

# Fingerprinting/indoor positioning using complex planar splines

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The rapid development of various LBS-based applications and services that operate on the basis of the user's current location, both global GPS and local LBS, today require the development of new and improved methods. This concerns, first of all, methods for determining the local location of LPS users in premises, if there is a high concentration of users and the presence of difficulties in the propagation of radio signals. The use of local methods of location determination based on the fingerprinting method is considered. It is shown that to improve the user positioning accuracy, it is expedient to use a combination of several methods. To determine the local location of the user, a method based on the finite element method and linear complex planar splines is proposed. The construction of linear complex planar splines is considered, their coefficients are found. Finding the error in determining the coordinates of the user's UE location is shown. The use of the proposed method will improve the accuracy of determining the coordinates of the user's location and will ensure the provision of LBS services and applications to users in the premises under various conditions of their provision.

**Key words:** position, wi-fi, RSSI, access point, power, complex planar spline

## 1 Introduction

The development in modern networks of services and LBS (location-based services) oriented applications based on the location of the subscriber is in high demand due to the growing popularity of services based on location and crowd sourcing. Today, LBS applications are actively used in the internet of things (IoT), which includes a wide range of technologies and IoT devices that have the ability to collect, manage and exchange data in monitoring services for employees, transport, prisoners, equipment in offices, industrial premises, logistics complexes, medical organizations, underground structures, parking lots. In mobile networks, UE (user equipment) subscriber devices are used to determine the user's location, geolocation, local search and targeted advertising. It is known that the user's location can be determined both in global GPS (global positioning system) and local LPS (local positioning system) coordinates. The undoubted advantage of GPS global positioning is a large coverage and high positioning accuracy, but only outdoors. On the contrary, local positioning systems LPS allow to determine the location accurately only in Indoor room. Mobile networks use positioning from the point of view of the location of the subscriber in the operator's cell when establishing a connection, but are not used for positioning, due to the high error of several hundred meters. However, there are still situations in which the required positioning accuracy cannot be ensured, for example, indoors, in parking lots, warehouses and work areas. The most commonly used equipment for positioning today is Wi-Fi

(IEEE 802.11n/ac/ad), ZigBee (IEEE 802.15.4) or Bluetooth (IEEE 802.15.1), RFID, UWB, *etc* [1-10].

It is known [1-5] that fingerprinting/indoor positioning is the most recognized and used method today, in comparison with methods based on triangulation and trilateration (AAA, TOA, TDOA, RSS) and proximity methods. The use of the AOA (angle of arrival) method, based on measuring the angle of arrival of a signal relative to the signal source, has a number of disadvantages associated with the need for additional antennas to measure angles. Also, AOA positioning accuracy is influenced by multipath propagation of signals and reflections from walls and other objects, including a decrease in positioning accuracy with increasing distance [4-5]. The TOA (time of arrival) and TDoA (time difference of arrival) methods, which use trilateration and measurements of the propagation delay of a radio signal, are quite difficult to implement and require accurate time synchronization of all devices, which is quite expensive. The received signal strength (RSS) method uses measurements of the received signal power level, but the presence of obstacles in the room can lead to significant positioning errors [4-5]. The proximity method is the simplest positioning method, it is based on determining the user's location in relation to the known closest position based on detectors, but does not allow achieving accuracy due to high variance [4-5].

The fingerprinting/indoor method (Fig. 1) of positioning works by measuring the values of the RSS signal strength of the user's device and comparing these values with a known set of measurements, which are collected and stored in a pre-created fingerprinting data base. The

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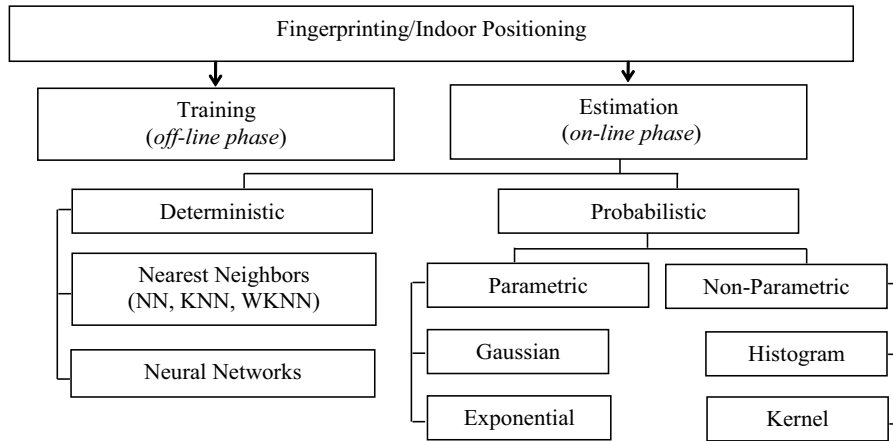


Fig. 1. Classification of fingerprinting/indoor positioning algorithms

functioning of the fingerprinting/indoor method is carried out at the first stage (off-line) by creating a fingerprinting data base in certain places of the RP (reference point) room, which are evenly and homogeneously distributed. The second stage (on-line) is implemented using deterministic and probabilistic algorithms to determine the user's location [4-5].

