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A Two Person Zero Sum Game Oriented to Integration of Objectives

Keywords

Multi-Objective Decision Analysis, Game Theory, Fuzzy Sets

Abstract

Decision making process is a process which includes decision makers, actors, environmental factors, objectives, strategies and criteria. In competitive environments, effectiveness of decision process depends on determining all environmental factors and evaluating them according to objectives. Decision makers aim to find optimal strategies for conflicting objectives. Game theory is an approach based on mathematics in which strategies of players are evaluated reciprocally by considering environmental effects.

In this study, a two-person zero-sum game approach is presented for choosing optimal strategies of actors in competitive environment by balancing objectives reciprocally. This approach refers to evaluation of each objective, creation of decision payoff matrixes by using fuzzy logic mathematical applications and their transformation to final decision payoff matrix subsequently. Finally, optimal strategies and their probabilities are found. A military case study is presented for illustrating the application of proposed approach. .

1. Introduction

Decision making process includes complexity and uncertainty. For choosing an appropriate alternative in decision process, organizational needs, objectives, risks, benefits and resources have to be taken into consideration. Decision making is not only process which is based on assessment of alternatives according to criteria but also objectives. Organizations may have more than one objective with different priorities. Objectives may be equally important or they may have different

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importance values. Decision makers have to balance the objectives when different kinds of strategies are executed.

Decision making in competitive environments has different kind of concerns. In this process, actors (organizations or individuals) need to decide by considering the decisions of other actors and one-sided decision process are not enough. Game theory is a decision making tool which brings mathematical solution and plays important role in competitive environments. It is an important approach for analyzing problems and finding the values of payoff matrixes which presents outcomes of players. However in real conditions, it is very difficult to have enough information, to assess players achievements and to formulate them with mathematical terms.

This study aims to provide a systematic approach for decision process in competitive environments which is dedicated to integrate objectives with their importance. Two-person zero-sum game approach is used to find optimal strategies of actors by balancing objectives reciprocally. According to proposed approach, firstly objectives are evaluated to determine their importance. Then, strategies of players are evaluated for all their combinations according to each objective and decision payoff matrixes are calculated by fuzzy logic mathematical applications. At the next phase, these matrixes are transformed to final decision payoff matrix. Finally, optimal strategies and their probabilities are found. For presenting the applicability of approach, a military case study is illustrated.

The paper is organized as follows. Literature review is given in Section 2. The methods which are used in the study, are explained in Section 3. Methodolology and case study are presented in Section 4. Finally, some conclusions are pointed out at the end of the paper.

2. Literature review

Game theory is an explanandum to formulate structure, analyze and understand strategic script (Turocy and Stengel, 2001). The problems in the concept of game theory are decision problems in which the person making decision is influenced by interaction and need to consider the situations of his/her rival and their benefits. The theory uses certain theoretical terms and mathematical tools with the aim of analyzing and defining real life problems in a simple way (McIntosh, 2002).

Classic game theory is related to how real people behave when they face to known payoff matrixes which generally are not known and obtained easily.

There are sample applications for obtaining the values of payoff matrixes in the literature. McIntosh (2002), examine the effects of degree and priority of information in strategic options and making decisions. In this study, the information degree and priority are used for calculating the values of two-person zero-sum game payoff matrix. Cantwell shows the application of game theory in decision making process in his study (Cantwell, 2003). Two person zero sum game is used to analyze strategies called course of action for each players. Mead examines Second Iraq War in the concept of game theory (Mead, 2005). In this approach, a two-person game focusing on the advantages of strategies for US and Iraq is formulated.

Brown et al. (2011) develop a two person zero sum game theoretic model called BASTION to guide the employment of antisubmarine warfare platforms (ships and aircraft). They propose a two person zero sum game example of with some additional variables that are under the control of the maximizing defender and minimizing attacker. Lee (2012) use game theory as an alternative tool for analyzing strategic interaction between economic development (land use and development) and environmental protection (water-quality). This study focuses on the development of a multi-objective game-theory model for balancing economic and environmental concerns in reservoir watershed management. Hämäläinen et al (2014) discuss about how qualitative and quantitative methods can be combined through the war game. They tried to identify the advantages of combining qualitative and quantitative studies in the context of a research war game.

It is clear that in addition to objective values, there is a strong need to formalize and convert subjective judgments to numerical values. Game theory, fuzzy mathematical applications which are used in this study are important tools for combining qualitative and quantitative methods to find optimal strategy in military decision making process.

3. Methods

Defining uncertainties in judgments of decision makers and expressing them with deterministic values are very difficult and important. In this study, analytic methods mentioned below are used to assess objectives and find out optimal strategies according to integration of them.

3.1. Fuzzy Set Theory

Fuzzy set theory is developed to solve the imprecise, uncertain environmental problems and allows mathematical operators to be applied to the fuzzy sets (Kaufmann & Gupta, 1991). A fuzzy set is characterized by a characteristic function, which assigns to each object a grade of membership ranging between zero and one. The fuzzy set can be shown with $\widetilde{A}$ and the membership function with $\mu_{\widetilde{A}}(x)$ (Chen & Hwang, 1992). $\mu_{\widetilde{A}}(x)$ is a real number between the interval of [0,1]. The membership of each element "x" is defined by fuzzy grade of this element. Ambiguous cases are assigned values between 0 and 1.

