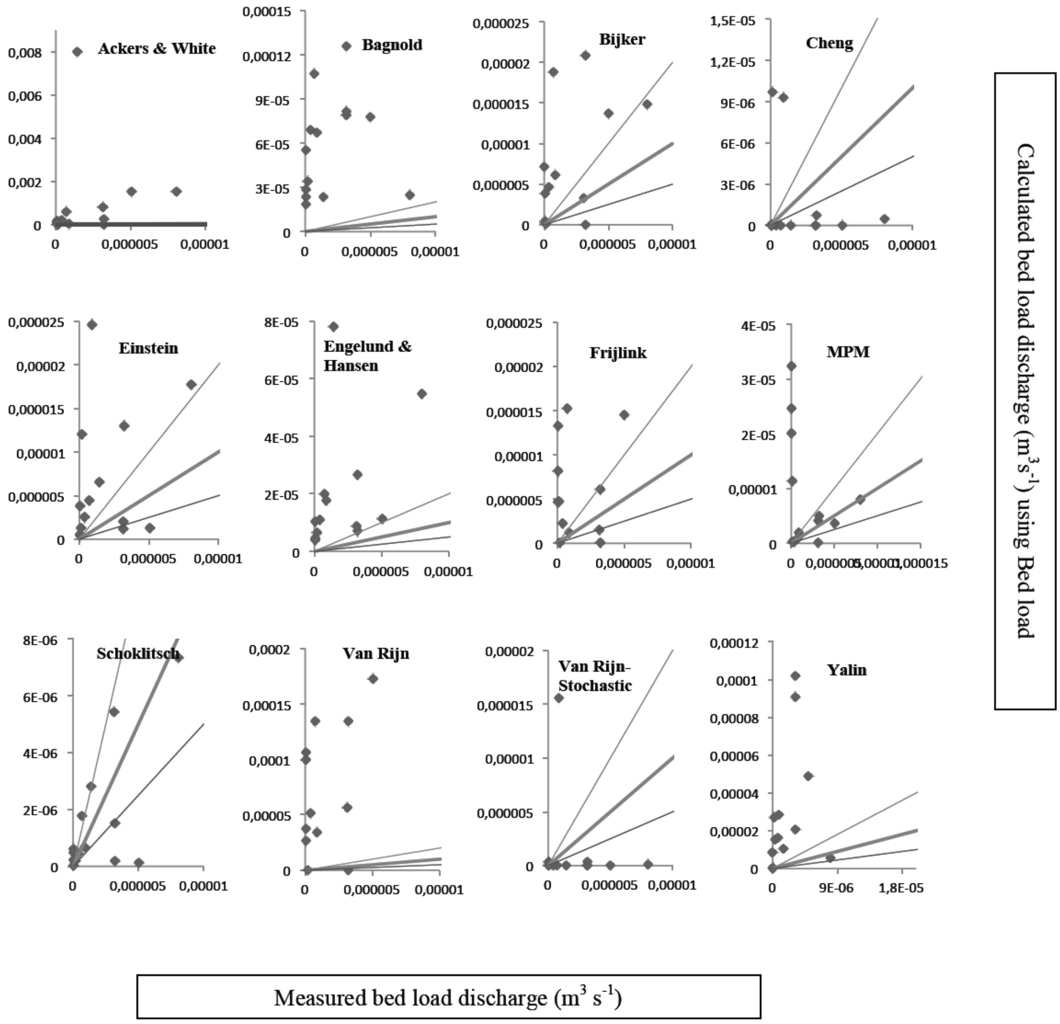

Fig. 5. Comparisons between calculated and measured bed load discharge (bed load grain size).

Habersack and Laronne (2002) compared measured and calculated values using the following bed load discharge formulas for equilibrium conditions: Meyer-Peter et al. (1934), Schocklitsch (1934, 1950) but formulated in 1943; Meyer-Peter and Mueller (1948), Einstein (1950), Yalin (1963), Ackers and White (1973), White and Day (1982), Bagnold (1980), Parker et al. (1982) and Zanke (1987) and for non-equilibrium conditions: Parker (1990), Zanke (1999) and Sun and Donahue (2000). The results of the discrepancy ratio in the interval of $0.5 < r < 2$ in descending order are Zanke (1987), Einstein (1950), Meyer-Peter et al. (1934), Schocklitsch (1950) to Ackers and White (1973) and White and Day (1982) respectively.

