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MULTI-OPTIONAL HYBRID FUNCTIONS ENTROPY DOCTRINE ADVANTAGES FOR A STATE MAXIMAL PROBABILITY DETERMINATION

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Abstract

The presented paper considers a comparison of the traditional methods for the state maximal probability determination to the proposed hybrid probabilistic and variational concept. It is shown the advantages of the described multi-optional hybrid-effectiveness functions uncertainty measure conditional optimization doctrine in the sense of avoiding the traditional ways analytical complicatedness concerning the maximal probability of the possible state determination. The results of the numerical example are presented.

Keywords: entropy, dynamic state, risk, Markovian random process.

INTRODUCTION

Reliability of engineering depends a lot upon the system of maintenance and maintainability [1]. Entropy approaches evolution [2]-[6] allows taking into account the uncertainty factors acting during operation, maintenance, and repair of any sorts of engineering objects, for example, aircraft [7]. Theoretically, the individual choices and expected utilities [8], [9] impacting such processes have their own uncertainties too. That is why the entropy research number has a significantly increasing rate lately [10]. Mathematically analyzing the structure of economics models [11], it can be noticed that reliability, maintainability, and risk issues [12] are indispensably connected with both economic levers [11] and engineering objects (such as aircraft engines and powerplants [13], for instance) technical operation. The complexity of the decision making process required axiomatic statements for obtaining the valuable numerical results [14], [15], however, the subjective analysis entropy maximum principle [6] made it possible to elaborate the same results in a theorem style based upon the a-priory optimization postulate. Such approach in combination with the entropy paradigm has already helped in, and will instigate, solving a lot of applicable problems [16]-[36].

The presented paper has an objective to demonstrate the multi-optional hybrid functions entropy doctrine advantages for a state maximal probability determination. One of the possible applications it is the aircraft maintenance optimal periodicity [37]. The comparison of the traditional mass service theory methods [38], [39] and the proposed multi-optional hybrid functions entropy doctrine one is going to be realized with the use of the nomenclature mathematics means [40].

PROBLEM STATEMENT

For example, considering a Markovian random process with discrete states and continuous time, for a general case with three states we have a graph shown in Fig. 1 [39].

Here, in Fig. 1. "0" designates the up state of the system; "1" − damage; "2" − failure. The corresponding values of the failure rates λ_{ij} and restoration rates μ_{ji} will determine the process going on in the system. For the substantiated reasons, for the state of "2" to be a state without an "exit", it has to be satisfied the conditions of $\mu_{20} = \mu_{21} = 0$. Then, it, the state of "2", will be a real failure.

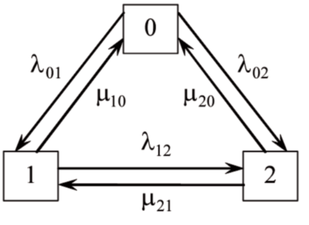

Fig. 1. Graph of three states of an aircraft functional system

The corresponding, to the graph of Fig. 1, system of ordinary linear differential equations of the first order by Erlang will have the view of [39]:

$$\left.\begin{aligned}\frac{dP_0}{dt} &= -(\lambda_{01}+\lambda_{02})P_0 + \mu_{10}P_1 + \mu_{20}P_2;\\ \frac{dP_1}{dt} &= \lambda_{01}P_0 - (\lambda_{12}+\mu_{10})P_1 + \mu_{21}P_2;\\ \frac{dP_2}{dt} &= \lambda_{02}P_0 + \lambda_{12}P_1 - (\mu_{20}+\mu_{21})P_2.\end{aligned}\right\} \tag{1}$$

Here, in the system of equations (1), P_0, P_1 and P_2 − probabilities of the corresponding states (see Fig. 1); t − time. The task is to find the maximal value for P_1, or optionally for P_2.

TRADITIONAL WAYS OF DETERMINATION

One way is through the characteristic equation. In accordance with [40, Chapter XIII, § 30, pp. 108-113], the *characteristic equation* for system (1) will be similarly (likewise) [40, Chapter XIII, § 30, p. 109, (5)]:

$$\begin{vmatrix} -(\lambda_{01}+\lambda_{02})-k & \mu_{10} & \mu_{20} \\ \lambda_{01} & -(\lambda_{12}+\mu_{10})-k & \mu_{21} \\ \lambda_{02} & \lambda_{12} & -(\mu_{20}+\mu_{21})-k \end{vmatrix} = 0. \tag{2}$$

