Regular Expression Quantifiers – at least m Occurrences

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Summary. This is the second article on regular expression quantifiers. [4] introduced the quantifiers m to n occurrences and optional occurrence. In the sequel, the quantifiers: at least m occurrences and positive closure (at least 1 occurrence) are introduced. Notation and terminology were taken from [8], several properties of regular expressions from [7].

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The notation and terminology used here are introduced in the following papers: [5], [1], [6], [2], [3], and [4].

1. PRELIMINARIES

For simplicity, we follow the rules: E, x denote sets, A, B, C denote subsets of E^ω , a, b denote elements of E^ω , and k, l, m, n denote natural numbers.

The following proposition is true

- (1) If $B \subseteq A^*$, then $(A^*) \cap B \subseteq A^*$ and $B \cap A^* \subseteq A^*$.

2. AT LEAST m OCCURRENCES

Let us consider E, A, n . The functor $A^{n,\cdots}$ yielding a subset of E^ω is defined as follows:

(Def. 1) $A^{n,\cdots} = \bigcup \{B : \bigvee_m (n \leq m \wedge B = A^m)\}$.

We now state a number of propositions:

- (2) $x \in A^{n,\dots}$ iff there exists m such that $n \leq m$ and $x \in A^m$.
- (3) If $n \leq m$, then $A^m \subseteq A^{n,\dots}$.
- (4) $A^{n,\dots} = \emptyset$ iff $n > 0$ and $A = \emptyset$.
- (5) If $m \leq n$, then $A^{n,\dots} \subseteq A^{m,\dots}$.
- (6) If $k \leq m$, then $A^{m,n} \subseteq A^{k,\dots}$.
- (7) If $m \leq n + 1$, then $A^{m,n} \cup (A^{(n+1),\dots}) = A^{m,\dots}$.
- (8) $A^n \cup (A^{(n+1),\dots}) = A^{n,\dots}$.
- (9) $A^{n,\dots} \subseteq A^*$.
- (10) $\langle \rangle_E \in A^{n,\dots}$ iff $n = 0$ or $\langle \rangle_E \in A$.
- (11) $A^{n,\dots} = A^*$ iff $\langle \rangle_E \in A$ or $n = 0$.
- (12) $A^* = A^{0,n} \cup (A^{(n+1),\dots})$.
- (13) If $A \subseteq B$, then $A^{n,\dots} \subseteq B^{n,\dots}$.
- (14) If $x \in A$ and $x \neq \langle \rangle_E$, then $A^{n,\dots} \neq \{\langle \rangle_E\}$.
- (15) $A^{n,\dots} = \{\langle \rangle_E\}$ iff $A = \{\langle \rangle_E\}$ or $n = 0$ and $A = \emptyset$.
- (16) $A^{(n+1),\dots} = (A^{n,\dots}) \cap A$.
- (17) $(A^{m,\dots}) \cap A^* = A^{m,\dots}$.
- (18) $(A^{m,\dots}) \cap (A^{n,\dots}) = A^{(m+n),\dots}$.
- (19) If $n > 0$, then $(A^{m,\dots})^n = A^{m \cdot n,\dots}$.
- (20) $(A^{n,\dots})^* = (A^{n,\dots})?$.
- (21) If $A \subseteq C^{m,\dots}$ and $B \subseteq C^{n,\dots}$, then $A \cap B \subseteq C^{(m+n),\dots}$.
- (22) $A^{(n+k),\dots} = (A^{n,\dots}) \cap A^k$.
- (23) $A \cap (A^{n,\dots}) = (A^{n,\dots}) \cap A$.
- (24) $(A^k) \cap (A^{n,\dots}) = (A^{n,\dots}) \cap A^k$.
- (25) $(A^{k,l}) \cap (A^{n,\dots}) = (A^{n,\dots}) \cap A^{k,l}$.
- (26) If $\langle \rangle_E \in B$, then $A \subseteq A \cap (B^{n,\dots})$ and $A \subseteq (B^{n,\dots}) \cap A$.
- (27) $(A^{m,\dots}) \cap (A^{n,\dots}) = (A^{n,\dots}) \cap (A^{m,\dots})$.
- (28) If $A \subseteq B^{k,\dots}$ and $n > 0$, then $A^n \subseteq B^{k,\dots}$.
- (29) If $A \subseteq B^{k,\dots}$ and $n > 0$, then $A^{n,\dots} \subseteq B^{k,\dots}$.
- (30) $(A^*) \cap A = A^{1,\dots}$.
- (31) $(A^*) \cap A^k = A^{k,\dots}$.
- (32) $(A^{m,\dots}) \cap A^* = (A^*) \cap (A^{m,\dots})$.
- (33) If $k \leq l$, then $(A^{n,\dots}) \cap A^{k,l} = A^{(n+k),\dots}$.
- (34) If $k \leq l$, then $(A^*) \cap A^{k,l} = A^{k,\dots}$.
- (35) $A^{mn,\dots} \subseteq A^{m \cdot n,\dots}$.
- (36) $A^{mn,\dots} \subseteq (A^{n,\dots})^m$.
- (37) If $a \in C^{m,\dots}$ and $b \in C^{n,\dots}$, then $a \cap b \in C^{(m+n),\dots}$.
- (38) If $A^{k,\dots} = \{x\}$, then $x = \langle \rangle_E$.