3.1.1. Fuzzy Linguistic Variables

Human judgments are so vague that it may not be possible to represent them by accurate numbers. Generally, crisp data are inadequate to model real-life situations. The ratings and weights of the criteria can be assessed by means of linguistic variables (Bellmann & Zadeh, 1970). Based on fuzzy set theory introduced by Zadeh, a fuzzy set becomes linguistic variable when it is modified with descriptive words. This enables the fuzzy logic to function in a way to formulate the human reasoning.

Linguistic variable is a variable whose values (interpretation) are natural language expressions referring to the contextual semantics of the variable (Ross, 2004). Zadeh (1975) described this notion as follows;

> "*The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. In particular, treating Truth as a linguistic variable with values such as true, very true, completely true, not very true, untrue, etc., leads to what is called fuzzy logic (Zadeh,1975).*"

A linguistic variable are defined as a variable whose values are linguistic terms (Zimmermann, 1985). The linguistic variables are useful in situations where decision problems are too complex or not well-defined with conventional quantitative expressions (Li & Yang, 2004). Linguistic values can be represented using different kinds of fuzzy numbers. Conversion scales are applied to transform linguistic terms to fuzzy numbers (Chen & Hwang, 1992).

3.1.2. Fuzzy Numbers

Fuzzy numbers are standard fuzzy sets defined on the set of real numbers, whose α-cuts for all $\alpha \in (0,1]$ are closed intervals of real numbers. Triangular fuzzy numbers (TFN) are a special type of fuzzy numbers and characterized by the triple of real numbers (a_L, a_M, a_U). The parameter a_M gives the maximal grade of $\mu_{\tilde{A}}(x)(i.e., \mu_{\tilde{A}}(a_M) = 1)$, "$a_L$" and "$a_U$" are the lower and upper bounds which limit the field of the possible evaluation.

3.1.3. Aggregation of Fuzzy Numbers (Chen et al., 2006)

If we assume that a decision group has "***k***" decision makers, then the fuzzy rating of each decision maker can be represented as a triangular fuzzy number (TFN) with membership function $\mu_{\tilde{R}_k}(x)$. An effective aggregation method should be considered the range of fuzzy rating of each decision maker. The range of aggregated fuzzy rating must include the ranges of all decision-makers' fuzzy ratings. If the fuzzy ratings are TFNs $\tilde{R}_k(a_k, b_k, c_k),\ k = 1,2,..K$, then the aggregated fuzzy rating can be defined as;

$$\tilde{R}(a,b,c), \quad k = 1,2,..K \quad a = \min_k \{a_k\}, \quad b = \frac{1}{K}\sum_{k=1}^{K} b_k, \quad c = \max_k \{c_k\} \quad (1)$$

3.1.4. Defuzzification of TFNs

Defuzzification is used to convert fuzzy numbers to crisp values. Chou et al. (2008) use the graded mean integration representation method developed by Chen and Hsieh (2000) in their study. This method can be used for defuzzication of linguistic variables which are used for evaluating the significance of objectives and comparing strategies according to these objectives. Based on this method, defuzzified value can be obtained for TFN $\tilde{A} = (a, b, c)$ as;

$$R(A_i) = \frac{a + 4b + c}{6} \quad (2)$$

3.2. Game Theory

Game theory is a multi-person decision making process or decision analysis. It is assumed that players have knowledge about situation, opponent player's strategies and preferences in game theory. But most of the times, it is not possible to formulate all strategies implicitly because of the complexity of real situation (Law and Pan, 2008). A strategic game consists of set of players, strategies for each player and payoff for each strategy combination. If there is an interactive situation described as a game, a formal analysis should find optimal strategies for players and determine an expected outcome of game. A solution to a game is a certain combination of strategies. This combination is called as Nash equilibrium if no player can gain by unilaterally deviating from it. Two-person non-constant sum game is one of the matrix games which represent the aspects of non-cooperate games.

3.2.1. Payoff Matrix and Optimal Strategies

Because of easy presentation and calculation convenience, two person zero sum game is used as the most common method in game theory literature. It is a mathematical representation of situation in which there are two players and one player's outcome is equal to the other's losses (Table 1).

Table 1. Game Payoff Matrix

			Column Player			
		Probability	y_1	y_2	...	y_n
	Probability	Strategy	1	2	...	n
Row Player	x_1	1	a_{11}	a_{12}	...	a_{1n}
	x_2	2	a_{21}	a_{22}	...	a_{2n}
	:	:	:	:	:	:
	x_m	m	a_{m1}	a_{m2}	. . .	a_{mn}

A game which has Nash equilibrium is called stable game. Nash equilibrium is a saddle point which no player can gain more by a change of strategy as long as the other player remains unchanged. Nash equilibrium is a point in which following equation is satisfied.

$$\text{Maximum (Row Minimum)} = \text{Minimum (Column Maximum)} \quad (3)$$

A game which has not Nash equilibrium is called unstable game. In unstable games instead of pure strategies players have mixed strategies. The payoff to player 1 when both players play optimally is referred to as the *value of the game* (Hillier and Lieberman, 2001).