Determinant (2) yields:

$$\begin{gathered}\left[-(\lambda_{01}+\lambda_{02})-k\right]\left[-(\lambda_{12}+\mu_{10})-k\right]\left[-(\mu_{20}+\mu_{21})-k\right]+\lambda_{01}\lambda_{12}\mu_{20}+\\ +\lambda_{02}\mu_{10}\mu_{21}-\left\{\lambda_{02}\left[-(\lambda_{12}+\mu_{10})-k\right]\mu_{20}\right\}-\left\{\lambda_{01}\mu_{10}\left[-(\mu_{20}+\mu_{21})-k\right]\right\}-\\ -\left\{\left[-(\lambda_{01}+\lambda_{02})-k\right]\lambda_{12}\mu_{21}\right\}=0.\end{gathered} \tag{3}$$

From (3) it can be found the roots of $k_{1,2,3}$. For each root k_i of Eq. (2), (3), namely k_1, k_2, k_3 we will write down the system of linear uniform (homogenous) algebraic equations with respect to their coefficients $\alpha_1^{(i)}$, $\alpha_2^{(i)}$, $\alpha_3^{(i)}$, [40, Chapter XIII, § 30, p. 108, (3)]. The system derives from an assumption of a partial solution existence in the view of [40, Chapter XIII, § 30, p. 108, (2)] for the system of Eq. (1). Since having three roots in the stated problem setting, we obtain, [40, Chapter XIII, § 30, p. 109], the solution of the system of Eq. (1).

The other method of the system of Eq. (1) solution is represented with the *Laplace transformations* in the *operational calculus* [40, Chapter XIX, pp. 400-432].
The system of Eq. (1) is transformed with [40, Chapter XIX, § 1, p. 401, (4)]:

$$F(p)=\int_{0}^{+\infty} e^{-pt} f(t)\,dt, \tag{4}$$

where p – complex parameter (variable) of the Laplace transformation.

The function $F(p)$ is called the *Laplace transformant* (*image*) of the function $f(t)$, which is called the *initial function*, or *original*. The indication is [40, Chapter XIX, § 1, pp. 401, 402, (7)]:

$$L\{f(t)\}=F(p). \tag{5}$$

In accordance with the theorem for transformants of derivatives [40, Chapter XIX, § 8, P. 409, (27)], the system of Eq. (1), taking into account the initial conditions of the problem, that is for the probabilities of the system's possible states: $P_0|_{t=t_0}=1$, $P_1|_{t=t_0}=P_2|_{t=t_0}=0$, $t_0=0$, will have the corresponding algebraic system.

$$\left.\begin{aligned}
&\left(pF_0(p)-\left[P_0|_{t_0=0}=1\right]=L\left\{\frac{dP_0}{dt}\right\}\right)=-(\lambda_{01}+\lambda_{02})\left[F_0(p)=L\{P_0\}\right]+\\
&+\mu_{10}\left[F_1(p)=L\{P_1\}\right]+\mu_{20}\left[F_2(p)=L\{P_2\}\right];\\
&\left(pF_1(p)-\left[P_1|_{t_0=0}=0\right]=L\left\{\frac{dP_1}{dt}\right\}\right)=\lambda_{01}\left[F_0(p)=L\{P_0\}\right]-\\
&-(\lambda_{12}+\mu_{10})\left[F_1(p)=L\{P_1\}\right]+\mu_{21}\left[F_2(p)=L\{P_2\}\right];\\
&\left(pF_2(p)-\left[P_2|_{t_0=0}=0\right]=L\left\{\frac{dP_2}{dt}\right\}\right)=\lambda_{02}\left[F_0(p)=L\{P_0\}\right]+\\
&+\lambda_{12}\left[F_1(p)=L\{P_1\}\right]-(\mu_{20}+\mu_{21})\left[F_2(p)=L\{P_2\}\right].
\end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned}
&pF_0(p)-1=-(\lambda_{01}+\lambda_{02})F_0(p)+\mu_{10}F_1(p)+\mu_{20}F_2(p);\\
&pF_1(p)-0=\lambda_{01}F_0(p)-(\lambda_{12}+\mu_{10})F_1(p)+\mu_{21}F_2(p);\\
&pF_2(p)-0=\lambda_{02}F_0(p)+\lambda_{12}F_1(p)-(\mu_{20}+\mu_{21})F_2(p).
\end{aligned}\right\} \tag{7}$$

The obtained algebraic equations system, Eq. (7), solving is possible in different ways. One of them is a matrix-vector.