Traditional deterministic positioning algorithms [4-5] can be implemented based on the method of the nearest neighbors NN (nearest neighbor), K-nearest neighbors KNN (K-nearest neighbor), K-nearest neighbors using the weighting coefficients WKNN (weighted K-nearest neighbor), other more complex deterministic algorithms such as neural networks show better localization accuracy at higher computational costs. However, due to random fluctuations in the RSSI (received signal strength indication) values in the room, measurement errors are inevitable, therefore the use of probabilistic algorithms (extrapolation, Gaussian, histogram, kernel) store and use the RSSI probability distribution to estimate the user's location. Often, such algorithms are quite critical to insufficient data, in such cases, most of the results may turn out to be unreliable. According to [11-12], positioning algorithms based on a neural network can achieve the highest accuracy, but such a network must be trained with each change in the network, which can cause certain difficulties in the positioning process. Therefore, an urgent task is to find a new positioning algorithm that would be easier to implement and would simplify the solution of the problem of positioning the user in the WiFi/indoor network.

Such a method can be the fingerprinting/indoor method, which uses a positioning algorithm based on the finite element method using linear complex planar splines [13]. Earlier, in the works of the authors [14-17], it was proposed to use real splines (linear, quadratic, cubic, B-splines) to solve problems of data recovery and evaluation, signals and traffic, problems of improving QoS/QoE quality characteristics and predicting traffic characteristics, which made it possible to solve or simplify the solution of many problems. The use of complex planar splines

[18-19] when solving problems of user positioning in the WiFi/indoor network will allow determining the user's location with the required accuracy, thereby increasing the positioning accuracy.

## 2 Development of a modified method

Consider a WiFi/indoor network (Fig. 2), which is a set of  $AP_i$ , access points, where  $i$  is the number of  $AP_i$  access points in the radio access network, and  $i = (1, \dots, m)$ ,  $AP_i(x, y)$  are the coordinates of the  $i$  access point  $AP_i$ . Considering the finite element method, we use triangulation to determine the user's location in the WiFi/indoor network. According to triggered positioning, we will assume that at each point of the considered WiFi/indoor network, the user's device is within the range of at least three  $AP_i$  access points. Then the access points  $AP$  have coordinates:  $AP_1(x_1, y_1)$ ,  $AP_2(x_2, y_2)$ ,  $AP_3(x_3, y_3)$ , and the coordinates of the user are  $M(x, y)$ . Distances from access points  $AP_i$ ,  $i = 1, 2, 3$  to user  $M$  are equal  $r_1, r_2$  and  $r_3$ , respectively, [13].

To determine the distance between the user's device and the  $AP_i$ ,  $i=1,2,3$  access points, the value of the received power level indicator RSSI is used according to the formula [4]:

$$P_{d_i} = P_0 - 10n \log \frac{d_i}{d_0}, \quad (1)$$

where  $P_{d_i}$  is the value of the received signal strength RSSI of the corresponding access point  $AP_i$ ,  $d_i$  – is the distance from the user device  $M$  to the transmitter of the access point  $AP_i$ ,  $d_0$  – is the distance from the user device  $M$  to the access point  $AP_i$ , at which the signal strength was measured,  $P_0$  is the power signal,  $n$  is the power loss factor of the signal during propagation in the medium.