Probability distribution of players' strategies (possibility of use for each strategy) and expected value of the game are calculated by using following equations.

x_i = probability of i^{th} strategy for row player; (i=1, 2, ..., m)
y_j = probability of j^{th} strategy for column player; (j=1, 2, ..., n)

Since the players should choose one strategy,

$$\sum_{i=1}^{m} x_i = 1, \qquad \sum_{j=1}^{n} y_j = 1 \qquad \text{(Total probability of strategies)} \qquad (4)$$

The solution of game with mixed strategies are found according to maximin and minimax criteria. So, the expected payoff value of player A is maximized in columns and the expected payoff value of player B is maximized in rows. The models for player are presented as follows (Arslan, 2006);

$$\max_{X_i}\left[\min\left(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, \ldots\ldots \sum_{i=1}^{m} a_{im}x_i\right)\right] \text{ (Strategies of row player)} \qquad (5)$$

$$\min_{y_j}\left[\max\left(\sum_{i=1}^{n} a_{1j}y_j, \sum_{i=1}^{n} a_{2j}y_j, \ldots\ldots \sum_{i=1}^{n} a_{nj}y_j\right)\right] \text{(Strategies of column player)} \qquad (6)$$

After calculating expected maximin and minimax values, x_i^* and y_j^* are presenting the optimal values.

$$v^* = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_i^* y_j^* \qquad \text{(Value of the game)} \qquad (7)$$

While generating payoff matrixes generally only one objective considered. But in reality, players may have more than one objective with different criteria weights and different priorities in competitive circumstances.

4. Methodology and Case Study

In the study, a fuzzy logic game theoretical methodology is developed for decision makers to analyze the situation and find optimal strategies in accordance with players' conflicting objectives in competitive environments. A generic military tactical scenario is designed to demonstrate proposed method's application in which two enemy forces (red and blue) confronted in a field. Although, the scenario is generic, experts were asked to evaluate situation according to their tactical knowledge and experiences.

4.1. Methodology

In this study, a methodology has been proposed to integrate player's conflicting objectives. Decision makers' assessments produced by game theory approach, are combined to find optimal strategies. Subjective assessments and opinions related to experts experiences are converted to numerical values by fuzzy logic mathematical applications. The process of proposed method is presented in Figure 1.

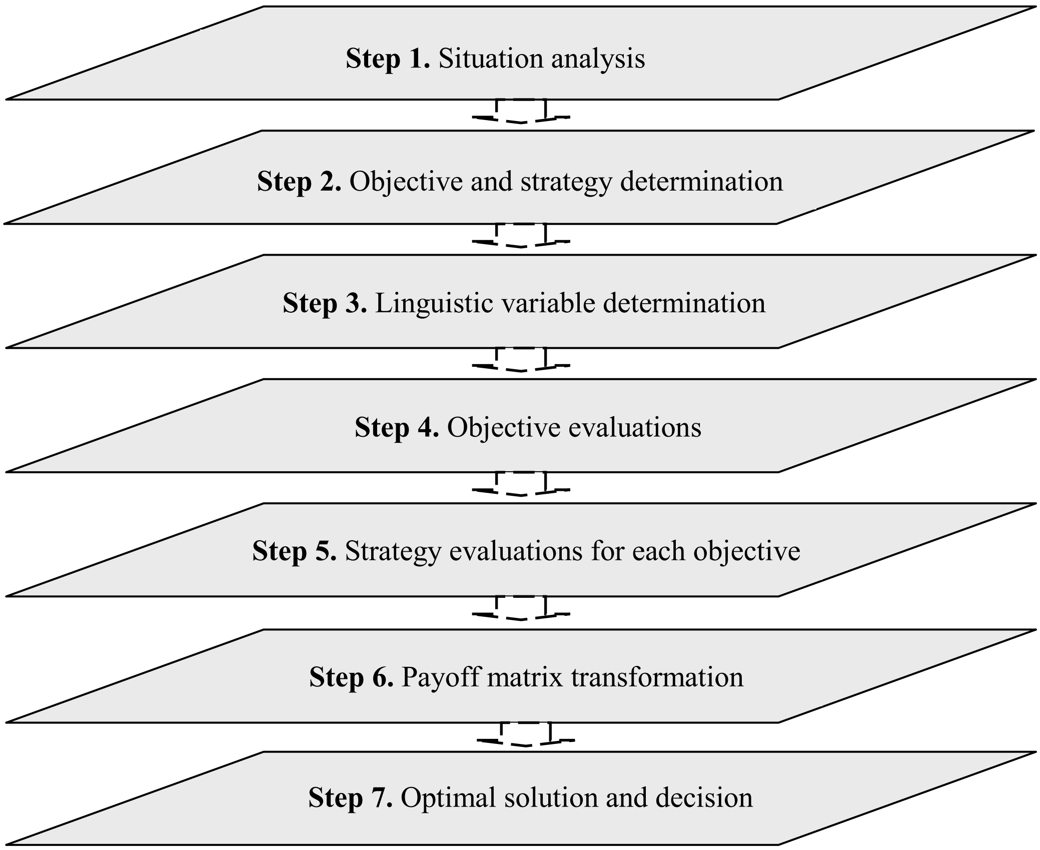

Figure 1. Objective Integrated Two Person Zero Sum Game Theoretic Approach

4.2. Case Study

Case study for presenting the application of methodology comprises a generic tactical combat scenario. According to situation in scenario, blue troops are in attack position while red ones are defending a terrain. The environmental effects are terrain and weather conditions.

Step 1. Situation Analysis

In military decision making process, situation analysis has vital importance for finding best course of action according to circumstances. This step includes the situational awareness and the evaluation of all the environmental factors according to combat intelligence.