Let us rewrite the system of Eq. (7) in the following style:

$$\left.\begin{array}{llll} \left[p+\left(\lambda_{01}+\lambda_{02}\right)\right]F_0 & -\mu_{10}F_1 & -\mu_{20}F_2 & =1; \\ -\lambda_{01}F_0 & +\left[p+\left(\lambda_{12}+\mu_{10}\right)\right]F_1 & -\mu_{21}F_2 & =0; \\ -\lambda_{02}F_0 & -\lambda_{12}F_1 & +\left[p+\left(\mu_{20}+\mu_{21}\right)\right]F_2 & =0. \end{array}\right\} \tag{8}$$

The matrix for the transformation of the system of Eq. (8) will be [40, Chapter XXI, § 1, p. 510, (5)]:

$$\mathbf{M}=\begin{Vmatrix} p+\left(\lambda_{01}+\lambda_{02}\right) & -\mu_{10} & -\mu_{20} \\ -\lambda_{01} & p+\left(\lambda_{12}+\mu_{10}\right) & -\mu_{21} \\ -\lambda_{02} & -\lambda_{12} & p+\left(\mu_{20}+\mu_{21}\right) \end{Vmatrix}. \tag{9}$$

The needed (unknown/wanted/sought) vector-column of transformants is:

$$\mathbf{F}=\begin{Vmatrix} F_0 \\ F_1 \\ F_2 \end{Vmatrix}. \tag{10}$$

Then the transformation of the system of Eq. (8) is [40, Chapter XXI, § 8, p. 522, (5)]:

$$\begin{Vmatrix} p+\left(\lambda_{01}+\lambda_{02}\right) & -\mu_{10} & -\mu_{20} \\ -\lambda_{01} & p+\left(\lambda_{12}+\mu_{10}\right) & -\mu_{21} \\ -\lambda_{02} & -\lambda_{12} & p+\left(\mu_{20}+\mu_{21}\right) \end{Vmatrix} \cdot \begin{Vmatrix} F_0 \\ F_1 \\ F_2 \end{Vmatrix} = \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}. \tag{11}$$

Or, to make it shorter it is [40, Chapter XXI, § 8, p. 523, (6)]:

$$\mathbf{M}\cdot\mathbf{F}=\mathbf{B}, \tag{12}$$

where $\mathbf{B}$ – vector-column of free members of the system of Eq. (8):

$$\mathbf{B}=\begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}. \tag{13}$$

The required solution of Eq. (10) will be [40, Chapter XXI, § 9, p. 523, (2)] found with the use of the inverse matrix $\mathbf{M}^{-1}$:

$$\mathbf{F}=\mathbf{M}^{-1}\cdot\mathbf{B}. \tag{14}$$

The last equation (14) with taking into account [40, Chapter XXI, § 7, p. 521, (5)]:

$$\mathbf{M}^{-1}=\frac{1}{\Delta(\mathbf{M})}\cdot\tilde{\mathbf{M}}, \tag{15}$$

where: $\Delta(\mathbf{M})$ – determinant of matrix $\mathbf{M}$, Eq. (14), [40, Chapter XXI, § 7, p. 520, (2)], compare the determinants of the matrix $\mathbf{M}$, Eq. (14) with the one of the Eq. (7); $\tilde{\mathbf{M}}$ – adjacent matrix to matrix $\mathbf{M}$, [40, Chapter XXI, § 7, p. 521, (4)]; can be written as [40, Chapter XXI, § 9, p. 523, (3)]:

$$\mathbf{F} = \frac{1}{\Delta(\mathbf{M})} \cdot \tilde{\mathbf{M}} \cdot \mathbf{B}, \tag{16}$$

or, in the developed view [40, Chapter XXI, § 9, p. 523, (4)]:

$$\begin{Vmatrix} F_0 \\ F_1 \\ F_2 \end{Vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{Vmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}, \tag{17}$$

where M_{ij} – algebraic addition of the element of m_{ij}, [40, Chapter XXI, § 2, p. 512], of the initial matrix Eq. (9).

