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Desired work-leisure balance in a partial equilibrium job search model with multiple job holding

Abstract

Work-leisure balances are beneficial to society. A partial equilibrium job search model is developed to explain desired work-leisure tradeoffs for single-job holders and multiple-job holders. Significant work-leisure mismatches are found: 63% of the observations underwork by an average of 17 hours per week, while 37% overwork by 8.5 hours. The value of leisure is approximately four times the average hourly real wage when a single job is held, and it drops by one-third when multiple jobs are held. Models ignoring possibilities of multiple jobholding overstate the elasticity of leisure and understate the value of leisure.

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1 Introduction

Work-leisure balances are beneficial to society (Driver et al., 1991; Guest 2002; Krueger, 2009), as they improve employees' well-being (Judge and Watanabe, 1993; Pouwels et al., 2008; Erdogan et al., 2012; Bannal and Tamakoshi, 2014) and enhance labor productivities (White et al, 2003; Beauregard and Henry, 2009; Oswad et al., 2015; Pencavel, 2015; Collewet and Sauermann, 2017). Previous surveys reported that workers were generally dissatisfied with their working hours (Best, 1980; Cogan, 1981; Moffit, 1983; Kahn and Lang, 1991; Zabel, 1993; Feather and Shaw, 1999; Merz, 2002; Bloemen, 2008; Reynolds and Aletraris, 2010). Policymakers should know how individuals value their leisure and how many hours they desire to work. In this paper we answer these questions in the context of a partial equilibrium job search model. The model assumes that job providers determine work schedules. Job seekers accept a job offer provided that the offered wage is high enough to compensate the cost induced by any undesirable working time. Having said that, workers can adjust their working hours to their desired level by searching for a new job or holding more than one job at the same time (multiple job holding). By extending the job search model (Burdett, 1978; Flinn and Heckman, 1982) to incorporate desired working hours and multiple jobholding, we identify how individuals value their leisure time and estimate empirically the gaps between the desired and the actual working hours. In short, this paper builds on and contributes to three strands of literature, including the job search model, multiple jobholding, and valuation of leisure time.

Studies on the value of leisure dated back to the neoclassical static labor supply model (Heckman, 1974; Blundell and MaCurdy, 1999). It assumes that job seekers have discretion to choose their working hours. Observed hours in the market reflect workers' optimal choices and the market wages represent the value of leisure. Parameters revealing leisure preferences are identified from the observed wages and hours. This model was challenged empirically by a series of survey studies that found persistent work-leisure mismatches in the past decades. An early survey in 1978 found that 28% of the respondents in the U.S. preferred to work more and earn more, while 11% preferred to work less (Best, 1980). Feather and Shaw (1999) found that half of the respondents desired longer working hours, while another half desired fewer. Bloemen (2008) noticed that more than 30% of the workers were not satisfied with their working times. Reynolds and Aletraris (2010) observed that only 20% of the workers wanted the same hours. Work-leisure mismatches happened in other countries as well (Kahn and Lang, 1991; Merz, 2002; and Reynolds and Aletraris, 2010). For instance, 33% of Canadian workers reported working too short, while 17 % too long (Kahn and Lang, 1991). These survey data, though insightful, reported workers' subjective assessments, which were vulnerable to measurement errors and the outcomes varied considerably by questionnaire designs and interviewees' interpretations (Paxon and Sicherman, 1994; Kahn and Lang, 2001; Bloemen, 2008). The goal of this paper is to estimate work-leisure mismatches empirically based on a structural model.

