

# Modelling a pumped storage power plant on the example of the Porąbka Żar power plant

**Bernard Twaróg**

btwarog@pk.edu.pl |  <https://orcid.org/0000-0003-3150-1409>

Department of Geoengineering and Water Management,  
Faculty of Environmental and Energy Engineering,  
Cracow University of Technology

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**Received:** January 9, 2023

**Accepted:** January 31, 2023

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**Data Availability Statement:** All relevant data are within the paper and its Supporting Information files.

**Competing interests:** The authors have declared that no competing interests exist.

**Citation:** Twaróg, B. (2023). Modelling a pumped storage power plant on the example of the Porąbka Żar power plant. *Technical Transactions*, e2023001. <https://doi.org/10.37705/TechTrans/e2023001>

## Abstract

This article presents the idea and mathematical model of a pumped storage power plant. PSPS Porąbka Żar was selected as the real object for modelling. Due to the specificity of the operation of an intervention-regulatory and scheduled nature related to the coverage of power demand in the power system, the possibility of different modes of operation was taken into account in the model. Porąbka-Żar power plant is a pearl of hydro-engineering on the global stage. It is the first underground and the second largest pumped storage power plant in Poland. It is located in Międzybrodzie Bialskie, in the Silesian province. The short distance between the upper reservoir of the power station, located on Mt. Żar, and the lower reservoir of the międzybrodzkie lake, the high average head of the power station of 432 m above sea level, create great opportunities for accessibility and interference in the operation of the power station. It is equipped with four reversible Francis turbine sets, which operate simultaneously with a total capacity of 500 MW in the generator mode and 542 MW in the pumping mode. Due to its high potential and short turn-on and turn-off times of the turbine sets, it plays an important role in the national power system. The mathematical model was made in Matlab - Simulink software. An important role in the control process of the facility is played by the forecast of both power demand and contingencies. The forecasting model is equipped with elements of artificial intelligence. In addition, the article shows the possibility of supplementing an operating power plant with hybrid elements. The analysis is supported by commentary and a number of charts.

**Keywords:** pumped storage power plant, optimisation, control criteria, mine usage

## 1. Introduction

The energy sector, especially since 24 February 2022, has been under intense pressure from the need to re-evaluate and build new criteria affecting market, geopolitical and regulatory conditions. The previous emphasis resulting from the reduction of greenhouse gas emissions, the development of RES and the phasing out of conventional sources is now changing to the equal development of conventional energy sources along with renewables. In addition, there is a need to develop new nuclear power plant technologies in Poland, which have not been used so far. The obligation to diversify is an overriding element of the country's energy security policy (Sakowska, 2017, Min En. Projekt, 2019). The axiom of diversification implies the need for energy storage. One possible alternative is pumped storage power plants. Despite the rapid development of other storage technologies, they are still the only way to store electricity on a large scale (Beibei et al., 2019). Pumped storage power plants are a proven technology that continues to develop and offer new opportunities for the electricity system, support the development of renewable energy sources and will also support the development of nuclear power in the future.

PSPS (pumped storage power system) services make this type of power plant extremely attractive to the NPS (The National Power System) in terms of the implemented scheduled, regulatory, compensatory and contingency operation.

The basic possibilities of PSPS cooperation with the NPS are listed below:

- ▶ the rapid switching of generator and pumping mode cycles (in the order of a few minutes);
- ▶ the supply of power and reactive energy during compensatory operation to regulate voltage levels in individual power nodes;
- ▶ initiating the reconstruction of the national power system after the so-called "power blackout";
- ▶ the flexibility of PSPS operation – the expansion of regulation ranges, the acceleration of PSPS response to network conditions and the increase of the realised daily number of operation cycles;
- ▶ reducing PSPS operating costs – extending periods of failure-free operation;
- ▶ reducing the negative phenomena of cavitation and vortex propagation through modern calculation techniques and diagnostics;
- ▶ in addition, new localization possibilities in mine workings appear.

The Porąbka Żar hydroelectric power plant is located in Międzybrodzie Bialskie. It is the first underground power plant in Poland and the second largest pumped storage power plant in Poland with a total generating capacity of 500 MW and a pumped storage capacity of 542 MW (Hydroprojekt, 2001). It was commissioned in 1979. The lower reservoir of the power plant is Międzybrodzkie Lake with a dam in Porąbka. The upper reservoir is located on top of Mt. Zar.

## 2. Operating characteristics of the pumped storage power plant

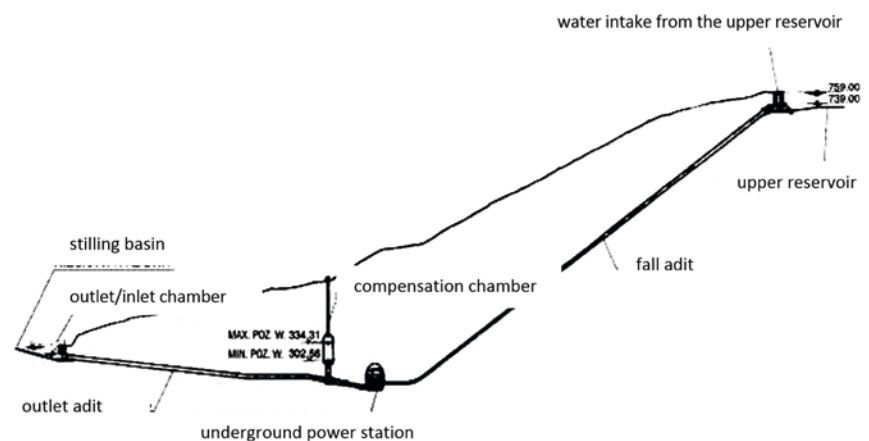
In scheduled operation, PSPS at times of low load on the power grid, i.e. at night when electricity is cheap, the power plant operates in a pumped storage regime filling the upper reservoir. During peak load hours on the power grid, the power plant goes into generator mode producing electricity. Analysing the actual power demand of the national electricity system, it can be seen that the highest values occur in the winter months, such as December, January, February and March. The lowest power consumption occurs in April, May and November. The load on the power grid is lowest from 1:00 a.m. to 5:00 a.m. The low demand for energy during these hours generates the lowest regulated energy prices, while during the highest demand for energy in the afternoon and evening hours, energy prices are highest.