Fulfilling multiplying the matrixes in the right hand part of Eq. (17) we will obtain [40, Chapter XXI, § 9, p. 523, (5)]:

$$\begin{Vmatrix} F_0 \\ F_1 \\ F_2 \end{Vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{Vmatrix} M_{11} \cdot 1 + M_{21} \cdot 0 + M_{31} \cdot 0 \\ M_{12} \cdot 1 + M_{22} \cdot 0 + M_{32} \cdot 0 \\ M_{13} \cdot 1 + M_{23} \cdot 0 + M_{33} \cdot 0 \end{Vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{Vmatrix} M_{11} \\ M_{12} \\ M_{13} \end{Vmatrix}. \tag{18}$$

In accordance with the initial matrix Eq. (9).

$$\begin{gathered} M_{11} = (-1)^{1+1} \cdot \left\{ \left[p + (\lambda_{12} + \mu_{10}) \right] \left[p + (\mu_{20} + \mu_{21}) \right] - \lambda_{12}\mu_{21} \right\} = \\ = p^2 + p(\mu_{20} + \mu_{21}) + (\lambda_{12} + \mu_{10}) p + (\lambda_{12} + \mu_{10})(\mu_{20} + \mu_{21}) - \lambda_{12}\mu_{21} = \\ = p^2 + p(\mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10}) + \lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}. \end{gathered} \tag{19}$$

$$\begin{gathered} M_{12} = (-1)^{1+2} \cdot \left\{ -\lambda_{01} \left[p + (\mu_{20} + \mu_{21}) \right] - \lambda_{02}\mu_{21} \right\} = \\ = p\lambda_{01} + \lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}. \end{gathered} \tag{20}$$

$$\begin{gathered} M_{13} = (-1)^{1+3} \cdot \left\{ \lambda_{01}\lambda_{12} + \lambda_{02} \left[p + (\lambda_{12} + \mu_{10}) \right] \right\} = \\ = p\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10}. \end{gathered} \tag{21}$$

$$\begin{gathered} \Delta = \Delta(\mathbf{M}) = \left[p + (\lambda_{01} + \lambda_{02}) \right] \left[p + (\lambda_{12} + \mu_{10}) \right] \left[p + (\mu_{20} + \mu_{21}) \right] + \\ + (-\lambda_{01})(-\lambda_{12})(-\mu_{20}) + (-\lambda_{02})(-\mu_{10})(-\mu_{21}) - \\ - (-\lambda_{02})(-\mu_{20}) \left[p + (\lambda_{12} + \mu_{10}) \right] - (-\lambda_{12})(-\mu_{21}) \left[p + (\lambda_{01} + \lambda_{02}) \right] - \\ - (-\lambda_{01})(-\mu_{10}) \left[p + (\mu_{20} + \mu_{21}) \right]. \end{gathered} \tag{22}$$

$$\begin{gathered} \Delta = p \left\{ p^2 + p(\mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} + \lambda_{01} + \lambda_{02}) + \right. \\ + \left[(\lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}) + (\lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}) + \right. \\ \left. \left. + (\lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10}) \right] \right\}. \end{gathered} \tag{23}$$

Compare the last determinants of the Eq. (22) and (23) with the determinants of the Eq. (3).

Now, applying the matrix-vector approach of Eq. (8)-(23), in accordance with Eq. (14)-(18), it yields for the transformant of F_0 the following expression:

$$F_0 = \frac{M_{11}}{\Delta(\mathbf{M})} =$$

$$= \frac{p^2 + p(\mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10}) + \lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}}{\left(\begin{gathered} p\{p^2 + p(\mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} + \lambda_{01} + \lambda_{02}) + \\ + [(\lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}) + (\lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}) + \\ + (\lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10})]\} \end{gathered}\right)}. \tag{24}$$

Let us designate for members in Eq. (19)-(21) and Eq. (22)-(24).

$$a_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10}, \quad b_1 = \lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}. \tag{25}$$

$$c_1 = \lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}. \tag{26}$$

$$d_1 = \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10}. \tag{27}$$

$$e_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} + \lambda_{01} + \lambda_{02}. \tag{28}$$

Then Eq. (24) will get the view of:

$$F_0 = \frac{M_{11}}{\Delta(\mathbf{M})} = \frac{p^2 + pa_1 + b_1}{p(p^2 + pe_1 + b_1 + c_1 + d_1)}. \tag{29}$$

The other two transformants of F_1 and F_2 will be, correspondingly with the expressions of Eq. (14)-(18) and Eq. (20)-(23) and notations of Eq. (25)-(28), written down as follows:

$$F_1 = \frac{M_{12}}{\Delta(\mathbf{M})} = \frac{p\lambda_{01} + \lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}}{p(p^2 + pe_1 + b_1 + c_1 + d_1)}. \tag{30}$$

$$F_2 = \frac{M_{13}}{\Delta(\mathbf{M})} = \frac{p\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10}}{p(p^2 + pe_1 + b_1 + c_1 + d_1)}. \tag{31}$$