- (39) If $A \subseteq B^*$, then $A^{n,\dots} \subseteq B^*$.
- (40) $A? \subseteq A^{k,\dots}$ iff $k = 0$ or $\langle \rangle_E \in A$.
- (41) $(A^{k,\dots}) \cap A? = A^{k,\dots}$.
- (42) $(A^{k,\dots}) \cap A? = A? \cap (A^{k,\dots})$.
- (43) If $B \subseteq A^*$, then $(A^{k,\dots}) \cap B \subseteq A^{k,\dots}$ and $B \cap (A^{k,\dots}) \subseteq A^{k,\dots}$.
- (44) $A \cap B^{k,\dots} \subseteq (A^{k,\dots}) \cap (B^{k,\dots})$.
- (45) $(A^{k,\dots}) \cup (B^{k,\dots}) \subseteq (A \cup B)^{k,\dots}$.
- (46) $\langle x \rangle \in A^{k,\dots}$ iff $\langle x \rangle \in A$ but $\langle \rangle_E \in A$ or $k \leq 1$.
- (47) If $A \subseteq B^{k,\dots}$, then $B^{k,\dots} = (B \cup A)^{k,\dots}$.

3. POSITIVE CLOSURE

Let us consider E, A . The functor A^+ yielding a subset of E^ω is defined as follows:

(Def. 2) $A^+ = \bigcup \{B : \bigvee_n (n > 0 \wedge B = A^n)\}$.

Next we state a number of propositions:

- (48) $x \in A^+$ iff there exists n such that $n > 0$ and $x \in A^n$.
- (49) If $n > 0$, then $A^n \subseteq A^+$.
- (50) $A^+ = A^{1,\dots}$.
- (51) $A^+ = \emptyset$ iff $A = \emptyset$.
- (52) $A^+ = (A^*) \cap A$.
- (53) $A^* = \{\langle \rangle_E\} \cup A^+$.
- (54) $A^+ = A^{1,n} \cup (A^{(n+1),\dots})$.
- (55) $A^+ \subseteq A^*$.
- (56) $\langle \rangle_E \in A^+$ iff $\langle \rangle_E \in A$.
- (57) $A^+ = A^*$ iff $\langle \rangle_E \in A$.
- (58) If $A \subseteq B$, then $A^+ \subseteq B^+$.
- (59) $A \subseteq A^+$.
- (60) $A^{*+} = A^*$ and $A^{++} = A^*$.
- (61) If $A \subseteq B^*$, then $A^+ \subseteq B^*$.
- (62) $A^{++} = A^+$.
- (63) If $x \in A$ and $x \neq \langle \rangle_E$, then $A^+ \neq \{\langle \rangle_E\}$.
- (64) $A^+ = \{\langle \rangle_E\}$ iff $A = \{\langle \rangle_E\}$.
- (65) $A^+? = A^*$ and $A^{+?} = A^*$.
- (66) If $a, b \in C^+$, then $a \cap b \in C^+$.
- (67) If $A \subseteq C^+$ and $B \subseteq C^+$, then $A \cap B \subseteq C^+$.
- (68) $A \cap A \subseteq A^+$.

- (69) If $A^+ = \{x\}$, then $x = \langle \rangle_E$.
- (70) $A \cap A^+ = A^+ \cap A$.
- (71) $(A^k) \cap A^+ = A^+ \cap A^k$.
- (72) $(A^{m,n}) \cap A^+ = A^+ \cap A^{m,n}$.
- (73) If $\langle \rangle_E \in B$, then $A \subseteq A \cap B^+$ and $A \subseteq B^+ \cap A$.
- (74) $A^+ \cap A^+ = A^{2,\dots}$.
- (75) $A^+ \cap A^k = A^{(k+1),\dots}$.
- (76) $A^+ \cap A = A^{2,\dots}$.
- (77) If $k \leq l$, then $A^+ \cap A^{k,l} = A^{(k+1),\dots}$.
- (78) If $A \subseteq B^+$ and $n > 0$, then $A^n \subseteq B^+$.
- (79) $A^+ \cap A? = A? \cap A^+$.
- (80) $A^+ \cap A? = A^+$.
- (81) $A? \subseteq A^+$ iff $\langle \rangle_E \in A$.
- (82) If $A \subseteq B^+$, then $A^+ \subseteq B^+$.
- (83) If $A \subseteq B^+$, then $B^+ = (B \cup A)^+$.
- (84) If $n > 0$, then $A^{n,\dots} \subseteq A^+$.
- (85) If $m > 0$, then $A^{m,n} \subseteq A^+$.
- (86) $(A^*) \cap A^+ = A^+ \cap A^*$.
- (87) $A^{+k} \subseteq A^{k,\dots}$.
- (88) $A^{+m,n} \subseteq A^{m,\dots}$.
- (89) If $A \subseteq B^+$ and $n > 0$, then $A^{n,\dots} \subseteq B^+$.
- (90) $A^+ \cap (A^{k,\dots}) = A^{(k+1),\dots}$.
- (91) $A^+ \cap (A^{k,\dots}) = (A^{k,\dots}) \cap A^+$.
- (92) $A^+ \cap A^* = A^+$.
- (93) If $B \subseteq A^*$, then $A^+ \cap B \subseteq A^+$ and $B \cap A^+ \subseteq A^+$.
- (94) $(A \cap B)^+ \subseteq A^+ \cap B^+$.
- (95) $A^+ \cup B^+ \subseteq (A \cup B)^+$.
- (96) $\langle x \rangle \in A^+$ iff $\langle x \rangle \in A$.

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