**Fig. 1.** Power demand of the National Power System for the selected 1-week period and regulated electricity prices [own study]

### 3. Basic parameters of the power plant

There are four Francis reversible turbines in operation at Porąbka-Żar PSPS. The installed capacity of the turbines for generator operation is 500 MW. The installed power for the pumping mode is 542 MW. The average gross drop of the power plant is 432 m. The turbines have a start-up time of only 180 seconds and a shutdown time of 142 seconds. Transition from one operating state to another requires a time of about 10 min. The power plant produces an average of 640 GWh of energy per year, and consumes 850 GWh of energy for pumping. The cycle efficiency of the power plant is about 75% (Hydroprojekt, 2001, IIiGWPK, 1984).



**Fig. 2.** Cross section through the Żar rock mass - schematic diagram of Porąbka-Żar PSPS (IIiGWPK, 1984)

### 4. Description of selected facilities of Porąbka-Żar PSPS

#### 4.1. The upper tank

The upper reservoir is an earthen, artificial reservoir with no natural inflow. It is located at the top of Mt. Żar. The total capacity of the upper reservoir is 2.31 million m<sup>3</sup> of water. Leakage is estimated at 0.002 m<sup>3</sup>/day. The water table of the upper reservoir varies in the operating range from 739.00 m above sea

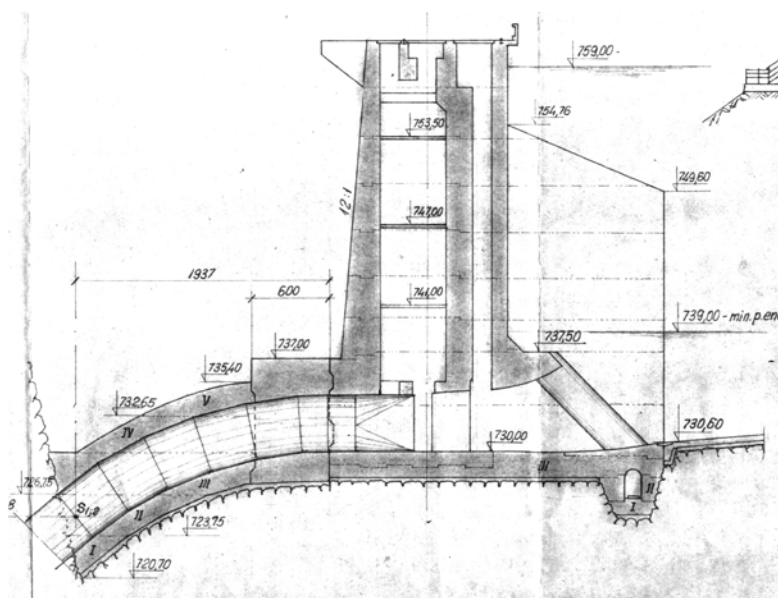


**Fig. 3.** The upper reservoir of Porąbka-Żar PPS [by M. Mleczko]

level to 759.00 m above sea level. The filling of the upper reservoir takes 5 h and 30 min (pumping mode). The emptying time of the upper reservoir is about 4 h (generator mode) (Hydroprojekt, 2001, IIIGWPK, 1984).

#### 4.2. Water intake from the upper reservoir

The upper reservoir is a 34-metre-high facility of reinforced concrete construction, through which water is drawn through two intake vents. The water, flowing at a rate of about 146 m<sup>3</sup>/s through the intake, is directed to high-pressure collapse adits that feed it to four turbine units (Hydroprojekt, 2001, IIIGWPK, 1984).



**Fig. 4.** Inlet to the collapse adit from the upper reservoir (IIiGWPK, 1984).

### 4.3. Lower reservoir

The Międzybrodzkie Dam Lake, which was created as a backwater after the Porąbka Dam was built, serves as the Lower Reservoir. The Międzybrodzkie retention reservoir, also known as the Porąbka reservoir, was built in the period 1928-1936. Its total capacity was originally  $32.2 \cdot 10^6 \text{ m}^3$ , but when serving as a second reservoir in the Soła River cascade,  $5.6 \cdot 10^6 \text{ m}^3$  became silted over



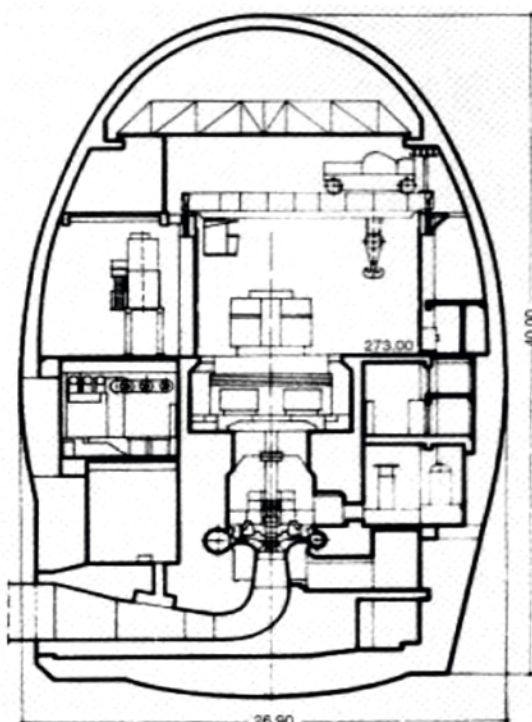


**Fig. 5.** Porąbka Żar PPS lower reservoir, Mt. Żar on the horizon [by M. Mleczo]

several decades of operation. The normal feathering level of the NWL (normal water level) is 322.00 metres above sea level. The lake serves as a retention reservoir in the cascade, protects the operation of the Porąbka power plant and is the lower equalisation reservoir for the operation of the Tresna power plant and Porąbka Żar PPS (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 4.4. Power plant building