Applying the same designations as of the expressions of (25)-(28) to the nominators of the fractions of Eq. (30) and (31) we obtain:

$$F_1 = \frac{p\lambda_{01} + c_1}{p(p^2 + pe_1 + b_1 + c_1 + d_1)}. \tag{32}$$

$$F_2 = \frac{p\lambda_{02} + d_1}{p(p^2 + pe_1 + b_1 + c_1 + d_1)}. \tag{33}$$

Finally, the ratios for the transformants of the expressions of (29)-(33) represented in the view of the simplest fractions (elementary ratios) will be found with taking into account the k_1, k_2 and k_3:

$$k_3 = 0, \tag{34}$$

$$k_1 = \frac{-e_1 + \sqrt{e_1^2 - 4 f_1 g_1}}{2 f_1}, \qquad k_2 = \frac{-e_1 - \sqrt{e_1^2 - 4 f_1 g_1}}{2 f_1}, \tag{35}$$

where $f_1 = 1$ and $g_1 = b_1 + c_1 + d_1$ correspondingly with the denominators of Eq. (24)-(33), i.e. of the determinant Eq. (22) or Eq. (23); – corresponding roots, here; and with corresponding coefficients of the decomposition.

At last for the image (transformant):

$$F_0 = \frac{\dfrac{b_1}{k_1 k_2}}{p} + \frac{1 - \dfrac{k_2 + a_1 + \dfrac{b_1}{k_2}}{k_2 - k_1} - \dfrac{b_1}{k_1 k_2}}{\left(p - k_1\right)} + \frac{\dfrac{k_2 + a_1 + \dfrac{b_1}{k_2}}{k_2 - k_1}}{\left(p - k_2\right)}. \tag{36}$$

And for the probability (original) of Eq. (1):

$$P_0\left(t\right) = \frac{b_1}{k_1 k_2} + \left(1 - \frac{k_2 + a_1 + \dfrac{b_1}{k_2}}{k_2 - k_1} - \frac{b_1}{k_1 k_2} \right) e^{k_1 t} + \left(\frac{k_2 + a_1 + \dfrac{b_1}{k_2}}{k_2 - k_1} \right) e^{k_2 t}. \tag{37}$$

Or on the other hand:

$$P_0\left(t\right) = \frac{k_1 e^{k_1 t} - k_2 e^{k_2 t}}{k_1 - k_2} + a_1 \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} +$$

$$+ \frac{b_1}{k_1 k_2} + \left(- \frac{b_1}{k_2\left(k_2 - k_1\right)} - \frac{b_1}{k_1 k_2} \right) e^{k_1 t} + \left(\frac{b_1}{k_2\left(k_2 - k_1\right)} \right) e^{k_2 t}. \tag{38}$$

$$P_1\left(t\right) = P_1^{(1)}\left(t\right) + P_1^{(2)}\left(t\right) = \lambda_{01} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} +$$

$$+ \frac{c_1}{k_1 k_2} + \left(- \frac{c_1}{k_2\left(k_2 - k_1\right)} - \frac{c_1}{k_1 k_2} \right) e^{k_1 t} + \left(\frac{c_1}{k_2\left(k_2 - k_1\right)} \right) e^{k_2 t}. \tag{39}$$

$$P_2\left(t\right) = P_2^{(1)}\left(t\right) + P_2^{(2)}\left(t\right) = \lambda_{02} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} +$$

$$+ \frac{d_1}{k_1 k_2} + \left(- \frac{d_1}{k_2\left(k_2 - k_1\right)} - \frac{d_1}{k_1 k_2} \right) e^{k_1 t} + \left(\frac{d_1}{k_2\left(k_2 - k_1\right)} \right) e^{k_2 t}. \tag{40}$$

OPTIMAL (MAXIMAL PROBABILITY) VALUES

In accordance with the graph (see Fig. 1) one may state that the developed aircraft given functional system maintenance improvement influences the corresponding values of the failure rates λ_{ij} and restoration rates μ_{ji} determining the process going on in the system.

The problem might be, for instance, to choose an optimal maintenance periodicity for the aircraft given functional system. In its turn, it might be done likewise in example described in reference [37].