Gomez and Church (1989) tested the performances of 12 bed load sediment transport formulas developed for use in gravel bed streams. A total of 88 sets of river data were used in the analyses. These river data were collected from four gravel bed rivers in the USA. No formula performs consistently well, but the Bagnold, Schoklitsch' and Einstein's equations yield the highest degree of accuracy and the least deviation from measured river data.

Evaluation based on absolute values

One of the ways to make comparisons is by plotting the observed values against the calculated ones. In this method, result from only one formula can be shown in any one graph but the observed results can be obtained from any number of streams.

A comparison between 12 formulas and the measured bed load discharge utilizing grain size of the bed material and bed load is demonstrated in Figs 4 and 5, respectively. Middle lines represent equality between measured and calculated bed load discharges (r = 1). Meanwhile, the two thin lines depict half values and double values of the discrepancy ratio (r = 0.5 or 2), respectively.

In Fig. 4 a noticeable distinction exists between calculations and most of the measurements. Einstein-Brown (1950), Van Rijn (1984a), and Engelund and Hansen (1967) are frequently within $0.5 < r < 2$ range, while Yalin (1963) and Bagnold (1980) formulas overestimate, and all other equations considerably underestimate with using bed material grain size.

Utilizing bed load grain size to calculate bed load discharge gives better results than using bed material grain size in evaluation of most equations.

Van Rijn (1984a) and Engelund and Hansen (1967) that have reasonable results with bed material grain size, tend to overestimate with bed load grain size. Obviously, Schoklitsch (1950) and Meyer-Peter and Mueller (1948) have points between two thin lines of half values and double values of discrepancy ratio. In Fig. 5 Cheng (2002) equation still underestimate bed load discharge (like in Fig. 4), but the results of Cheng equation with using bed load grain size as representative size are much better than using bed material as representative size.