To explain the discrepancies between the actual and desired working hours, static labor supply models incorporating minimum hours constraints were developed (Cogan, 1981; Moffit, 1982; Zabel, 1993). In these models, job seekers enter into the labor market only if their desired working hours are longer than their required minimum. Identifications of these Tobit-type models require exclusion restriction: different sets of covariates are used to explain the desired hours and the offered hours (Zabel, 1993). Alternative strategies reckoned on available information about multiple jobholding were introduced (Shishko and Rostker, 1976; Krishnan, 1990; Abdukadir, 1992; Paxon and Sichermen, 1996; Feather and Shaw, 1999). These models assume that people use the second job to adjust their working time when their first job does not provide enough wage income. Under this circumstance, observed hours for the second job coincide with the desired hours. Identifications are achieved by deriving the optimal hours for the second job and fitting them empirically with the observed hours. Although hours constraint is deemed as the only reason for multiple job holdings in these models, other reasons were suggested in the literature (Conway and Kimmel, 1998; Averett, 2001; Kimmel and Conway, 2001; Dickey et al., 2011). As an example, people hold the second job to obtain nonpencuniary benefits unprovided by the first job (e.g., enjoyment or insurance against unemployment risks). Kostyshyna and Lalé (2022) found that there was an increasing trend of multiple job holdings, two-thirds of which were caused by hours constraints and one-third were caused by nonpencuniary benefits. Models considering both reasons for multiple job holdings were discussed in Oaxaca and Renna (2006) and Hlouskova et al. (2017).

All models discussed above took the static neoclassical labor supply model as the point of departure. In this paper we apply the dynamic job search model that shares some similarities and differences with the static models. Similar to above, we assume that multiple job holding is a device to adjust working hours to the desired level, and we use the observed wages and hours of the second jobs to identify individual preference of leisure. However, workers make their optimal choice once and for all at the prevailing market wage in the static models. The optimal solution depends crucially on whether the first job is hours constrained or not (Conway and Kimmel, 1998). In the dynamic models, in contrast, workers have the option to perform job search continuously (Burdett, 1978; Flinn and Heckman, 1982). Wages and hours of the arriving new offers are not fixed but are drawn randomly from some distribution functions. This option has two implications. First, optimal hours are not constant values but change with the offered wage of the new jobs. Whether the new jobs are hours constrained or not is an endogenous outcome generated in the dynamic model, but not an assumption required to set up the model. Differences in the motivations for multiple jobholding play no role here. Second, the possibility of job search creates option value that improves workers' utility. This option value increases the values of leisure in the dynamic model and the optimal working hours are shorter relative to the static models. Ignoring job searching behaviors understates the value of leisure and overstates the wage elasticities of labor supply.

There are some recent works analyzing multiple jobholding in the framework of job search models (Compton, 2019; Mancino and Mullins, 2019; Lalé, 2020). As far as we are aware, identifications of leisure value and desired leisure time in this setting have not been fully addressed along the lines that we set out below. Compton (2019) developed a general equilibrium model that permits multiple jobholding. Optimal working hours are not the subject of interest in his model, as working hours are fixed exogenously at 20 hours a week for part-time jobs and 40 hours a week for full-time jobs. The general equilibrium model introduced by Lalé (2020), in contrast, considers working hours as endogenous, which are chosen jointly by the job seekers and job providers to maximize their joint surplus. The resulting working times are generally different from the job seekers' desired hours implied by the model's structural parameters. Comparisons of these two figures reveal the extent of work-leisure mismatches. The key structural parameter that measures leisure preference was calibrated to generate a predefined high (0.60) and low (0.30) values of the Frisch elasticity of labor supply. As a result, the estimated proportion of workers wanting more hours and fewer hours varied considerably by the chosen calibrated values. For instance, the proportion of male single-job holders wanting more hours increased from 8% to 26% when the assumed Frisch elasticity increased from its low to high value, while the proportion of female multiple-job holders wanting more hours increased from 20% to 48%. All structural parameters are estimated directly without calibration in our partial equilibrium model.