Step 2. Objective and Strategy Determination

After revealing the situation, mission is analyzed and stated. The important part of mission analysis is to describe objectives and strategies (course of actions). According to results of situation analysis, objectives and strategies for realization of objectives are determined. The objectives of blue forces can be listed as follows;

O_1: Maximizing losses of red forces.
O_2: Capturing terrain possessed by red forces.
O_3: Minimizing personnel, weapon and material losses.

While blue forces want to maximize realization of these objectives, on the contrary, red forces aim to minimize them. The strategies (course of actions-COA) of players (Blue and Red) for the success of objectives are presented below.

Blue's strategies (BS) are;

- **BS_1**: Attack by fire
- **BS_2**: Attack from left flank
- **BS_3**: Attack form right flank
- **BS_4**: Attack by air forces
- **BS_5**: Frontal attack

Red's strategies (RS) are;

- **RS_1**: Defense in depth
- **RS_2**: Counter attack
- **RS_3**: Defense forward
- **RS_4**: Delay

Step 3. Linguistic Variable Determination

In this study, linguistic variables are used for converting subjective judgments and considerations. These variables are presented by TFNs and range from ‘‘no importance” to ‘‘very important”. The scale for TFN is also calculated by taking the opinions of decision makers since they are the group of evaluators. This group are asked to assign minimum, common and extreme values for linguistic variables. After getting inputs, the average of all judgments are calculated and assigned to create the scale which is given in Table 2.

Table 2. Fuzzy Linguistic Terms and TFN

Importance		Fuzzy Numbers		
Very Important	VI	(0.85	0.95	1.00)
Important	I	(0.65	0.77	0.86)
Medium	M	(0.46	0.54	0.65)
Less Important	L	(0.26	0.38	0.46)
No Importance	NI	(0.12	0.21	0.27)

The values of some linguistic variables are overlapping in the table 2. For example, minimum value of "very important" is "0.85" and the maximum value of "important" is 0.86. Since these are the presentation of linguistic variables with fuzzy numbers, some values can be used for successive variables. On the other hand, for successive variables, the values should be overlapped or at least meet at upper and lower limits. In addition, there is an assumption for the lower limit of variable "no importance". There is an assumption here. The lower limit of this variable is calculated by the judgments and instead of zero, its value is assumed as 0.12.

Step 4. Objective Evaluations

In the methodology, the significant assumption is the variety of objectives in importance. According to the our assumption, objectives should have different weights and can be determined by all environmental factors related to situation. So, objectives are evaluated by decision making group with linguistic variables in table 2. These evaluations are presented below in table 3.

Table 3. Evaluation of objectives with linguistic variables by Decision Makers (DM)

	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	DM_8	DM_9	DM_{10}	DM_{11}
O_1	L	NI	L	I	M	M	M	M	I	M	L
O_2	I	VI	I	VI	VI	VI	I	I	M	I	VI
O_3	VI	I	VI	I	VI	VI	I	VI	I	M	M

After evaluation, these linguistic variables are converted to TFNs and by using eq. 1 and 2. Fuzzy weights of objectives "O_r" are calculated and defuzzified (Table 4).

As an example for these calculations, the mathematical operations for O_1 are presented below.

$$O_1(a,b,c), \qquad DM_k = 1,2,...11 \quad a = \min_k\{a_k\}, \quad b = \frac{1}{K}\sum_{k=1}^{K} b_k , \qquad c = \max_k\{c_k\}$$

$$a = \min\{0.26 \;\; 012 \;\; 0.26 \;\; 0.65 \;\; 0.46 \;\; 0.46 \;\; 0.46 \;\; 0.46 \;\; 0.65 \;\; 0.46 \;\; 0.26\} = \underline{0.12}$$

$$b = \frac{1}{10}(0.38 + 0.21 + 0.38 + 0.77 + 0.54 + 0.54 + 0.54 + 0.54 + 0.77 + 0.54 + 0.26) = \underline{0.48}$$

$$c = \max\{0.46 \;\; 0.27 \;\; 0.46 \;\; 0.86 \;\; 0.65 \;\; 0.65 \;\; 0.65 \;\; 0.65 \;\; 0.86 \;\; 0.65 \;\; 0.46\} = \underline{0.86}$$

$$O_1 = \frac{a+4b+c}{6} \quad \Rightarrow \qquad O_1 = \frac{0.12 + (4 \times 0.48) + 0.86}{6} = \underline{0.482}$$

Table 4. Objective fuzzy weighted matrix and defuzzification values (DV_i)

Or	Weights			DVr
O_1	0.12	0.48	0.86	***0.482***
O_2	0.46	0.83	1.00	0.797
O_3	0.46	0.79	1.00	0.772

As shown in table 4, O_2 is the most important one (DV_2=0.797) while O_1 has less importance (DV_1=0.482).

Step 5. Strategy Evaluations for Each Objective

After calculating importance of objectives, next step is evaluation of strategies and determination of their performance regarding to realization of each objective. Decision making group is asked to compare and evaluate the success of strategies for each objective. Strategies of blue and red player are compared mutually with game theoretical approach.