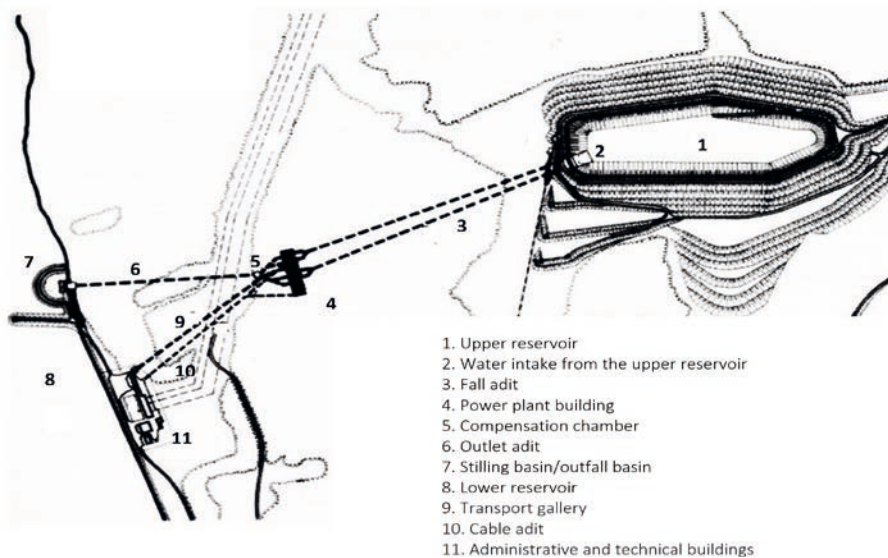
The headquarters of the Porąbka Żar hydroelectric power plant are located on the right bank of Lake Międzybrodzkie, while the power plant, machinery hall and control room are located inside Mt. Żar in a hollow chamber measuring 124x37x40m (Hydroprojekt, 2001, IIiGWPK, 1984).



**Fig. 6.** Vertical cross section of the object (IIiGWPK, 1984)

#### 4.5. Francis turbines

The Porąbka Żar PPS has installed four reversible (reversible), high-speed, Francis-type turbines in a vertical arrangement. For turbine operation, the total power of the Francis turbines is 500 MW, which means that each of the four turbines operates at 125 MW. For pumped operation, their total output is 542 MW, meaning that each of the four turbines operates at 135.5 MW. The electricity produced is supplied to the power grid (Hydroprojekt, 2001, IIiGWPK, 1984).



**Fig. 7.** Upper reservoir, power plant, collapse and outflow adits (IIiGWPK, 1984)

#### 4.6. Fall adit

There are two high-pressure collapse adits with concrete casing and steel armouring, each 872 m long, made at an angle of  $36^\circ$  to ground level. Their diameters are 4.3 m in the upper part, 3.9 m, 3.6 m or 3.2 m in the lower part. The water velocity in the collapse adits ranges from 5 to 9 m/s. The thickness of the armour ranges from 15 to 52 mm. The maximum pressure occurring in the collapse adits is 65 atm. Each of the two collapse adits splits into two turbine adits (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 4.7. Turbine adit

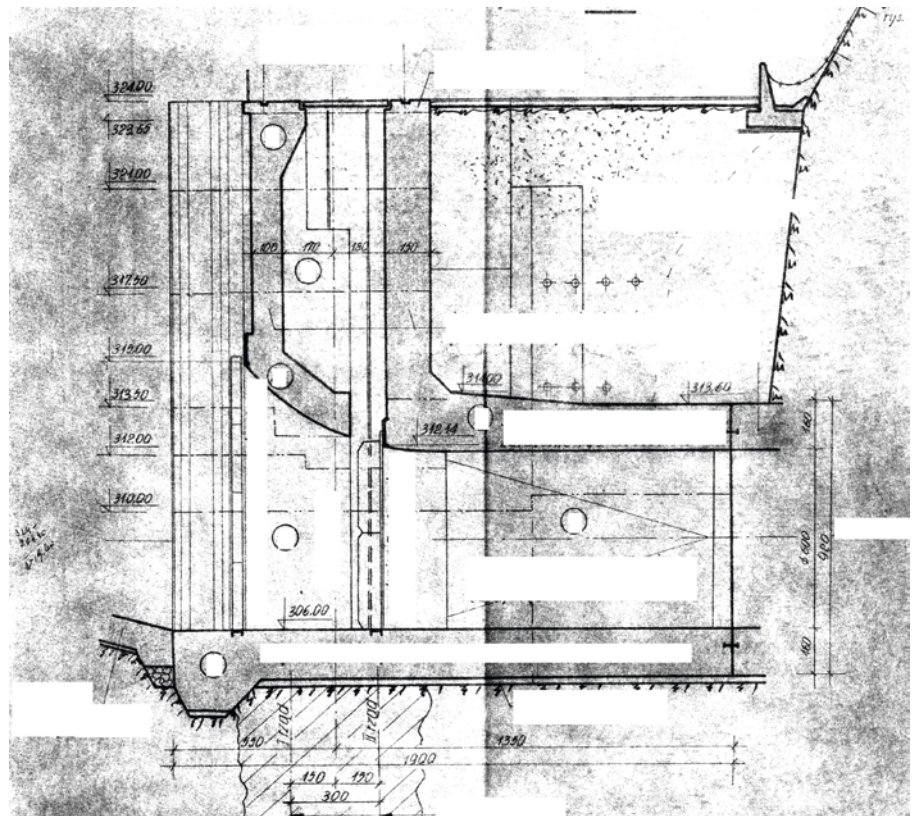
There are four turbine adits, each 2.3 m in diameter, which are connected to Francis pump-turbines. They drain water directly from the turbines toward the lower reservoir (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 4.8. Outlet adit

There is one outflow adit, which has a length of 436 m. Its diameter in the lumen of the casing is 6 m. The water velocity in the outflow adit is 5.1 m/s. The thickness of the reinforced concrete casing is 0.6 m. At the start of this adit, there is an underground compensation chamber of a cylindrical type with a diameter of 14 m. At the end of the adit, by the shore of the Międzybrodzki reservoir, an outlet chamber is located (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 4.9. Outlet chamber

From the outlet chamber, water flows out of the outflow adit into the lower reservoir. Water is also pumped to the upper reservoir this way during periods of low load on the power grid (Hydroprojekt, 2001, IIiGWPK, 1984).



**Fig. 8.** Outlet from the outfall adit to the lower reservoir of the power station (IIiGWPK, 1984)

#### 4.10. Outfall basin

This is a facility that prevents sediment from entering the bottom of the lower reservoir while water is being pumped into the upper reservoir, as well as being pumped to other hydrotechnical facilities (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 4.11. Compensation chamber

The compensation chamber has a diameter of 14 m and a height of 42 m. The thickness of the casing is 0.8 m. The diameter of the inflow shaft is 4.5 m. The diameter of the aeration shaft is 3 m. The ratio of the length of the derrick to the slope is 3:5 (Hydroprojekt, 2001, IIiGWPK, 1984).