Different from the two general equilibrium job search models discussed above, job profiles are determined by the job providers and are not negotiated with the job seekers in the partial equilibrium models. The standard partial equilibrium models usually ignore working hours in the job profiles. Exceptions are Gørgens (2002) and Bloemen (2008). However, they did not consider multiple jobholding, which makes their models different from ours in two important ways. First, multiple job holdings are in all respects different from single job holdings. According to our results, multiple jobholders value their leisure 33% less than single jobholders, whereas their demand for leisure is 50% more elastic. Ignoring multiple jobs would underestimate the value of leisure by almost one-third. Second, the single-job models cannot exploit information from multiple jobs for identifications. Identifications are accomplished by assuming that there are no hours constraints in the single jobs, of which the observed hours are equal to the optimal hours. Work-leisure mismatches cannot be explained under this assumption. To solve this problem, Bloemen (2008) used self-reported desired hours available in his dataset as the second identification strategy. Results obtained from these two methods were pretty different. The estimated optimal hours ranged from 48 to 60 hours a week, while the self-reported desired hours ranged from 36 to 40 hours a week. Bloemen's results provide further insights on the (un)reliability of subjective data. He found that interviewees declared dissatisfied with their working hours only when the difference between their desired and actual hours was larger than some threshold values. The estimated threshold values had a mean of 16 hours a week with a standard deviation of 13 hours. We conclude that surveys' results using subjective assessments quite likely underestimate the degree of work-leisure mismatches and the outcomes vary considerably by individual's interpretations. Closest to our paper is Mancino and Mullins (2019), who used partial equilibrium model to study multiple jobs. Their model is identified by the indirect inference method that necessitates more assumptions than ours. In their model, working hours are assumed to take two values only, i.e., 20 hours a week for part-time jobs and 40 hours for full-time jobs. Moreover, the offered wages for part-time jobs relative to full-time jobs are assumed to be a fixed ratio that applies to all individuals. Since working hours are fixed exogenously, identifying optimal working hours is out of scope of their model.

To our knowledge, our paper is the first to consider multiple jobholding in the partial equilibrium job search model that investigates the value of leisure and optimal working times. We believe that this makes a useful contribution to the literature concerning work-life balance. We applied our model to panel data collected from the National Longitudinal Surveys in the U.S. covering 1997 to 2015 for young adults aged 25 to 35 years. People in their early stages of adult life are more likely to take a second job (Wu et al., 2009; Dickey et al., 2011), so this data sample is appropriate for analyzing multiple jobs. We found evidence of remarkable work-leisure mismatches, both underwork and overwork. The estimated value of leisure is approximately three times the average hourly wage, and it drops by one-third when multiple jobs are taken. Workers are more willing to sacrifice their leisure time for obtaining multiple jobs than single jobs. Consistent with previous findings (Blundell, et al., 1998; Blundell and MaCurdy, 1999; Kudoh and Sasaki, 2011; Bils et al., 2012; Attanasio et al., 2018), age, education, and industry were the most important factors in determining leisure values, while gender and having kids play secondary roles. In particular, female, parents, older employees with more education who work in public or professional industries value leisure more than the others and are less elastic in their demand for leisure. Policies promoting more flexible work schedules (for reviews of similar policies see Lewis, 2003; Plantenga et al., 2009; Messenger, 2018) should facilitate desirable work-life balances.

The remainder of this paper is organized as follows: Section 2 introduces the theoretical model; Section 3 discusses the data and estimation methods; Section 4 provides the main results and extensions; the conclusion is presented in Section 5.

2 The partial equilibrium job search model

2.1 Time constraints and utilities

Central to our model is the optimal time allocation devoted to work and leisure. Available hours for each individual are $24 \times 7 = 168$ hours a week. Week is used as the time unit because the dataset contains employees' weekly working hours but not daily working hours. We normalize the weekly time endowment to one, which is allocated as working hours h and leisure time l with $0 \leq h, l \leq 1$.

Individuals can hold more than one job simultaneously. When an individual holds one job only, it is a 'single job spell'. When an individual holds more than one job at the same time, it is a 'multiple job spell'. Let $j = 1,\ldots, J$ be the job index. The working hours of job j is h_j and the total working hours are $h = \sum_{j=1}^{J} h_j$. The time constraint implies that $h + l = 1$. Approximately 15% of the working spells in our dataset hold multiple jobs, while the majority (13.5%) hold at most two jobs at the same time and the remaining (1.5%) hold three or more jobs. We simplify the model by setting $J = 2$. An extension to consider $J > 2$ is straightforward (see Section 4.2.3 for the case of $J = 3$), although it makes the model analytically complicated without providing additional useful insights.