In this phase, strategies are matched and compared for each objective. Every possible combination is tested. As shown in appendix (table 8), TFN values for each case are calculated by using eq.1. Then, for each case assessments, fuzzy weights of strategies ($\widetilde{WSO}_{ijr}$) are obtained by scalar multiplication of objective and strategy evaluation values. Finally, by using eq. 2 and defuzzified values of strategies ($DWSO_{ijr}$) are obtained. Let;

$\widetilde{WSO}_{ijr}$ = Fuzzy weights of players' strategies (Regarding to objective r, if Alpha plays its ith and Beta plays its jth strategy)

$DWSO_{ijr}$ = Defuzzified values of $\widetilde{WSO}_{ijk}$ (The performance of Alpha's strategies over Beta' strategies regarding to objectives)

For explaining the applications, the calculations and mathematical operations for BS_1& RS_1 case (Blue and Red plays their first strategies), are presented below.

$$\widetilde{WSO}_{111} = (a(AS_1BS_1)\times aO_1 \quad b(AS_1BS_1 \times bO_1 \quad c(AS_1BS_1)\times cO_1)$$
$$\widetilde{WSO}_{111} = (0.12\times0.12 \quad 0.68\times0.48 \quad 1.00\times0.86) = (0.014 \quad 0.324 \quad 0.863)$$

$$DWSO_{111} = 0{,}014 + (4 * 0{,}324) + 0{,}863 = 0{,}363$$

The other results for 2nd and 3rd objectives are given in appendix (Table 9). Then, by transforming all values to new table, payoff matrixes pertaining to objectives are formed. All the values including the value of $DWSO_{111}$ (0,363) is presented in table 5.

Table 5. The payoff matrix pertaining to all objectives $DWSO_{ijr}$

Player-2 (Red

		RS$_1$			**RS$_2$**			**RS$_3$**			**RS$_4$**		
		O$_1$	**O$_2$**	**O$_3$**	**O$_1$**	**O$_2$**	**O$_3$**	**O$_1$**	**O$_2$**	**O$_3$**	**O$_1$**	**O$_2$**	**O$_3$**
Player1 (Blue)	BS$_1$	**0.363**	0.573	0.614	0.360	0.553	0.700	0.384	0.603	0.605	0.377	0.612	0.635
	BS$_2$	0.306	0.661	0.644	0.312	0.641	0.602	0.382	0.696	0.640	0.330	0.591	0.635
	BS$_3$	0.339	0.517	0.520	0.319	0.493	0.558	0.390	0.606	0.488	0.362	0.465	0.581
	BS$_4$	0.407	0.624	0.593	0.412	0.529	0.544	0.418	0.635	0.638	0.394	0.588	0.543
	BS$_5$	0.335	0.453	0.568	0.355	0.462	0.504	0.417	0.513	0.449	0.433	0.600	0.488

Step 6. Payoff Matrix Transformation

This step requires the integration of objectives in one basis. As explained in previous steps of methodology, objectives have different importance weights and strategies are evaluated by their contributions to them. The main idea for decision makers is to realize all objectives simultaneously to achieve overall objective. So, the outputs (table 5) which are obtained separately, should be merged in one basis. We can call this unification process as the integration of objectives with their importance's.

Since all of evaluations are conducted with linguistic variables and calculated with their corresponding values, $\widetilde{WSO}_{ijr}$ values of are merged and $\widetilde{RO}_{ijr}$ (Integrated objective TFNs) are calculated by using eq. 1. Then, they are defuzzified with eq. 2 and VRO_{ij} (values of integrated objectives) are found (Appendix Table 10).

As an example for these calculations, the mathematical operations for BS_1& RS_1 case (Blue and Red plays their first strategies), are presented below.

($\widetilde{WSO}_{111}$= 0.014 0.325 0.864) ($\widetilde{WSO}_{112}$= 0.055 0.595 1.000) ($\widetilde{WSO}_{113}$= 0.214 0.618 1.000)

$$a = \min\{0.014 \quad 0.055 \quad 0.864\} = \underline{0.0140}$$

$$b = ort\{0.325 \quad 0.595 \quad 0.618\} = \underline{0.5127} \qquad \Rightarrow \qquad \widetilde{RO}_{ijk} = \{\ 0.0140 \ \ 0.5127 \ \ 1.0000\}$$

$$c = \max\{0.864 \quad 1.000 \quad 1.000\} = \underline{1.0000}$$

$$VRO_{11} = \frac{a+4b+c}{6} \quad \Rightarrow \quad VRO_{11} = \frac{0.0140+(4\times 0.5127)+1.0000}{6} = \underline{0.511}$$

Payoff matrix which is obtained by using VRO_{ij} is presented in Table 6.

Table 6. The payoff matrix according to integration of objectives (VRO_{ij})

		Player-2 (Red)			
		RS_1	**RS_2**	**RS_3**	**RS_4**
Player-1 (Blue)	**BS_1**	**<u>0.511</u>**	0.525	0.521	0.531
	BS_2	0.529	0.512	0.562	0.513
	BS_3	0.456	0.464	0.502	0.473
	BS_4	0.531	0.487	0.556	0.507
	BS_5	0.459	0.434	0.456	0.512

Step 7. Optimal Solution and Decision

Lastly, the linear programming model is built by the perspective of eq. 3-7. and according to payoff matrix according to integration of objectives (table 6).

v: value of game

x_i*: The probability of blue player i*th *strategy*

Max v

St;

$0.511x_1+0.529x_2+0.456x_3+0.531x_4+0.459x_5-v\geq 0$

$0.525x_1+0.512x_2+0.464x_3+0.487x_4+0.434x_5-v\geq 0$

$0.521x_1+0.562x_2+0.502x_3+0.556x_4+0.456x_5-v\geq 0$

$0.531x_1+0.513x_2+0.473x_3+0.507x_4+0.512x_5-v\geq 0$

$x_1+x_2+x_3+x_4+x_5=1$

$x_i\geq 0$ $(i=1, 2, \ldots, 5)$; v unlimited

By using Lindo program, optimal solutions for each objective separately and integrated objectives are found and presented in Table 7.