### 5. Characteristics of turbine sets

#### 5.1. Start-up engine

The start-up motor drives the hydropower units to synchronous rotation when they are put into pumping operation. It has a short-term power of 8 MW and a voltage of 13.8 kV. The synchronous speed of the motor is 600 rpm (Hydroprojekt, 2001, IIiGWPK, 1984).

#### 5.2. Synchronous generator

The synchronous generator consists of a rotor and a stator. Powered by direct current, the rotor poles move toward the stator windings due to the rotating turbine. The mechanical energy of the turbine is converted into electrical energy. Generators with a rated capacity of 150 MVA and a voltage of 13.8 kV were used. The sweeping torque is 1,380 Tm<sup>2</sup>. The power factor of the generator is 0.9 (Hydroprojekt, 2001, IIiGWPK, 1984).

### 5.3. Francis pump-turbine

Four reversible, high-speed, Francis-type reaction water turbines in a vertical arrangement were used. The speed of the turbines used is 600 rpm. They operate at an average net drop of 420 m. The rotor diameters of the Francis turbines are 3.82 m. The steering apparatus takes 15 seconds to close'. The nominal gullet of the turbine is 36 m<sup>3</sup>/s. The start-up time of the hydrosystem is 10.41 seconds. Each turbine operates at a capacity of 125 MW. The efficiency of the turbine operation is 91.6% (Hydroprojekt, 2001, IIiGWPK, 1984).

### 5.4. Pumped operation characteristics

The total capacity of Francis turbines for pumped operation is 542 MW, which means that each turbine operates at 135.5 MW. The gullet for pumped operation is about 28.8 m<sup>3</sup>/s. The useful head is 440 m. The suction height is 50 m (Hydroprojekt, 2001, IIiGWPK, 1984).

### 5.5. Suction pipe

The hydraulic system of the turbines is equipped with suction pipes that enable partial recovery of the kinetic energy of the water flowing out of the turbine rotor (Hydroprojekt, 2001, IIiGWPK, 1984).

## 6. Simulation model of the operation of the Porąbka Żar power plant

The model was developed in the Matlab – Simulink interface. In developing the model, the following factors were focused on:

- ▶ input to the system, which reads information about the power demand of the NPS, and the size of the turbine/pump start-up criteria, this can be a forecast;
- ▶ building a control signal for reversible turbines, turning them on and off appropriately for both pump and generator operation;
- ▶ modification of the control signal by restrictions on the retention of the upper reservoir, possibly due to the impulse of emergency operation, this can be a forecast;
- ▶ pump-turbine operation;
- ▶ economic calculations – revenue-cost, taking into account current energy prices or their forecast;
- ▶ the output graphically presenting the results of the simulation or allowing the saving of results in an appropriate format, e.g. ASCII

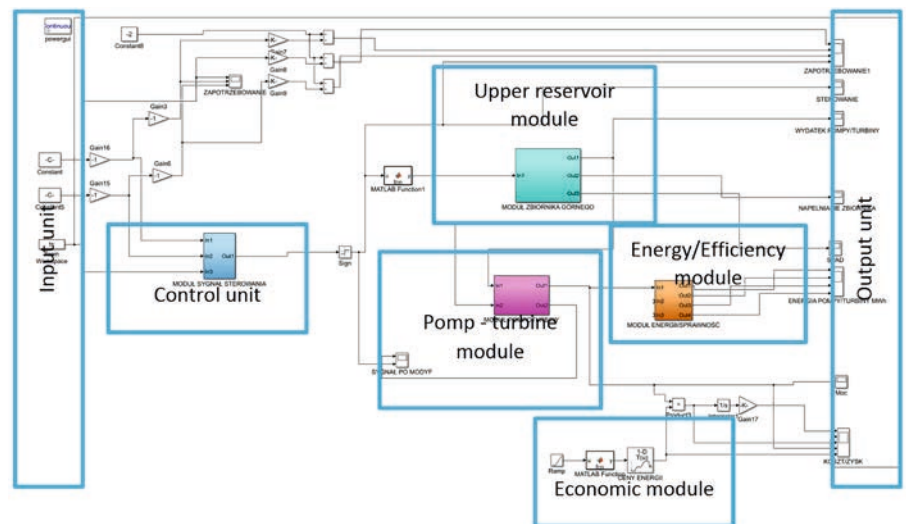
For the analysed example, a seven-day simulation horizon was assumed. The power demand was assumed on the basis of history from 2021-03-01 to 2021-03-07. Regulated energy prices are for November 2022.

### 6.1. Input module

In the construction of this module, the value of power demand is read in ASCII format. In this format, it is possible to obtain information on the current/historical or forecasted power demand from the following page: <https://www.pse.pl/dane-systemowe/funkcjonowanie-kse/raporty-dobowe-z-pracy-kse/zapotrzebowanie-mocy-kse>

In addition, the value of the criterion for the construction of the pump-turbine control signal, turbine startup/shutdown of the generator mode and turbine startup/shutdown of the pump mode is read.





**Fig. 9.** Simulation model of operation of Porąbka-Żar PSPS [own study]

## 6.2. Pump-turbine on/off signal building module and operating mode

The signal-building module is based on defined on/off criteria, which are given a priori by the system dispatcher (e.g. NDC – National Power Dispatch). They determine the on/off moments and the pump-turbine operating mode. The simplest system has been adopted, in which the intersection points between the adopted on/off levels and power demand information determine these decisions (Kaczmarek, 1984; Malinowski, 1995; Słota, 1983; Padiyar, 2008).

## 6.3. On/Off signal modification module and operating mode

Modification of the control signal is due to the limitations of the system's operating capabilities. Assuming that the retention capacity of the lower reservoir is not a problem for the system, the capacity of the upper reservoir and the times required to fill/empty the reservoir make it necessary to verify the signal and modify it. In addition, there is a need to take into account so-called intervention (i.e. a change in the operating mode resulting from the needs of the NPS). The interpenetration of planned operation and intervention requires the reconstruction of the system operation control signal. Planned operation, i.e., energy production, and intervention possibilities (the value of energy potential that can be available at a given moment), draws attention to the search for intervention forecasting techniques so as to minimise pumping costs during periods of high energy prices and, in principle, to maintain maximum availability of PSPS power for moments of intervention.

## 6.4. Pump-turbine operation

The pump-turbine operation module calculates the power and energy of turbines operating in the generator mode and in the pump mode. It takes into account the current filling of the upper reservoir. The water table of the lower reservoir is assumed in the model at the normal level of impoundment (NPP). The drop and the amount of energy consumed/produced are calculated while taking into account the filling/emptying of the upper reservoir.