On condition that probability of the failure state "2": P_2 does not exceed (go beyond) the accepted limit (level), whereas (while) the up (normal operation conditions) state of the system designated as "0" probability: P_0 is not lower than the accepted level (limit), the corresponding maximum of the damage state P_1 can be considered as an optimum for the aircraft given functional system maintenance periodicity; that has been considered, discussed, and disputed in references [17, 20, 22, 25].

Let us consider the firs derivative of the probability of the damaged but not failure (ruined, crash, break, fracture, split, crack, rupture) state P_1, Eq. (39), with respect to time t:

$$\frac{dP_1(t)}{dt} = \frac{\lambda_{01}}{k_1 - k_2}\left(k_1 e^{k_1 t} - k_2 e^{k_2 t}\right) + k_1\left(-\frac{c_1}{k_2(k_2 - k_1)} - \frac{c_1}{k_1 k_2}\right)e^{k_1 t} + k_2\left(\frac{c_1}{k_2(k_2 - k_1)}\right)e^{k_2 t}. \tag{41}$$

$$\begin{gathered}\frac{dP_1(t)}{dt} = \frac{-k_1 k_2 \lambda_{01}}{k_1 k_2 (k_2 - k_1)}\left(k_1 e^{k_1 t} - k_2 e^{k_2 t}\right) + \\ + \left(-\frac{k_1 k_1 c_1}{k_1 k_2 (k_2 - k_1)} - \frac{k_1 c_1 (k_2 - k_1)}{k_1 k_2 (k_2 - k_1)}\right)e^{k_1 t} + \left(\frac{k_1 k_2 c_1}{k_1 k_2 (k_2 - k_1)}\right)e^{k_2 t}.\end{gathered} \tag{42}$$

Equalizing Eq. (42) to zero yields:

$$\begin{gathered}\frac{dP_1(t)}{dt} = \\ = \frac{-k_1 k_2 \lambda_{01}\left(k_1 e^{k_1 t} - k_2 e^{k_2 t}\right) - k_1 k_1 c_1 e^{k_1 t} - k_1 c_1 (k_2 - k_1) e^{k_1 t} + k_1 k_2 c_1 e^{k_2 t}}{k_1 k_2 (k_1 - k_2)} = 0.\end{gathered} \tag{43}$$

$$-\cancel{k_1} k_2 \lambda_{01}\left(k_1 e^{k_1 t} - k_2 e^{k_2 t}\right) - \cancel{k_1} k_1 c_1 e^{k_1 t} - \cancel{k_1} c_1 (k_2 - k_1) e^{k_1 t} + \cancel{k_1} k_2 c_1 e^{k_2 t} = 0. \tag{44}$$

$$-\left(\lambda_{01} k_2 k_1 e^{k_1 t} - \lambda_{01} k_2 k_2 e^{k_2 t}\right) - \cancel{k_1 c_1 e^{k_1 t}} - c_1 k_2 e^{k_1 t} + \cancel{c_1 k_1 e^{k_1 t}} + k_2 c_1 e^{k_2 t} = 0. \tag{45}$$

$$-\lambda_{01} \cancel{k_2} k_1 e^{k_1 t} + \lambda_{01} \cancel{k_2} k_2 e^{k_2 t} - c_1 \cancel{k_2} e^{k_1 t} + \cancel{k_2} c_1 e^{k_2 t} = 0. \tag{46}$$

$$\begin{gathered}-\lambda_{01} k_1 e^{k_1 t} + \lambda_{01} k_2 e^{k_2 t} - c_1 e^{k_1 t} + c_1 e^{k_2 t} = 0, \\ \left(\lambda_{01} k_2 + c_1\right) e^{k_2 t} = \left(\lambda_{01} k_1 + c_1\right) e^{k_1 t}.\end{gathered} \tag{47}$$

$$e^{(k_2 - k_1)t} = \frac{\lambda_{01} k_1 + c_1}{\lambda_{01} k_2 + c_1}, \qquad (k_2 - k_1)t = \ln\frac{\lambda_{01} k_1 + c_1}{\lambda_{01} k_2 + c_1}. \tag{48}$$

Thus, we have come to the optimal solution, maintenance periodicity, expressed with:

$$t_p^* = \frac{\ln\left(\lambda_{01} k_1 + c_1\right) - \ln\left(\lambda_{01} k_2 + c_1\right)}{k_2 - k_1}. \tag{49}$$

EXPERIMENTATIONS (TRIALS ETC.)

Now, it is suggested to conduct experimentations (tests, trials, mathematical modeling, computer simulation, numerical or calculation experiments [37] etc.) with the obtained above theoretical studies, mathematical derivations, and statistical search results.