Table 7. The probabilities of strategies according to objectives

(%)	**Player-1 (Blue)**					**Player-2 (Red)**			
	BS_1	BS_2	BS_3	BS_4	BS_5	RS_1	RS_2	RS_3	RS_4
O_1				87.7	13.3	35.3			64.7
O_2	64.3	35.7				18.8			81.2
O_3	28.3	71.7					25.9	74.1	
VRO_r	54.8	45.2				41.9	58.1		

4.3. Discussion

The proposed methodology is consisted of fuzzy mathematical and game theoretical applications. In table 7, it can be easily observed that players may have different preferences for each objective. It makes the situation and decision making process more complicated. The following results are discussed by the objective's priorities and integration.

According to results, for 1st objective (maximizing losses of red forces), blue player will choose its 4th and 5th strategies with the probability of 87.7% and 13.3% subsequently. It means that most probably blue player will choose the strategy of attacking by air force to maximize red forces' losses. Against this strategy, for minimizing its losses, red player will choose its 1th (defense in depth) and 4th (delay) strategies with the probability of 35.3% and 64.7% subsequently.

For the second objective (capturing terrain possessed by red forces) blue player will choose its 1th (attack by fire) and 2nd (attack from left flank) strategies with the probability of 64.3% and 35.7% subsequently. Red player will select again the same strategies as first objective but with different probabilities (18.8% and 81.2%). Although red player is still on the same route, the strategy "delay" is more appropriate for the second objective which is protecting terrain.

For the third and last objective (minimizing personnel, weapon and material losses), blues will use again 1th (attack by fire) and 2nd (attack from left flank) strategies but with different probability of 28.3% and 71.7%. In this case, red will change its strategies and use 2nd (counter attack) and 3rd (defense forward) strategies with the probability of 25.9% and 74.1%.

On the other hand, the results for the integration of objectives with their importance weights, are different from previous ones. Blue player will play its 1th (Attack by fire) with 54.8% probability and its 2nd (Attack from left flank) with 45.2% when all objectives are considered simultaneously with their weights. Despite, blue's first strategy is not in optimal solution according to first objective (Maximizing losses of red forces), it will be the strongest strategy in overall objective evaluation.

Same kinds of results are obtained for red player. For both 1st and 4th strategies are optimal for O_1 and O_2, for O_3, 2nd and 3rd strategies have probabilities to use. According to integration of objectives, 1st strategy (defense in depth) is with 41.9% probability and 2nd strategy (counter attack) with 58.1% probability are considered as optimal strategies.

According to the results, it is clear that optimal strategies and their probabilities are different for situations in which payoff matrixes for objectives are separately and integrated. It is definite that each objective has a weight or priority of importance. If the situation requires more than one objective for achieving main goal, decision makers should consider the importance or priority of objectives and include them for assessments. For these kind of situations, finding best solutions regarding to one objective might not give the optimal solution. These decision processes requires overall evaluation of objectives. Key consideration is how these objectives can be included to assessment with their severity.

One of advantages can be derived from payoff matrix and the probabilities of strategies is sensitivity analysis. Especially, additional intelligence and information could enhance decision process and change the decisions. Players will have the ability to reevaluate their strategies if there is a circumstantial evidence about a preparation in opposite side.

5. Conclusion

Game theoretical approaches in the literature have significant contributions to decision processes in competitive environments. In reality, actors might have more than one objective in their decision challenges. This objectives may have relative importance to each other and it is not possible to realize all objectives simultaneously. In this case, players will try to balance all objectives and optimize them according to their importance. On the other side, creation of payoff matrix according to mutual strategies, is very complicated and difficult process. Especially, converting subjective assessments to objective values need a systematic approach.

In comparison of the strategies, decision makers need some subjective evaluations pertaining to their experiences. Fuzzy logic applications and linguistic variables are important tools which provides transformation from verbal to analytics in evaluation process.

The proposed methodology requires multi objective approach in two person zero sum game perspective and main focus is the integration of objectives. Most important assumption for methodology is about importance of objectives which are maximization for row player and minimization for column player. The study is based on idea that optimal solutions can be different regarding to evaluations of objectives separately or integrated. To introduce the methodology with numbers and to find an empirical solution, generic military scenario is used. All the calculations are done according to verbal evaluations about events and incidents in the scenario. Linguistic variables and fuzzy mathematical operations are used to convert subjective assessments to objective values. The performance of corresponding strategies of players are evaluated for each objective separately. Then, evaluations are conducted regarding to integration of objectives. Finally, optimal strategies with their probabilities are found.

As a consequence, the proposed methodology provides a systematic approach for multi person decision making process with more than one objective and strategies in competitive environment. We believe that this study can be improved by supporting verbal assessments with quantitative values such as specific numerical indicators. Application of this methodology to n-person or/and non-constant games may provide an important contribution for decision making process in competitive environments. Moreover, this methodology may and can be applied to different areas which require competitiveness.