## 6.5. Economic module

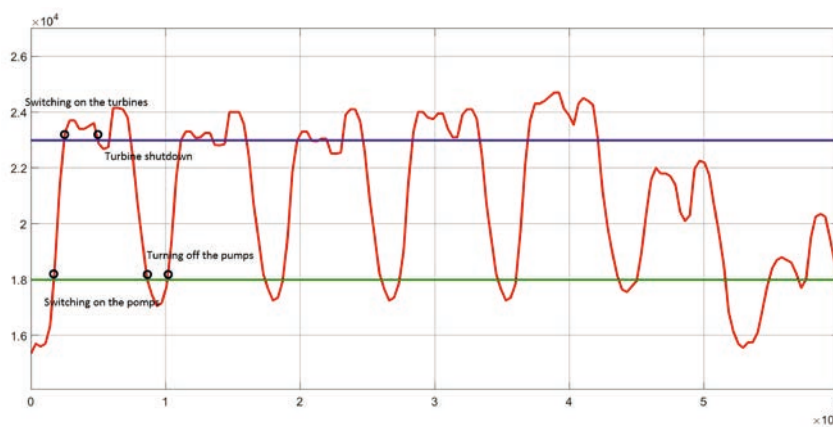
The economic (financial) module enables assessment of the value of revenue from the sale of energy produced and the costs associated with buying the necessary energy for pumping. The cost balance is presented cumulatively.

## 6.6. Output module

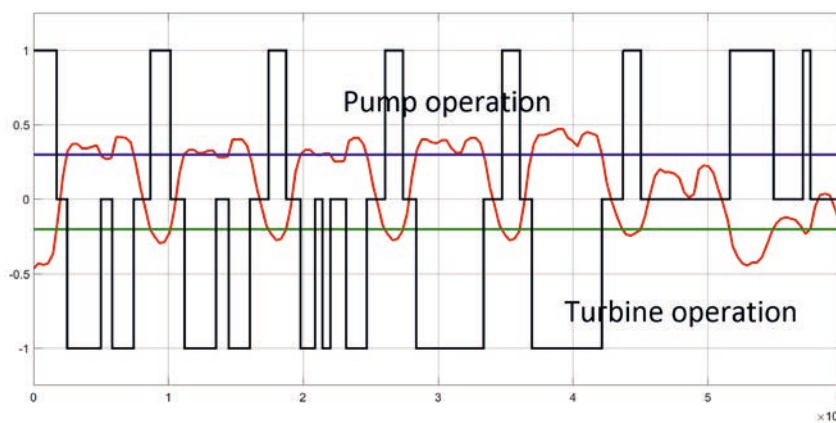
This module allows the presentation of all quantities that are calculated during the simulation of the Porąbka Żar PSPS. At the same time, it is possible to save them in any form.

## 7. Model simulation results

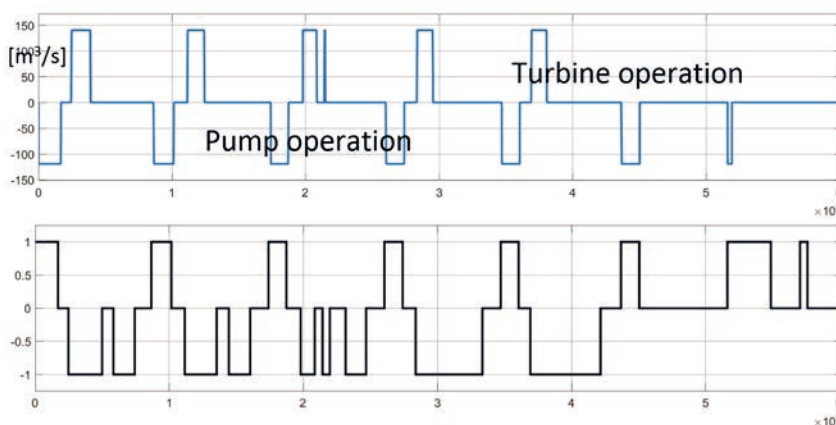
The results are presented for a short simulation period of seven days. They are readable and one can intuitively assess the correctness of the working model. The results of the calculations reveal the relationships between the behaviour of individual objects/modules. The simulation results are presented below and a detailed description of the results is included.