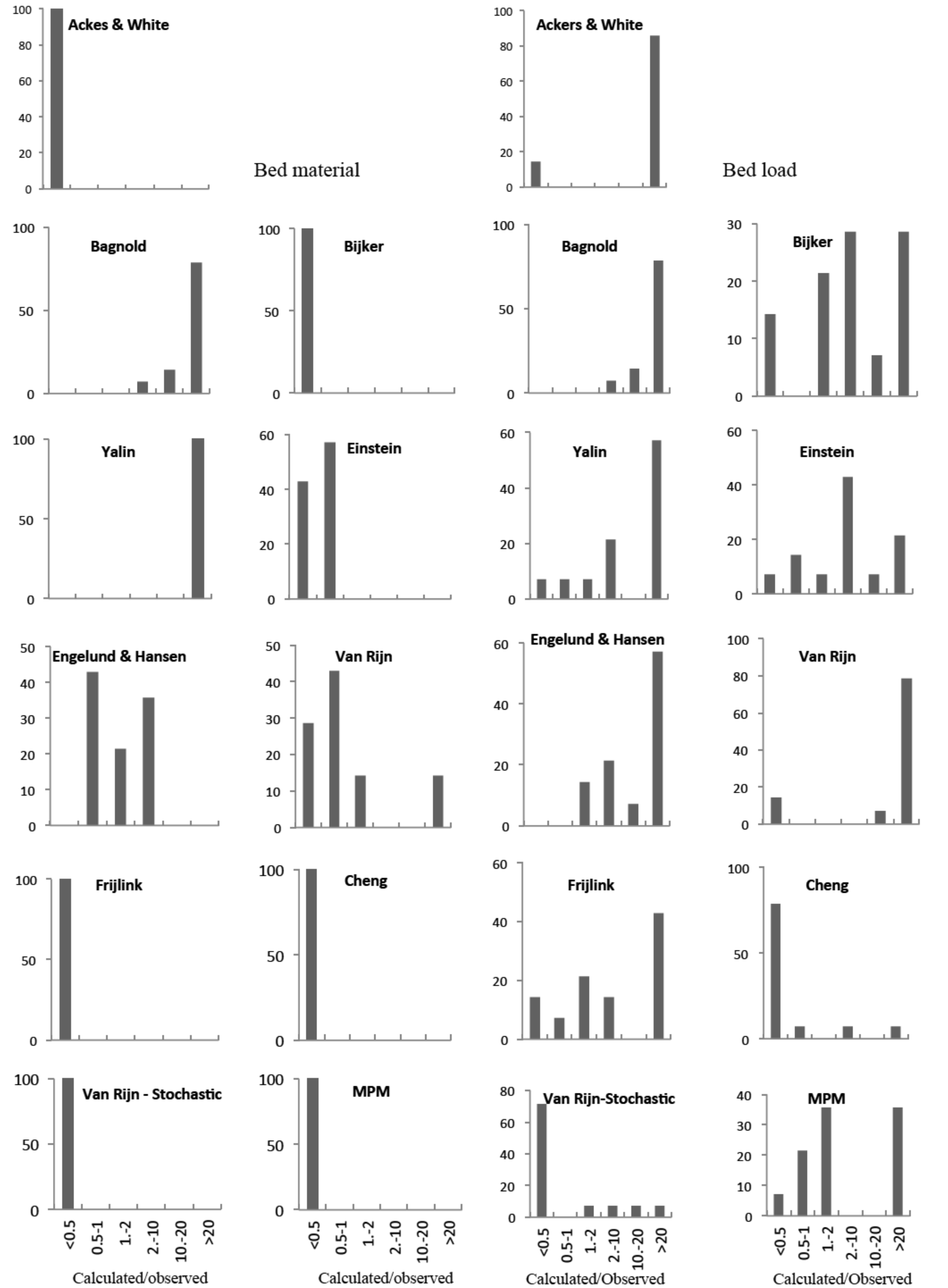

Fig. 6. Ranking of bed load discharge formulas in terms of calculated to measured bed load discharge (r) for bed material grain size (right side) and bed load grain size (left side).

Evaluation based on frequency of *r* values

The accuracy of the computed bed load transport rate is described in terms of the discrepancy ratio, defined as:

$$r = \frac{q_{b,computed}}{q_{b,measured}}$$

Previous comparisons have shown that none of the formulas achieved an *r* values that was very close to unity for all measurements (Bravo-Espinosa et al. 2003; Yang and Simoes, 2005).

Fig. 6 shows the calculated and measured bed load discharge using grain size of the bed load and bed material. As the use of finer texture (the grain size of bed load is finer than bed material load (see Fig. 3)) leads to better calculated bed load discharge, formulas like Frijlink (1952), Meyer-Peter and Mueller (1948), and Bijker (1971) that under estimate bed load discharge with bed material grain size, thereby improve substantially with using bed load grain size.

In comparison, bed material as the input parameter with bed load, the Ackers and White (1973) equation under predicts for the former and over predict for the latter. For Cheng equation using both grain sizes under predicts the measured bed load discharge, while using both bed load and bed material load as representative size, achieve to the same results and over predict the observed values in the Bagnold equation, since there is not sediment particle parameter in this equation. Einstein equation has better results with the bed load grain size than the bed material grain size.

The percentage of *r* values within the discrepancy ratio classes 0.5–2.0 and 0.33–3 are presented in Table 5 for all 12 formulas with bed load and bed material grain size.

Table 5 demonstrates that the Engelund and Hansen (1967) equation provides the best predictions using a representative sediment size based on the bed material, while this formula using a representative sediment size based on the bed load provides poor predictions. This is also true of the Van Rijn formula, but generally, using bed load grain size to evaluate equations yields substantially better results than bed material grain size.

The accuracy of bedload discharge formulas utilizing grain size of the bed material in descending order are those by Engelund and Hansen (1967), Van Rijn (1984a), Einstein (1950) and other equations.