Let us consider the fingerprinting method (Fig. 2) [4-5] in relation to determining the user's location in the considered WiFi/indoor network. To determine the location,

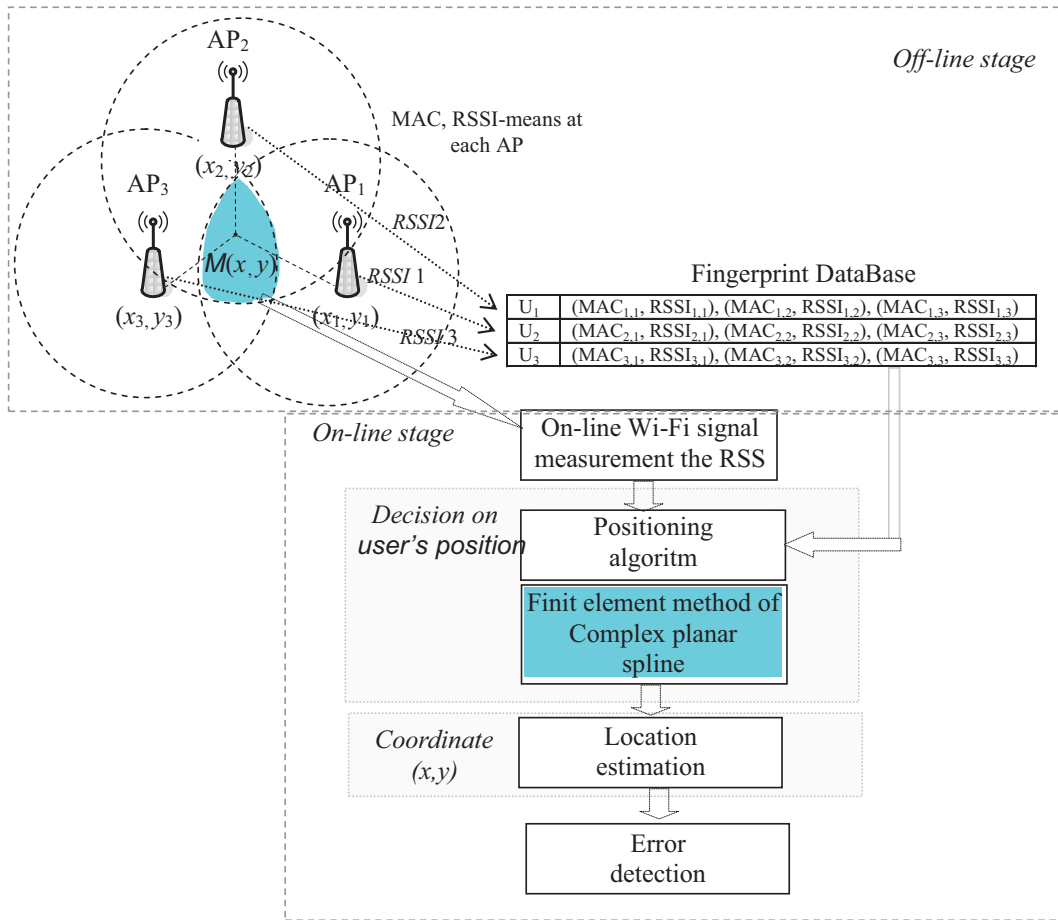


Fig. 2. Modified fingerprinting/indoor method based on the finite element method using linear complex planar splines

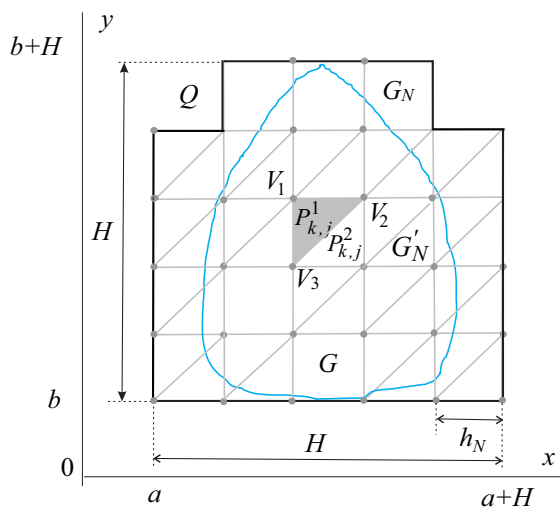


Fig. 3. Construction of the mesh area G

the fingerprinting method uses a positioning algorithm based on the method of finding the nearest neighbor NN (nearest neighbor) or KNN (K-nearest neighbor), which determines the user coordinates as the arithmetic mean of the coordinates of the corresponding points, or the nearest neighbor method with the weighting factors WKNN

(weighted K-nearest) using weights. However, these methods have an error in determining the user's location. We use a different approach in the positioning algorithm based on the finite element method of the considered area using complex planar splines (Fig. 2), which will increase the positioning accuracy. Find the coordinates of user M in the WiFi/indoor network, according to the following:

- At the initial stage (off-line stage), the data base of data of the RSSI power values from the access points  $AP_i$ ,  $i = 1, 2, 3$ , is formed, at various pre-determined points in the RP room. For each RP in the WiFi/indoor network, the fingerprinting data base stores MAC-address data and RSSI received strength values from access points  $AP_i$ ,  $(MAC_{1,1}, RSSI_{1,1}), (MAC_{1,2}, RSSI_{1,2}), (MAC_{1,3}, RSSI_{1,3})$ .
- The position request from the subscriber unit of the user UE arrives at the nearest three access points  $AP_i$ , (off-line stage). Each  $AP_i$ , upon receiving a position request, determines the value of the signal strength measurement the RSS and sends the values of the strengths of the RSSI signals from all available  $AP_i$ ,  $i = (1, m)$  for comparison, to the previously created fingerprint data base.
- Determination of the coordinates of the user's UE location using the position algorithm proposed in the

work, based on the finite element method using the complex planar spline. As a result of calculations and construction of a linear complex planar spline, the location of the user's UE in the WiFi/indoor network is determined with coordinates  $M(x, y)$ .

- Finding the error in determining the coordinates of the location of the UE user location error using Lemma 1.

### 3 Construction of linear complex planar splines

To determine the coordinates of the user, consider linear complex planar splines [18-19]. Let a region  $G$  of admissible values be given with nodes  $z_1, z_2, \dots, z_m$ . The outer boundary of the WiFi/indoor network area is set. Thus, an area of the WiFi/indoor network covered by a network of  $m$  access points  $AP_i$  and  $s$ -lines is created, divided into separate cells of arbitrary shape, including  $m$ -access points  $AP_i$ , and  $s$ -lines. At each access point  $z_j$  the signal strength  $f(z_j)$  is known, according to which the user's location in the WiFi/indoor network is determined.

Consider the process of constructing planar splines [18-19]. Let  $\bar{G} \subset Q$ , where  $\bar{G} = G \cup \partial G$ , and  $\partial G$ , is the  $G$  boundary of the domain,  $Q = [a, a + H] \times [b, b + H]$  is a square with side  $H > 0$ , (Fig. 3),  $N$  is a natural number,  $h_N = H/N$ ,  $x_k = a + kh_N$ ,  $y_j = b + jh_N$ ,  $k, j \in (0, N)$ . Then  $Q = \cup_{k,j=0}^{N-1} Q_{k,j}$ , where  $Q_{k,j}$  is square cells with a pitch  $h_N$ ,  $Q_{k,j} = \{z = x + iy : x \in [x_k, x_{k+1}] y \in [y_j, y_{j+1}]\}$ . This partition is denoted by  $\Delta_N$ , [13].

We define  $G_N$  as the union of all  $Q_{k,j}$  for which  $Q_{k,j} \cap G \neq \emptyset$ . We divide each square  $Q_{k,j} \subset G_N$  by the diagonal into two triangles  $P_{k,j}^1$  and  $P_{k,j}^2$ . We denote such a partition by  $\Delta'_N$ . The index  $N$  in  $h_N, \Delta_N, \Delta'_N$  will be omitted if a fixed partition is considered. Let us denote by  $G'_N$  the union of all  $P_{k,j}^1$  and  $P_{k,j}^2$ , for which  $P_{k,j}^1 \cap G \neq \emptyset, P_{k,j}^2 \cap G \neq \emptyset$ , [13].

Consider one of the triangles  $P_{k,j}, P_{k,j}$  included in  $G'_N$  with such vertices  $V_1, V_2, V_3$  that the conditions, [19]

$$\text{Im}\{V_1\} = \text{Im}\{V_2\}, \quad \text{Re}\{V_2\} = \text{Re}\{V_3\}$$

From the triangulated domain  $G'_N$ , we construct a linear planar complex spline  $S_\Delta(z)$ , interpolating the function  $f(z)$  at the vertices of the triangles,  $P_{k,j}, P_{k,j}$ , by setting

$$S_\Delta(z) = a + bz + c\bar{z}$$

where,  $z = x + iy$ , and  $\bar{z} = x - iy$ , and then

$$S_\Delta(z) = a + bx + cx + i(by - cy). \tag{2}$$

According to the interpolation condition at the points  $z_{k,j} = x_k + iy_j$ ,  $S_\Delta(z_{k,j}) = f(z_{k,j})$ ,  $f(z) = \text{Re}\{f(z)\} + i\text{Im}\{f(z)\}$