In order to illustrate this point for the example considered (see Fig. 1), the mathematical modeling has been realized for such initial data for the probabilities: $P_0\big|_{t=t_0} = 1$, $P_1\big|_{t=t_0} = P_2\big|_{t=t_0} = 0$, $t_0 = 0$, and other values: $\lambda_{01} = 5 \cdot 10^{-3}$ h^{-1}; $\lambda_{02} = 2 \cdot 10^{-4}$ h^{-1}; $\lambda_{12} = 1 \cdot 10^{-3}$ h^{-1}; $\mu_{10} = 1 \cdot 10^{-4}$ h^{-1}; $\mu_{20} = 3 \cdot 10^{-5}$ h^{-1}; $\mu_{21} = 5 \cdot 10^{-5}$ h^{-1}; $t = 0 \ldots 1.5 \cdot 10^3$ h. $\mathrm{t_{opt}} \approx 393$ h is found with the expression of Eq. (49).

OPTIONAL METHOD

Proposed Hypotheses (Statements, Problems)

Herein it is suggested to formulate the own concept (idea, problem, hypotheses).

In such respect [1-40], the considered example may be given an attention to in regards with the *Multi-Optional Hybrid-Effectiveness Functions Uncertainty Measure Conditional Optimization Doctrine* (method, approach, concept) applicable (used, implemented) to the aeronautical engineering optimal maintenance periodicities determination [17, 20, 22, 25].

The optimal values of aeronautical engineering maintenance periodicities can be obtained not only in the entire probabilistic way, but also in a hybrid partially probabilistic partially optional way [17, 20, 22, 25].

The essence of the doctrine (method, idea, approach, concept) is to consider the process developing in the system from the position of some hybrid optional functions distribution optimality.

Consider the options essential to the system.

Objective functional, like proposed in references [17, 20, 22, 25], is as follows:

$$\Phi_h = -\sum_{i=1}^{3}\left[xF_1^{(i)}\right]\ln\left[xF_1^{(i)}\right] - \frac{t_p^*}{\lambda_{01}}\sum_{i=1}^{3}\left[xF_1^{(i)}\right]\left(M_{12}^{(i)}\right) + \gamma\left[\sum_{i=1}^{3}\left[xF_1^{(i)}\right] - 1\right],$$
$$F_1^{(i)} = \frac{M_{12}^{(i)}}{\Delta(\mathbf{M})} = \frac{k_i\lambda_{01} + c_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)}, \tag{50}$$
$$M_{12}^{(i)} = k_i\lambda_{01} + c_1, \qquad \Delta(\mathbf{M}) = p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right),$$

where x is an unknown parameter; $h_i = xF_1^{(i)}$ is the multi-optional hybrid functions depending upon the options effectiveness functions of $F_1^{(i)}$; t_p^*/λ_{01} is the intrinsic parameter of the system and the process, which is the ratio of the optimal (delivering the sought maximal value to the probability) time t_p^* of the maintenance periodicity, it is unknown yet for such problem formulation and the time of t_p^* is going to be determined as a solution, i.e. it is not the Eq. (49) so far, however it will be, that is why the indication is the same, to the flow intensity λ_{01}; $M_{12}^{(i)}$, Eq. (20), is the algebraic addition of the initial elementary intensities matrix **M**, Eq. (9), formed in the style likewise from the Erlang's system, Eq. (1), element of m_{12}; γ is the parameter, coefficient, function (uncertain Lagrange multiplier, weight coefficient) for the normalizing condition.

Consider an extremum existence necessary conditions for the objective functional of (50), [17, 20, 22, 25]:

$$\frac{\partial \Phi_h}{\partial h_i} = \frac{\partial \Phi_h}{\partial \left[x F_1^{(i)} \right]} = 0, \qquad \forall i \in \overline{1,3}. \tag{51}$$

$$\ln\left[x F_1^{(1)} \right] + \frac{t_p^*}{\lambda_{01}} \left(\lambda_{01} k_1 + c_1 \right) = \gamma - 1 = \ln\left[x F_1^{(2)} \right] + \frac{t_p^*}{\lambda_{01}} \left(\lambda_{01} k_2 + c_1 \right). \tag{52}$$

From where:

$$\ln\left[x F_1^{(1)} \right] + \frac{t_p^*}{\lambda_{01}} \left(\lambda_{01} k_1 + c_1 \right) = \ln\left[x F_1^{(2)} \right] + \frac{t_p^*}{\lambda_{01}} \left(\lambda_{01} k_2 + c_1 \right). \tag{53}$$