Individuals derive their instantaneous utility U from their income and leisure. The utility function is quasilinear in the form of $U = Y + K$, where Y is a linear function for income and K is a strictly concave function of leisure. When working in job j, an employees' income is their wage rate w_j multiplied by their working time h_j. The instantaneous utility derived from their income is $Y = \sum_j w_j h_j$. An unemployed person has no wage income. It is called an 'unemployment spell'. In our relatively young sample with an average age of 29.2, only 10% of the unemployment spells collect unemployment benefits at an average weekly amount of \$229. Dividing it by the time endowment of 168 hours, the hourly rate is equivalent to \$1.36. It is an inconsiderable amount relative to the average nominal hourly wage of \$16.10. Unemployment benefits are not included in the model.

Let the dollar value per unit leisure time be $\kappa(l)$. The instantaneous utility derived from leisure is $K(l) = \kappa(l)l$. It is reasonable to assume that κ is positive and has diminishing returns, such that $\kappa(l) > 0$, $\kappa'(l) < 0$ and $\kappa''(l) > 0$ for all $l \in (0,1]$. In particular, when leisure time approaches one, the return of leisure drops to a minimum value, and when the leisure time reduces toward zero, the value of leisure increases to infinity at an increasing speed, as people have biological needs for leisure. One possible choice is $\kappa(l) = a(1 - \log l)$, where a (> 0) is an individual specific parameter that measures the minimum value of leisure (i.e., $\kappa(l) = a$). a can also be interpreted as the individual's preference for leisure, as $\kappa(l)$ increases with a at any $l \in (0,1]$. This specification ensures the strict concavity of $K(l)$ with respect to l, as $K'(l) = -a \log l > 0$ and $K''(l) = -a/l < 0$ for all $l \in (0,1]$.

Since leisure time is complemented to working time, i.e., $l = 1 - h$, $K(l) = K(1 - h)$ holds. The total instantaneous utility from work and leisure is $U = \sum_j w_j h_j + K(1-h)$, where $h = \sum_j h_j$.

2.2 Changes in job status

There are three job statuses in our model: single job spell, multiple job spell, and unemployment spell. Workers change their job status upon the arrival of stochastic events that arrive at a Poisson stream at different rates. To facilitate discussions, we need a system to define the job index.

In a single job spell workers hold one job, which is called job 1. The wage and working time associated with job 1 is defined as w_1 and h_1 respectively. The job status of single job holders have three possible changes. The first possibility is separating from job 1 with a separation rate of δ_1, and the single job spell switches to an unemployment spell. The second possibility is quitting the current job and accepting a new offer with an arrival rate of λ_1. We call it a 'single job offer'. In this case, workers switch from a single job spell to another single job spell. The new job is called job 1 again. The wage and the working time for a single job offer is denoted as w_1' and h_1' respectively, and their joint distribution is $J_1(w_1', h_1')$. The last possible change is continuing in job 1 while accepting a second job with an arrival rate of λ_2. We call it a 'second job offer' and the second job is named job 2. The wage and the working hours for a second job offer is denoted as w_2' and h_2' respectively and their joint distribution is $J_2(w_2', h_2')$. By accepting a second job offer, workers switch from a single job spell to a multiple job spell holding jobs 1 and 2.

We emphasize that jobs 1 and 2 are defined merely by chronological order, as job 2 is always accepted after job 1. This chronological order is essential in modeling the dynamic of changing job statuses. Our definitions of jobs 1 and 2 do not distinguish the differences between full-time and part-time jobs. For instance, workers can hold a part-time job in a single job spell (job 1) and switch to a multiple job spell by accepting another part-time second job (job 2) later. In fact, when we use 40 hours a week as the cutting line to distinguish a part-time and a full-time job, 20% of the multiple job spells in our dataset are holding two part-time jobs at the same time, while 27% of them are holding two full-time jobs simultaneously; and only 53% are holding one full-time and one part-time job.

Multiple job holders have two possible changes in job status. The first one is separating from job 1 with a separation rate of δ_1. After that, workers hold job 2 only. The second one is separating from job 2 with a rate of δ_2, and they hold job 1 afterward. The multiple job spell ends and is replaced by a single job spell in either case. Besides, it is possible that separations from both jobs occur simultaneously with an arrival rate of $\delta_1\delta_2$, when separation from these two jobs are independent events. In a short time period $\Delta t \to 0$, $\delta_1\delta_2 = o(\Delta t)$. Such a scenario accounts for 2% of all observations in our data and we disregard this possibility. Another rare occasion is a transition from a multiple job spell to another multiple job spell. For instance, a multiple job holder takes another second job at the same moment when he separates from either of the existing jobs. This possibility is discussed in Section 4.2.4 as an extension and we ignore it in our main model as it represents only less than 1% of all observations.