**Fig. 10.** Determination of switching points from turbine to pumped storage operation and vice versa [own study]

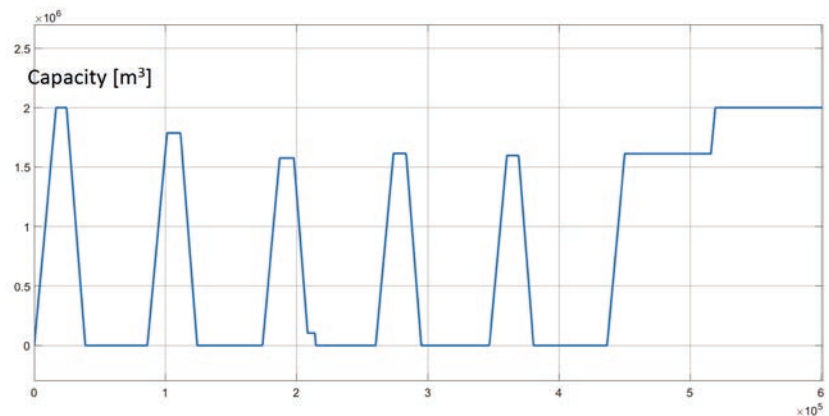


**Fig. 11.** Building a pump-turbine on/off signal [own study]

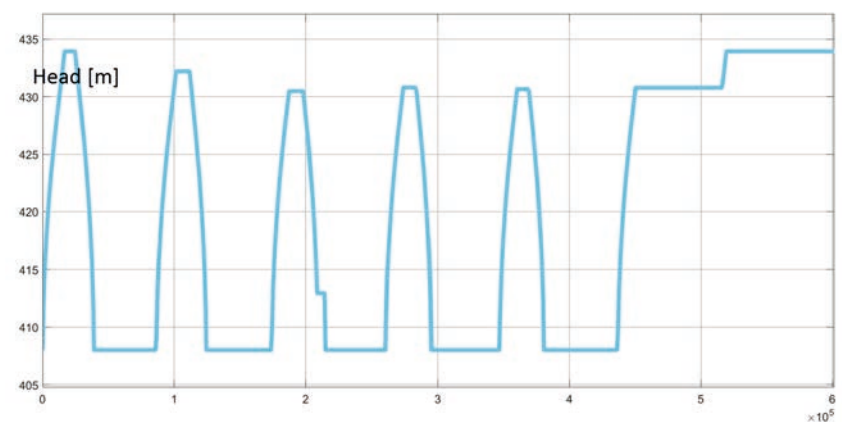


**Fig. 12.** Modification of pump-turbine on/off signal [own study]

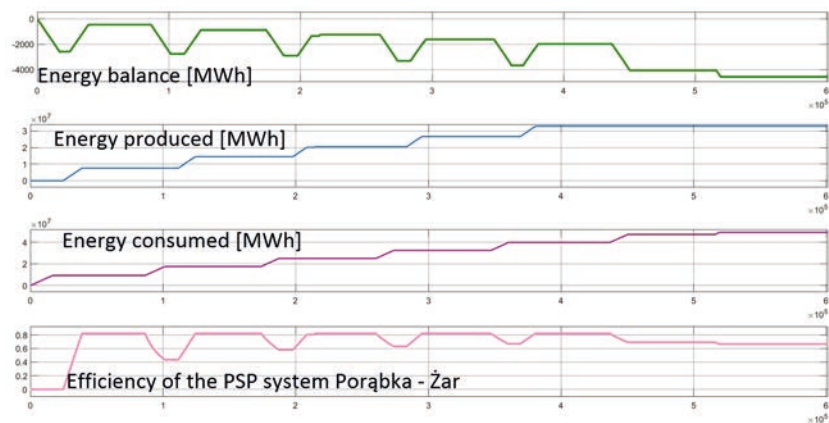
**Fig. 13.** Filling/emptying the upper tank  
[own study]



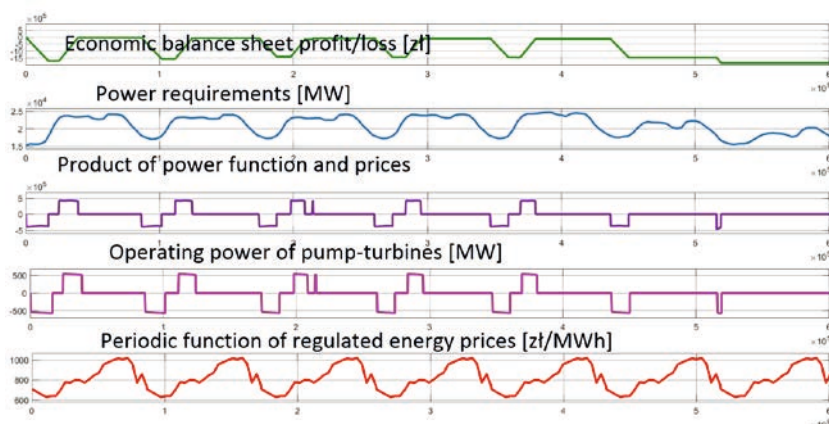
**Fig. 14.** Difference between upper and lower  
water table [own study]



**Fig. 15.** System efficiency, pumping energy  
loss, energy production, energy balance  
cumulative [own study]

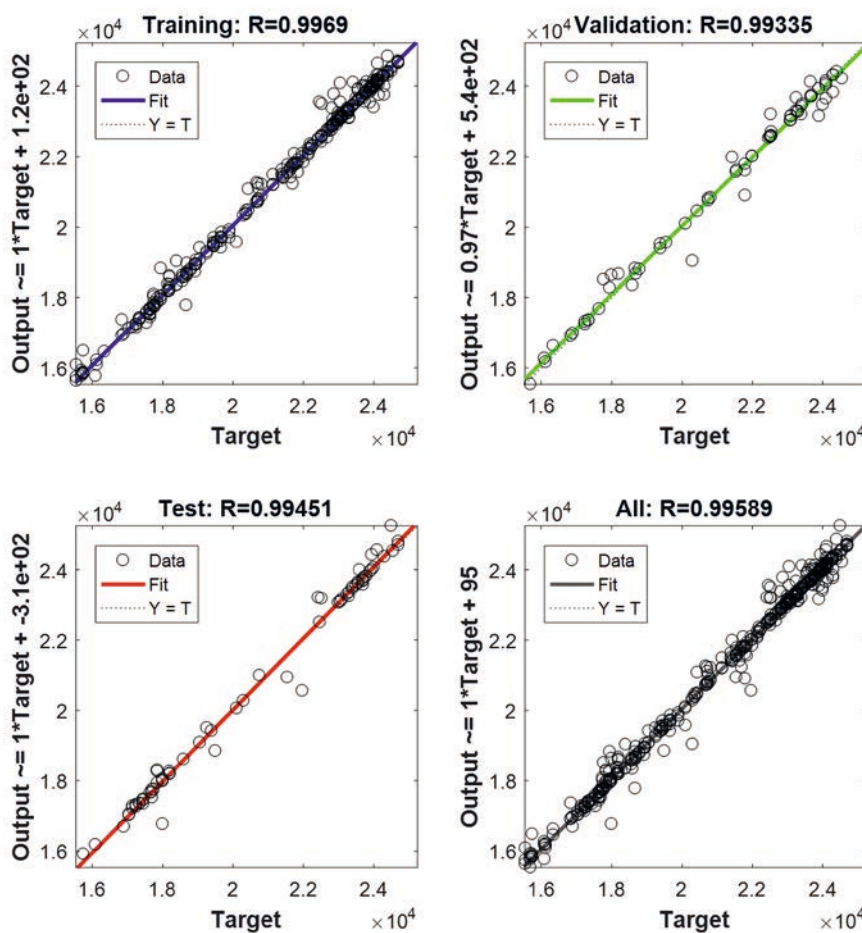


**Fig. 16.** Regulated energy prices, pump-  
turbine capacity, product of power and energy  
prices, power demand, cost of pumping  
– cumulative revenue from energy production  
[own study]