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APPENDIX

Table 8. Evaluation of strategies according to 1st objective (Maximizing losses of red forces)

Str.	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	Min	Av	Max	$\widetilde{WSO}_{ijr}$			$DWSO_{ijr}$	Rank
BS_1&RS_1	I	I	M	L	VL	I	VI	I	I	I	I	0.12	0.68	1.00	**<u>0.01</u>**	**<u>0.32</u>**	**<u>0.86</u>**	**<u>0.363</u>**	10
BS_1&RS_2	I	M	M	VI	L	M	I	VI	I	M	M	0.26	0.66	1.00	0.03	0.32	0.86	0.360	8
BS_1&RS_3	I	I	I	M	I	VI	I	M	M	I	I	0.46	0.72	1.00	0.05	0.35	0.86	0.384	13
BS_1&RS_4	M	M	I	I	I	M	VI	I	I	M	I	0.46	0.70	1.00	0.05	0.34	0.86	0.377	11
BS_2&RS_1	I	I	I	M	L	M	M	M	L	I	VL	0.12	0.56	0.86	0.01	0.27	0.75	0.306	1
BS_2&RS_2	M	I	M	M	L	M	M	I	I	L	M	0.26	0.57	0.86	0.03	0.27	0.75	0.312	2
BS_2&RS_3	I	M	I	M	M	M	VI	I	I	I	VI	0.46	0.72	1.00	0.05	0.34	0.86	0.382	12
BS_2&RS_4	M	M	I	I	M	L	M	I	I	I	M	0.26	0.63	0,86	0.03	0.30	0.75	0.330	4
BS_3&RS_1	L	M	L	L	I	M	VI	M	M	I	I	0.26	0.60	1.00	0.03	0.28	0.86	0.339	6
BS_3&RS_2	L	M	I	I	M	M	I	M	L	M	I	0.26	0.59	0.86	0.03	0.28	0.75	0.319	3
BS_3&RS_3	VI	M	I	L	VI	I	VI	I	VI	I	M	0.26	0.76	1.00	0.03	0.36	0.86	0.390	14
BS_3&RS_4	I	I	M	I	VI	L	M	M	I	M	I	0.26	0.67	1.00	0.03	0.32	0.86	0.362	9
BS_4&RS_1	I	I	M	VI	M	I	VI	VI	I	I	I	0.46	0.80	1.00	0.05	0.38	0.86	0.407	16
BS_4&RS_2	I	VI	VI	I	VI	I	I	VI	I	I	M	0.46	0.81	1.00	0.05	0.39	0.86	0.412	17
BS_4&RS_3	I	VI	VI	I	I	VI	VI	I	I	I	I	0.65	0.82	1.00	0.08	0.39	0.86	0.418	19
BS_4&RS_4	I	VI	I	I	VI	VI	I	M	M	I	M	0.46	0.76	1.00	0.05	0.36	0.86	0.394	15
BS_5&RS_1	VL	M	I	M	M	M	I	VL	M	I	VI	0.12	0.59	1.00	0.01	0.28	0.86	0.335	5
BS_5&RS_2	M	L	M	L	M	I	I	M	I	VI	VI	0.26	0.65	1.00	0.03	0.31	0.86	0.355	7
BS_5&RS_3	I	M	I	VI	VI	I	VI	I	VI	I	VI	0.46	0.83	1.00	0.05	0.40	0.86	0.417	18
BS_5&RS_4	VI	VI	I	I	VI	VI	I	VI	I	VI	VI	0.65	0.87	1.00	0.08	0.41	0.86	0.433	20