The accuracy of bed load discharge equations using bed load as representative size in decreasing order are Shocklitsch (1950), Meyer-Peter and Mueller (1948), Frijlink, (1952), Einstein (1950), Bijker (1971), Yalin (1963), Engeleund and Hansen (1967), Van Rijn – Stochastic (1987), Cheng (2002), Van Rijn (1984a), Bagnold (1980), Ackers and White (1973). In general, using bedload grain size yields remarkably better results than bed material grain size, since more formulas have discrepancy ratio in the range of 0.5 to 2 and 0.33 to 3. This result proves that although using bed load grain size as a representative diameter for predicting bed load with formulas is not logic. Because it needs intensive field and laboratory work and is a time consuming procedure. But using bed material grain sizes do not achieve to reliable results in most of the formulas. To solve this problem finding a relationship between bed load and bed material for each stream or using smaller diameters (for example representative diameter smaller than D_a for Meyer-Peter and Mueller (1948) or D_{35} in Ackers and White (1973)) can be proposed.

Table 5. Percentage of *r* values for all formulas with bed material grain size and bed load grain size.

Bed material		
Formulas	$0.5 < R < 2$	$0.33 < R < 3$
Engeleund and Hansen	64.3	71.5
Van Rijn	57.2	57.2
Einstein	57.2	64.3
Meyer-Peter and Mueller	0	0
Ackers and White	0	0
Bagnold	0	0
Yalin	0	0
Frijlink	0	0
Van Rijn – Stochastic	0	0
Bijker	0	0
Cheng	0	0
Shocklitsch	0	0

Bed load		
Formulas	$0.5 < R < 2$	$0.33 < R < 3$
Shocklitsch	64.3	71.5
Meyer-Peter and Mueller	57.2	57.2
Frijlink	33.3	35.7
Einstein	21.5	28.6
Bijker	21.5	35.7
Yalin	14.3	14.3
Engeleund and Hansen	7.2	21.5
Van Rijn – Stochastic	7.2	7.2
Cheng	7.2	7.2
Van Rijn	0	0
Bagnold	0	0
Ackers and White	0	0

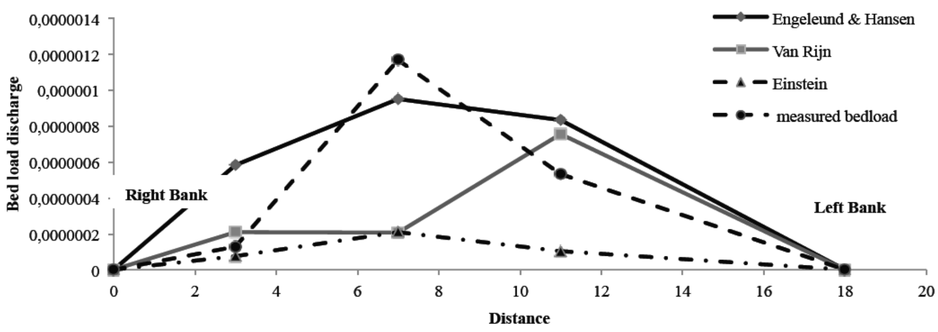

Fig. 7. Cross sectional variation of measured and calculated bed load discharge using best fitted equations with bed material grain size, Narmab River, 21 February 2010.

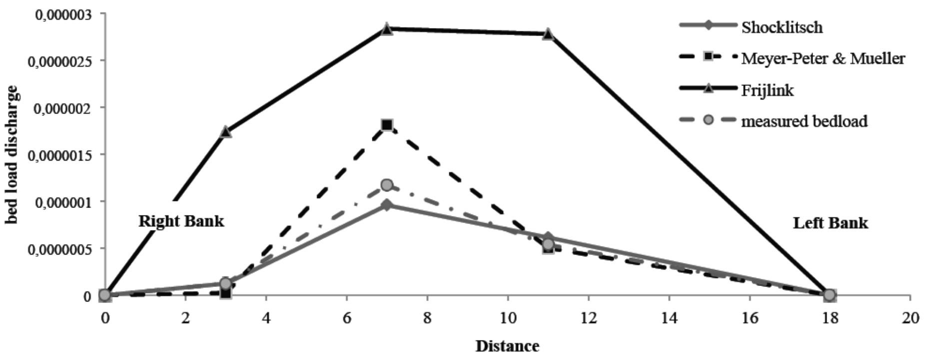

Fig. 8. Cross sectional variation of measured and computed bed load discharge using best fitted equations with bed load grain size, Narmab River, 21 February 2010.