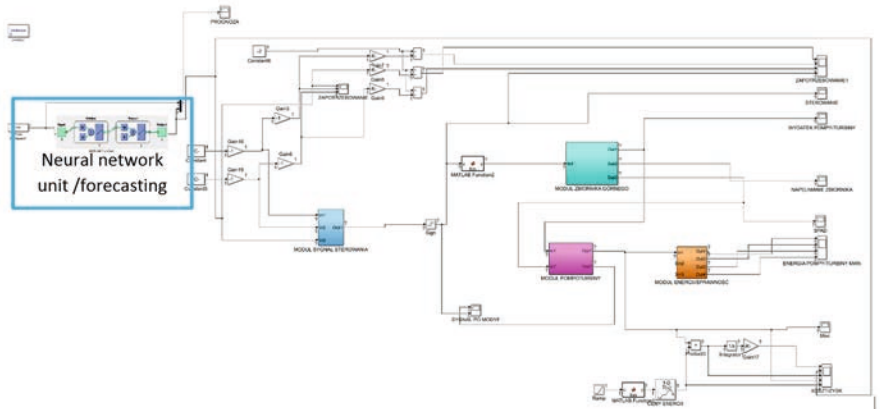
## 8. Forecasting module

The prediction model for power demand values is based on a simple feed-forward neural network (Twaróg, 2006). The results of learning, validation and testing are presented in the form of a regression coefficient in all cases at a level above the value of 0.99. The complete correspondence of the forecast value with the model value is confirmed by the position of points on a straight line with a directional coefficient of 1. In this example, the forecast of power demand is built one hour ahead. In the next step, the values are updated and the forecast cycle repeats. This example uses a two-layer feed-forward network with three sigmoidal hidden neurons and linear output neurons. The operation of this network enables the arbitrary adjustment of multidimensional mapping problems if the data is consistent and the number of neurons in the hidden layer is sufficient. From the point of view of the inertia of the operation of Porąbka-Żar PSPS (the time of filling/emptying the upper reservoir), a suitable horizon is at least a six-hour forecast time. There is no problem to expand the forecasting model based on the NARX network. It is a nonlinear autoregressive network with exogenous inputs (NARX) and it is a recursive dynamic network with feedback connections closing several layers of the network. The NARX model is based on the linear ARX model, which is commonly used in the modelling of time series.



**Fig. 17.** The obtained regression parameters after the learning process of the neural network for the forecast of energy demand [own study]

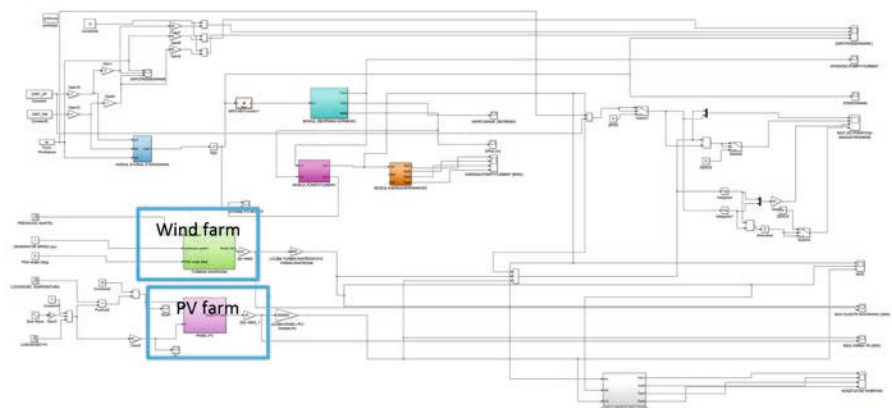




**Fig. 18.** Porąbka-Żar PSPS model with a forecasting module built on neural networks [own study]

## 9. Opportunities for support with hybrid models

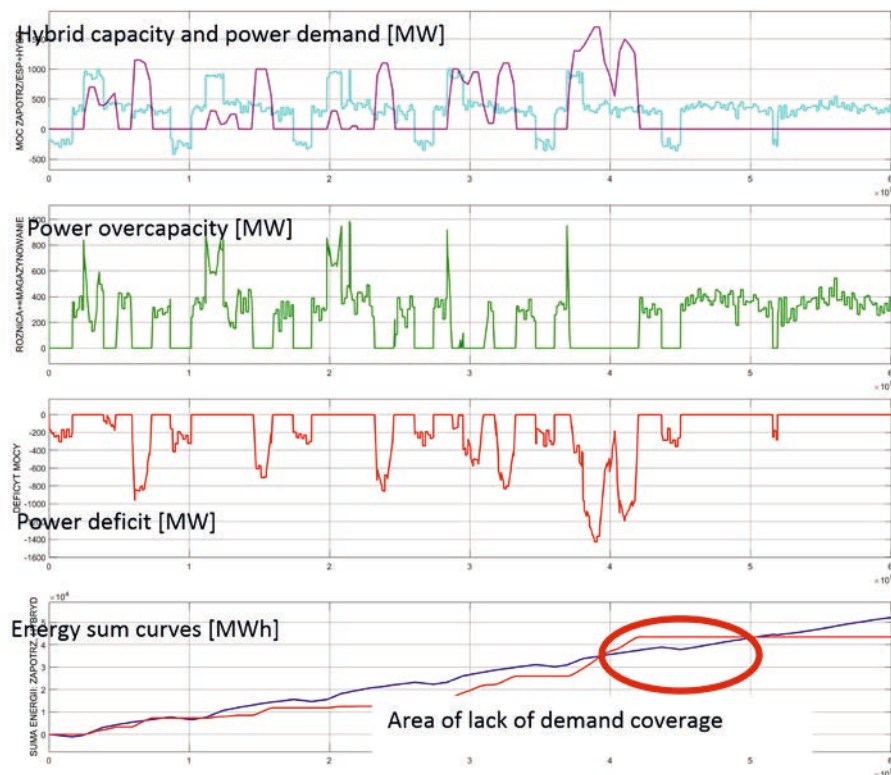
The energy production of the Porabka-Żar PSPS proposed in this example has been extended to include a hybrid system built from a PV farm (average power of 40MW) and a wind farm (average power of 300 MW). The random values of solar radiation, temperature and wind speed correspond to the actual random parameters occurring in the area of Mt. Zar.