After that, we have got the *law of subjective conservatism* on one hand and on the other hand the similar to Eq. (48) expression:

$$\ln\left[x F_1^{(1)} \right] - \ln\left[x F_1^{(2)} \right] = \frac{t_p^*}{\lambda_{01}} \left[\left(\lambda_{01} k_2 + c_1 \right) - \left(\lambda_{01} k_1 + c_1 \right) \right]. \tag{54}$$

At last, we obtain:

$$\ln\left[x F_1^{(1)} \right] - \ln\left[x F_1^{(2)} \right] = t_p^* \left[\left(k_2 + \frac{c_1}{\lambda_{01}} \right) - \left(k_1 + \frac{c_1}{\lambda_{01}} \right) \right]. \tag{55}$$

After that likewise Eq. (48), (49):

$$t_p^* = \frac{\ln\left[F_1^{(1)}(\cdot) \right] - \ln\left[F_1^{(2)}(\cdot) \right]}{k_2(\cdot) - k_1(\cdot)}. \tag{56}$$

And finally equivalent with Eq. (49) with taking into account Eq. (30), (32) for the roots, i.e. the second, third, and fourth expressions of the Eq. (50):

$$t_p^* = \frac{\ln \dfrac{k_1 \lambda_{01} + c_1}{p\left(p^2 + p e_1 + b_1 + c_1 + d_1 \right)} - \ln \dfrac{k_2 \lambda_{01} + c_1}{p\left(p^2 + p e_1 + b_1 + c_1 + d_1 \right)}}{k_2(\cdot) - k_1(\cdot)}. \tag{57}$$

$$t_p^* = \frac{\ln\left(k_1 \lambda_{01} + c_1 \right) - \ln\left(k_2 \lambda_{01} + c_1 \right)}{k_2(\cdot) - k_1(\cdot)} \tag{58}$$

DISCUSSION

Thus, the result of Eq. (49) is obtained in absolutely not probabilistic rather in the *Multi-optional hybrid-effectiveness functions uncertainty measure conditional optimization doctrine way* [17, 20, 22, 25].

The same approach is applicable to $F_2^{(i)}$ with yielding the parallel to the Eq. (49) and (59) results.

Now we ought to say that for the situation when the probability of $P_2(t)$ undergoes the extremum instead of the probability of $P_1(t)$, the problem, due to the symmetry, has a symmetrical solution:

$$t_p^* = \frac{\ln\left(\lambda_{02}k_1 + d_1\right) - \ln\left(\lambda_{02}k_2 + d_1\right)}{k_2 - k_1}. \tag{59}$$

CONCLUSION

That is the system according to the developing stationary Poison flow process has the possible states optimal options related with either the system of parameters $\{k_i, \lambda_{02}, d_1\}$ or $\{k_i, \lambda_{01}, c_1\}$ values for the initial moment probability of the state "0" being equaled to "1". The proposed optional method discovers an important property of the process (its optimality); moreover, the optional method is more compact and applicable, for example, for the aeronautical engineering optimal maintenance periodicities determination.

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MULTIOPCJONALNA HYBRYDOWA FUNKCJA ENTROPII JAKO ZASADA DLA OKREŚLENIA MAKSYMALNEGO PRAWDOPODOBIEŃSTWA STANU SYSTEMU DYNAMICZNEGO

Abstrakt

W prezentowanym artykule rozważono porównanie tradycyjnych metod określania maksymalnego prawdopodobieństwa stanu systemu dynamicznego z zaproponowaną hybrydową koncepcją probabilistyczną i wariacyjną. Pokazano zalety opisanej wielo opcjonalnej funkcji hybrydowo-efektywnościowej niepewności pomiaru i zasadę optymalizacji warunkowej w tym sensie, że prezentowana koncepcja unika tradycyjnych metod analitycznych dotyczących maksymalnego prawdopodobieństwa możliwego określenia stanu układu dynamicznego. Niezawodność urządzeń technicznych w dużym stopniu zależy od stanu systemu utrzymania ruchu i konserwacji. Entropia określenia stanu pozwala na uwzględnienie czynników niepewności działających podczas eksploatacji, obsługi technicznej i naprawy wszelkiego rodzaju obiektów inżynieryjnych, na przykład statków powietrznych. W końcowej części przedstawiono wyniki przykładu numerycznego.

Słowa kluczowe: entropia, system dynamiczny, ryzyko, procesy Markowa.