Local hydraulic conditions

One of the methods to improve the accuracy of bed load formulas is to use local hydraulic parameters (Carson and Griffiths 1989). Fig. 7 shows application of Engelund and Hansen (1967), Van Rijn (1984a), and Einstein (1950) equations to measurements of 28 May 2010 with discrepancy ratio of 0.83, 0.58, and 0.21, respectively. For this calculation we used the hydraulic parameters of the individual verticals in conjunction with grain size of the bed material. Also, in Fig. 8 lateral distribution of measured and calculated bed load discharge for Shocklitsch (1950), Meyer-Peter and Mueller (1948), and Frijlink (1952) with discrepancy ratio of 0.9, 0.98, and 3.3, respectively.

CONCLUSIONS

In this study, 12 bedload predictors are evaluated using fourteen series of measured bedload data in Narmab River, North Iran. An attempt was made to evaluate the formulas by comparing observed sediment discharges in a gravel bed river with values calculated by the formulas. The results presented in this paper can be used to find the suitable bedload formula for a gravel bed river with a mild slope. We conclude that:

1. In a given water discharge, two important input parameters that can affect the bedload discharges are river bed slope and representative grain size of sediments. In this mild slope river (S = 0.03%), using median size, D_{50} of bedload samples which is almost equal to D_{10} of bed material grain size leads to a more tolerable result than using median size of bed materials. This phenomenon is true in 11 out of 12 bedload formulas, except in Bagnold method that the particle size parameter does not exist, and the results of utilizing two types of grain sizes in evaluating bedload discharges are identical.
2. The equations by Engelund and Hansen (1967), Van Rijn (1984a), and Einstein (1950) adequately predicted bed load transport in this river with bed material grain size as representative diameter.
3. The accuracy of these 12 formulas in descending order with bed load grain size are: Shocklitsch (1950), Meyer-Peter and Mueller (1948), Frijlink (1952), Einstein (1950), Bijker (1971), Yalin (1963), Engeleund and Hansen (1967), Van Rijn – Stochastic (Van Rijn (1987)), Cheng (2002), Van Rijn (1984a), Bagnold (1980), Ackers and White (1973).
4. In general, more formulas predict bed load transport in the range of acceptable discrepancy ratio ($0.33 < R < 3$) using bed load grin size.
5. Higher overall accuracy of a formula does not guarantee that the formula is superior to the others under all flow and sediment conditions. The accuracy rating of a formula may vary depending on bed slope, grain size diameter, and other hydraulic and sedimentological data. This indicates that we cannot predict bed load transport discharge with any degree of reliability without an adequate number of observations.

REFERENCES

Ackers, P., White, W.R., 1973. Sediment transport: New approach and analysis. Journal of the Hydraulics Division, 99(11), 2041–2060.

Bagnold, R.A., 1980. An empirical correlation of bedload transport rates in flumes and natural rivers. Proceedings, Royal Society (London), 372A, 453–473.

Bijker, E.W., 1971. Longshore transport computations. Journal of the Water Ways, Harbours and Coastal Engineering Division, 97(4), 687–701.

Bravo-Espinosa, M., Osterkamp, W.R., Lopes, V.L., 2003. Bedload transport in alluvial channels. Journal of Hydraulic Engineering, 129(10), 783–795.

Brown, C.B., 1950. Sediment Transportation. Chapter XII Engineering Hydraulics. H. Rouse. New York, N.Y, John Wiley and Sons.