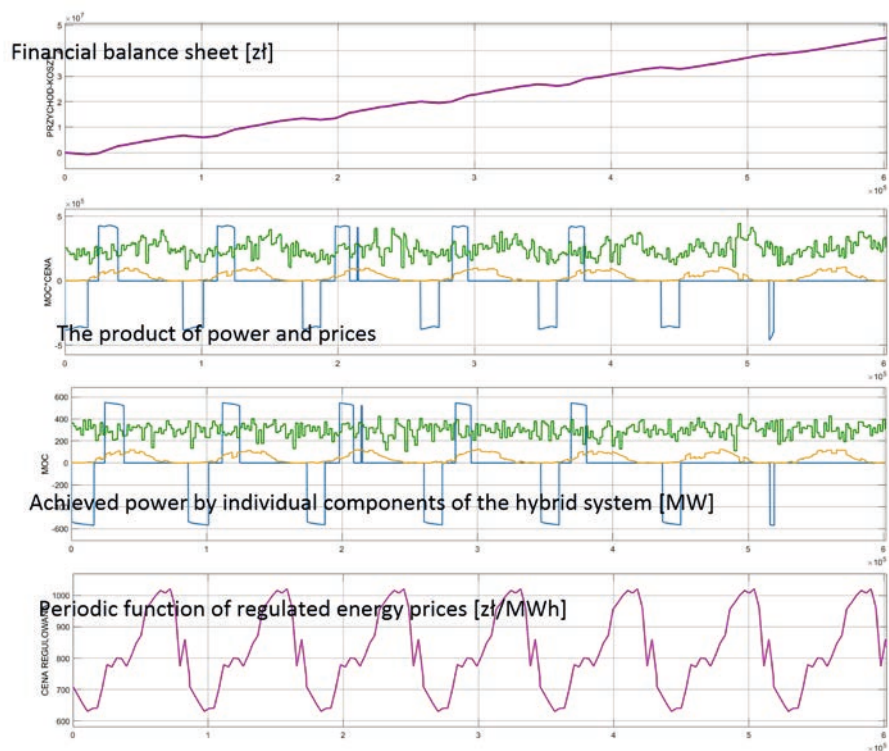
**Fig. 19.** Hybrid model: Porąbka-Żar PSPS + wind farm+ PV farm [own study]

The graphs above shows the energy balance for the example adopted parameters of the hybrid system (Beibei at all, 2019). The first graph shows the current power achieved by the hybrid system against the power demand above the assumed upper criterion (23 GW), which defines the moment of switching on and off of the turbines of Porąbka-Żar PSPS. Chart two shows the current overproduction of power, which through storage could be used in the future. Chart three shows the current power deficit. The last graph presents the energy balance assuming a possible energy storage. Two integral curves present the energy produced and energy consumed which reveals the moment when the sum of stored energy is not able to cover the energy demand.

The results of the REVENUE-COST module presents the results of the financial balance calculated on the basis of the adopted regulated energy prices. Subsequently, the periodicity of regulated prices, the power obtained by the various components of the hybrid system, the product of power and prices, and the achieved PROFIT= REVENUE-COST calculated on the basis of the energy produced are presented.



**Fig. 20.** Example energy balance for the adopted farm configuration [own study]



**Fig. 21.** Hybrid system - regulated energy prices, Porąbka-Żar PSPS + wind farm + PV farm – cumulative revenue from energy production [own study]

## 10. Elements of optimisation

The modular design, the selected Matlab – Simulink interface and the availability of all physical quantities of the built mathematical model of the Porąbka Żar PSPS and the hybrid system presents an easy opportunity to optimise selected

parameters of the system with the assumed objective function (Beibei et al., 2019; Kaczmarek, 1984; Malinowski, 1995; Słota, 1983; Twaróg, 2009). The tasks can be static or dynamic. The former boils down to determining the values of decision variables and the latter to determining and solving the function (Kaczmarek, 1984; Malinowski, 1995; Słota, 1983; Twaróg, 2009). Tasks can be formulated with one or multiple decision variables (Kaczmarek, 1984; Malinowski, 1995; Słota, 1983). The objective function can be one-dimensional, or the task can be extended to poly-optimisation using, for example, the Pareto approach (Malinowski, 1995; Słota, 1983; Twaróg, 2009). The whole can be solved based on deterministic parameters or the problem can be extended by changing their nature to probabilistic (Twaróg, 2008; Twaróg, 2017; Twaróg, 2016). The presented approach provides a whole range of possibilities for defining and solving PSPS – Porąbka-Żar problems. The discussed power plant is not the only pumped storage power plant in the NPS with such flexible capabilities. In optimising the operation of the NPS, it is necessary to consider a task that takes into account the operation of all facilities. Modern computational techniques provide opportunities to take into account all facilities producing and consuming energy in the NPS, taking into account energy exchange points with external power systems. This approach makes it possible to build on/off schedules for all NPS participants.

Example possibilities for solving simpler optimisation tasks are defined below:

- ▶ determination of the value of the upper criterion and the lower criterion (decision variables) enables the determination of the times of switching on and off the pump-turbines and the mode of operation when maximising the value of Revenue - Cost in the assumed time horizon, and the known form of power demand;
- ▶ the above task can be extended to forecast the randomness of demand;
- ▶ the next step can be extended to intervention decisions described in terms of a random variable (Twaróg, 2006) and the expected value of profit/loss can be proposed as a criterion;
- ▶ it is possible to determine what energy parameters should have the components of the hybrid power system to cover the emerging power deficit in a certain time horizon, taking into account the random nature of the factors affecting the achievement of current power;
- ▶ in addition, an answer can be obtained about the size of the energy storage capacity.

## 11. New directions for PSPS development - mines

The concept is to revitalise abandoned mines as massive “batteries” that will help offset growing shares of intermittent power generation, addressing emerging sustainability issues. The role of large-scale, long-life energy storage is becoming an increasingly important priority, especially in the face of rapid electrification and volatile energy prices. The revitalisation of abandoned mines requires a long-term undertaking, which can involve much higher capital costs than batteries. Assuming pumped storage power plants have a capacity of 15 MW to 400 MW and an energy potential at one cycle of 30 MWh to 800 MWh, there is the potential for profitable investment.

## 12. Summary

This article presents a mathematical model of the operation of Porąbka-Żar PSPS. The model was developed in the Matlab – Simulink interface. A modular approach to the model and full access to all physical quantities present in the model gives a wide range for applications of the model from simulation tasks to optimisation. The wide-ranging capabilities of the model are presented.

New, redefined conditions for the operation of energy systems with events after 24.02 this year affect the need to re-evaluate and build new criteria affecting market, geopolitical and regulatory conditions. The “environmental” policy, which has its genesis in the federal inclinations of the EU and the imperial ambitions of some countries, imposes an obligation to increase security through independence from selective sources and diversification. From the axiom of diversification comes the need for energy storage. One possible alternative is pumped storage power plants. The article shows the possibilities of PSPS operation and strengthening their contribution in the NPS through the development of hybrid systems.

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