Table 9. Evaluation of strategies according to 2nd and 3rd objective

	O_2 *(Capturing Terrain)*								**O_3** *(Minimizing Own Losses)*							
Str.	Min	Av	Max	Weights			DWO	R	Min	Av	Max	Weights			DWO	R
BS_1&RS_1	0.12	0.72	1.00	0.05	0.60	1.00	0.573	9	0.46	0.74	1.00	0.21	0.62	1.00	0.614	14
BS_1&RS_2	0.26	0.66	1.00	0.12	0.55	1.00	0.553	8	0.65	0.87	1.00	0.30	0.72	1.00	0.700	20
BS_1&RS_3	0.46	0.72	1.00	0.21	0.60	1.00	0.603	13	0.46	0.73	1.00	0.21	0.60	1.00	0.605	13
BS_1&RS_4	0.46	0.74	1.00	0.21	0.61	1.00	0.612	15	0.46	0.78	1.00	0.21	0.65	1.00	0.635	15
BS_2&RS_1	0.46	0.83	1.00	0.21	0.69	1.00	0.661	19	0.46	0.80	1.00	0.21	0.66	1.00	0.644	18
BS_2&RS_2	0.46	0.79	1.00	0.21	0.66	1.00	0.641	18	0.46	0.72	1.00	0.21	0.60	1.00	0.602	12
BS_2&RS_3	0.65	0.87	1.00	0.30	0.72	1.00	0.696	20	0.28	0.81	1.00	0.13	0.68	1.00	0.640	17
BS_2&RS_4	0.46	0.70	1.00	0.21	0.58	1.00	0.591	11	0.46	0.78	1.00	0.21	0.65	1.00	0.635	19
BS_3&RS_1	0.26	0.60	1.00	0.12	0.49	1.00	0.517	6	0.28	0.60	1.00	0.13	0.50	1.00	0.520	5
BS_3&RS_2	0.26	0.59	0.86	0.12	0.49	0.6	0.493	4	0.46	0.64	1.00	0.21	0.53	1.00	0.558	8
BS_3&RS_3	0.26	0.76	1.00	0.12	0.63	1.00	0.606	14	0.13	0.56	1.00	0.06	0.47	1.00	0.488	2
BS_3&RS_4	0.12	0.56	0.86	0.05	0.47	0.86	0.465	3	0.46	0.68	1.00	0.21	0.57	1.00	0.581	10
BS_4&RS_1	0.46	0.76	1.00	0.21	0.63	1.00	0.624	16	0.46	0.70	1.00	0.21	0.59	1.00	0.593	11
BS_4&RS_2	0.26	0.66	0.86	0.12	0.55	0.86	0.529	7	0.28	0.68	0.86	0.13	0.57	0.86	0.544	7
BS_4&RS_3	0.46	0.78	1.00	0.21	0.65	1.00	0.635	17	0.46	0.79	1.00	0.21	0.65	1.00	0.638	16
BS_4&RS_4	0.46	0.70	1.00	0.21	0.58	1.00	0.588	10	0.13	0.66	1.00	0.06	0.55	1.00	0.543	6
BS_5&RS_1	0.12	0.54	0.86	0.05	0.45	0.86	0.453	1	0,28	0.69	1.00	0.13	0.57	1.00	0.568	9
BS_5&RS_2	0.12	0.56	0.86	0.05	0.46	0.86	0.462	2	0.28	0.61	0.86	0.13	0.51	0.86	0.504	4
BS_5&RS_3	0.26	0.63	0.86	0.12	0.52	0.86	0.513	5	0.13	0.53	0.86	0.06	0.44	0.86	0.449	1
BS_5&RS_4	0.46	0.72	1.00	0.21	0.60	1.00	0.600	12	0.13	0.60	0.86	0.06	0.50	0.86	0.488	3

Table 10. The payoff matrix according to all objectives (O_r) and integrated values (VRO_{ij})

	$W\tilde{S}O_{ij1}$			$W\tilde{S}O_{ij2}$			$W\tilde{S}O_{ij3}$			**Integrated Obj. ($\tilde{R}O_{ijk}$)**			VRO_{ij}
BS_1&RS_1	0.01	0.32	0.86	0.05	0.60	1.00	0.21	0.62	1.00	0.014	0.455	1.000	0.5108
BS_1&RS_2	0.03	0.32	0.86	0.12	0.55	1.00	0.30	0.72	1.00	0.031	0.472	1.000	0.5250
BS_1&RS_3	0.05	0.35	0.86	0.21	0.60	1.00	0.21	0.60	1.00	0.031	0.456	1.000	0.5205
BS_1&RS_4	0.05	0.34	0.86	0.21	0.61	1.00	0.21	0.65	1.00	0.055	0.476	1.000	0.5310
BS_2&RS_1	0.01	0.27	0.75	0.21	0.69	1.00	0.21	0.66	1.00	0.014	0.473	1.000	0.5291
BS_2&RS_2	0.03	0.27	0.75	0.21	0.66	1.00	0.21	0.60	1.00	0.031	0.444	1.000	0.5121
BS_2&RS_3	0.05	0.34	0.86	0.30	0.72	1.00	0.13	0.68	1.00	0.031	0.497	1.000	0.5625
BS_2&RS_4	0.03	0.30	0.75	0.21	0.58	1.00	0.21	0.68	1.00	0.014	0.449	1.000	0.5125
BS_3&RS_1	0.03	0.28	0.86	0.12	0.49	1.00	0.13	0.50	1.00	0.031	0.401	1.000	0.4558
BS_3&RS_2	0.03	0.28	0.75	0.12	0.49	0.86	0.21	0.54	1.00	0.031	0.419	1.000	0.4644
BS_3&RS_3	0.03	0.36	0.86	0.12	0.63	1.00	0.06	0.50	0.86	0.031	0.450	1.000	0.5022
BS_3&RS_4	0.03	0.32	0.86	0.05	0.47	0.86	0.21	0.57	1.00	0.031	0.415	1.000	0.4731
BS_4&RS_1	0.05	0.38	0.86	0.21	0.63	1.00	0.21	0.59	1.00	0.031	0.465	1.000	0.5313
BS_4&RS_2	0.05	0.39	0.86	0.12	0.55	0.86	0.13	0.57	0.86	0.055	0.462	0.864	0.4873
BS_4&RS_3	0.08	0.39	0.86	0.21	0.65	1.00	0.21	0.65	1.00	0.014	0.482	1.000	0.5560
BS_4&RS_4	0.05	0.36	0.86	0.21	0.58	1.00	0.06	0.55	1.00	0.055	0.448	1.000	0.5068
BS_5&RS_1	0.01	0.28	0.86	0.05	0.45	0.86	0.13	0.57	1.00	0.014	0.398	1.000	0.4590
BS_5&RS_2	0.03	0.31	0.86	0.05	0.46	0.86	0.13	0.51	0.86	0.031	0.401	0.864	0.4338
BS_5&RS_3	0.05	0.40	0.86	0.12	0.52	0.86	0.06	0.44	0.86	0.055	0.419	0.864	0.4557
BS_5&RS_4	0.08	0.41	0.86	0.21	0.60	1.00	0.06	0.50	0.86	0.055	0.452	1,000	0.5123