According to [19] the values of the coefficients  $a, b, c$  of the linear planar complex spline  $S_\Delta(z)$  are determined as follows

$$a = \frac{1}{\delta} \{f_1(V_2\bar{V}_3 - \bar{V}_2V_3) + f_2(\bar{V}_1V_3 - V_1\bar{V}_3) + f_3(V_1\bar{V}_2 - \bar{V}_1V_2)\}, \tag{3}$$

$$b = \frac{1}{\delta} \{f_1(\bar{V}_2 - \bar{V}_3) + f_2(\bar{V}_3 - \bar{V}_1) + f_3(\bar{V}_1 - \bar{V}_2)\}, \tag{4}$$

$$c = \frac{1}{\delta} \{f_1(V_3 - V_2) + f_2(V_1 - V_3) + f_3(V_2 - V_1)\}, \tag{5}$$

where,  $\delta = 2i \text{Im}\{V_1\bar{V}_2 + V_2\bar{V}_3 + V_3\bar{V}_1\}$ .

LEMMA 1.

Let the function  $f(z)$  be continuous in the cellular region,  $\bar{G}_N$ , and  $S_\Delta(z)$  be a planar complex spline of the form (2) interpolating the function  $f(z)$  at nodes  $\{z_{k,j}\}$ . Then

$$|f(z) - S_\Delta(z)| \leq 2\omega(f, h)$$

where  $h = h_N$  and  $\omega(f, h)$  - is a module of a continuous function  $f(z)$  in  $\bar{G}_N$ .

PROOF.

Similarly to [19], dividing  $\text{Re}\{S_\Delta(z)\}$  and  $\text{Im}\{S_\Delta(z)\}$ , we obtain a spline of the first degree of one variable

$$\text{Re}\{S_\Delta(z)\} = a + bx + cx, \quad \text{Im}\{S_\Delta(z)\} = by - cy.$$

interpolating, respectively, functions  $\text{Re}\{f(z)\}$  and  $\text{Im}\{f(z)\}$  at the vertices of the squares  $Q_{k,j} \in \bar{G}_N$

Let be  $z \in Q_{k,j}$ . As shown in [20], the class of continuous functions satisfies the inequalities  $|\text{Re}f(z) - \text{Re}S_\Delta(z)| \leq \omega(f, h)$  and  $|\text{Im}f(z) - \text{Im}S_\Delta(z)| \leq \omega(f, h)$ ; whence it follows

$$|f(z) - S_\Delta(z)| \leq 2\omega(f, h)$$

with  $h = h_N$  and  $\omega(f, h)$  being a module of continuous function  $f(z)$  in  $\bar{G}_N$ .

Consider the use of linear complex planar splines to position a user in a WiFi/indoor network using a modified fingerprinted/indoor positioning method. Let us construct for the considered area shown in Fig. 2 and Fig. 3 linear complex planar spline (Fig. 4).

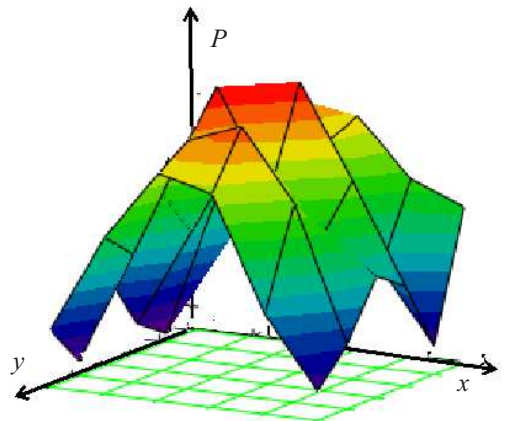


Fig. 4. Construction of a linear complex spline for the area G

## 4 Conclusions

The analysis of existing methods of user positioning in the WiFi/indoor network (triangulation and trilateration (AAA, TOA, TDOA, RSS), proximity and fingerprinting method) is carried out. It is shown that the admissible positioning accuracy can be achieved by the fingerprinting/indoor method. Deterministic and probabilistic positioning algorithms are considered. It was found that the existing positioning algorithms have limitations and do not allow determining the coordinates of the user with the required accuracy.

To improve the positioning accuracy in the WiFi/indoor network, a modified fingerprinting/indoor method based on the finite element method using linear complex planar splines is proposed. The coefficients are found and an example of constructing a linear complex planar spline is shown. It is shown finding the error in determining the coordinates of the user's UE location using Lemma 1.

The direction of further research is to consider a modified fingerprinting/indoor method based on complex planar splines (quadratic, cubic, B-splines, etc.) and a comparative analysis of the results obtained to improve the accuracy of positioning in a WiFi/indoor network.

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