Carson, M.A., Griffiths, G.A., 1989. Gravel transport in the braided Waimakariri River: Mechanisms, measurements and predictions. Journal of Hydrology, 109(3-4), 201–220.

Cheng, N.S., 2002. Exponential formula for bedload transport. Journal of Hydraulic Engineering, 128(10), 942–946.

Einstein, H.A., 1950. The bed-load function for sediment transportation in open channel flows. Technical Bulletin, No. 1026, U.S. Dep. of Agriculture, Washington, D.C.

Emmett, W.W., 1980. A field calibration of the sediment-trapping characteristics of the Helley-Smith bed load sampler. U.S. Geological Survey Prof. Paper 1139.

Engelund, F., Hansen E., 1967. A Monograph on Sediment Transport in Alluvial Streams. Teknisk Forlag, Copenhagen, Denmark.

Frijlink, H.C., 1952. Discussion of Bedload Movement Formulas of Kalinske, Einstein and Meyer-Peter and Müller and their application to recent measurements of bedload in the rivers in Holland. Delft Hydraulics, Delft, The Netherlands. X2344/LV.

Gomez, B., Church, M., 1989. An assessment of bed load sediment transport formulae for gravel bed rivers. Water Resources Research, 25(6), 1161–1186.

Habersack, H.M., Laronne, J.B., 2002. Evaluation and improvement of bed load discharge formulas based on Helley-Smith sampling in an alpine gravel bed river. Journal of Hydraulic Engineering, 128(5), 484–499.

Meyer-Peter, E., Mueller, R., 1948. Formulas for bed-load transport. Int. Assoc. Hydraulic Research, 2nd Mtg., Stockholm, 39–64.

Van Rijn, L.C., 1984a. Sediment transport, Part I: Bed load transport. Journal of Hydraulic Engineering, ASCE, 110, 10.

Parker, G., 1990. Surface-based bedload transport relation for gravel rivers. Journal of Hydraulic Research, 28(4), 417–436.

Parker, G., Klingeman P.C., McLean, D.G., 1982. Bed load and size distribution in paved gravel-bed streams. Journal of Hydraulic Division, 108(HY4), 544–571.

Schoklitsch, A., 1934. Specific bed load transport and bed load flux. Wass. Wasserwirtschaft, 39(4), 1–7.

Schoklitsch, A., 1950. Handbook of Hydraulic Engineering. New York, Springer.

Sun, Z., Donahue, J., 2000. Statistically derived bedload formula for any fraction of nonuniform sediment. Journal of Hydraulic Engineering, 126(2), 105–111.

Van Rijn, L.C., 1984a. Sediment transport, Part I: Bed load transport. Journal of Hydraulic Engineering, 110(10), 1431–1456.

Van Rijn, L.C., 1984b. Sediment transport, Part II: Suspended load transport. Journal of Hydraulic Engineering, 110(11), 1613–1641.

Van Rijn, L.C., 1987. Mathematical Modelling of Morphological Processes in the Case of Suspended Sediment Transport. Thesis, Dep. of Fluid Mech., Delft Univ. of Technology, Delft, The Netherlands.

White, W.R., Day, T.J., 1982. Transport of Graded Gravel Bed Material. Wiley, New York.

Yalin, M.S., 1963. An expression for bed load transportation. J. Hydraul. Div., Am. Soc. Civ. Eng., 89(HY3), 221–250.

Yang, C.T., 1973. Incipient motion and sediment transport. Journal of the Hydraulics Division, 99(10), 1679–1704.

Yang, C.T., Simoes, F.J.M., 2005. Wash load and bed-material load transport in the Yellow River. Journal of Hydraulic Engineering, 131(5), 413–418.

Zanke, U., 1987. Comparison of bed load transport formulas. Univ. Hannover, Mitt. Franzius-Instituts Wasserbau Küsteningenieurwesen, 64, 327–364.

Zanke, U., 1999. Analytical solution for bed load transport. Wasser Boden, 51(6), 46–52.

Received 18 August 2011
Accepted 6 December 2012