Unemployed people may accept a job offer. To indicate that it is a change from an unemployment spell to a single job spell, we call it a 'post-unemployment offer'. Since the unemployed people hold one job only after accepting this offer, we call it job 1 again. The wage and the working hours associated with this offer is denoted respectively as w_1' and h_1' as before. The joint distribution is, however, denoted differently as $J_0\left(w_1', h_1'\right)$ and the offer rate is denoted as λ_0. We do not assume that λ_0 is identical to λ_1, as the unemployed jobseekers may put more search efforts than that put in on-the-job search by the single job holders. Similarly, J_0 may not be the same as J_1 as firms' offers made to unemployed job seekers could be different from that to currently working peoples. Lastly, there is no unemployed people accepting two jobs at the same time in our dataset and we ignore the possibility for a direct switch from an unemployment spell to a multiple job spell.

We finish this section by introducing some notations. We define $J_j(w,h) = G_j(w|h)M_j(h)$, where $G_j(w|h)$ is the marginal distribution for the offered wage w conditional on the offered working time h, and $M_j(h)$ is the marginal distribution for the offered working time, with $j = 0,1,2$. We simplify the notation by denoting $G_j(w|h)$ as $G_j(w)$. The G_j and M_j are common knowledge to all job seekers. For all distribution functions, a capital letter refers to a cdf, whereas a small letter refers to a pdf. The partial derivative of $V(x,y)$ with respect to x is $\nabla_x\left[V\left(x,y\right)\right]$, and the second-order partial derivative is $\nabla_x^{(2)}\left[V\left(x,y\right)\right]$.

2.3 **Bellman equations**

In this section we discuss a variation of the standard partial equilibrium job search model (Burdett, 1978; Flinn and Heckman, 1982) that allows workers to hold multiple jobs. It extends the static multiple job model in Conway and Kimmel (1998) to a dynamic setting. The Bellman equations we discussed below are similar to those in Mancino and Mullins (2019). We let $V_1(w_1h_1)$ and $V_2(w_1h_1, w_2h_2)$ be the expected present value for single job holders and multiple job holders, respectively. The expected present value for unemployment jobseekers is V_0. ρ is the interest rate.

The Bellman equation for single job holders is

$$\begin{aligned}\left(\rho+\delta_1\right)V_1\left(w_1h_1\right) &= w_1h_1+K\left(1-h_1\right)+\delta_1V_0\\ &+\lambda_1\int_0^1\int_{\xi_1\left(w_1h_1,h_1'\right)}^{\infty}\left[V_1\left(w_1'h_1'\right)-V_1\left(w_1h_1\right)\right]dG_1\left(w_1'\right)dM_1\left(h_1'\right)\\ &+\lambda_2\int_0^{1-h_1}\int_{\xi_2\left(w_1h_1,h_2'\right)}^{\infty}\left[V_2\left(w_1h_1,w_2'h_2'\right)-V_1\left(w_1h_1\right)\right]dG_2\left(w_2'\right)dM_2\left(h_2'\right).\end{aligned} \tag{1}$$

The conditional wages $\xi_1\left(w_1h_1, h_1'\right)$ and $\xi_2\left(w_1h_1, h_2'\right)$ are solved from the following equations:

$$V_1\left(\xi_1\left(w_1h_1,h_1'\right)h_1'\right)=V_1\left(w_1h_1\right), \tag{2}$$

$$V_2\left(w_1h_1,\xi_2\left(w_1h_1,h_2'\right)h_2'\right)=V_1\left(w_1h_1\right). \tag{3}$$

The conditional wages are functions depending on the current wage income w_1h_1 and the offered hours from the new job h_1' or h_2'. To simplify notations, we suppress the arguments of these functions and write ξ_1 and ξ_2 instead when no ambiguity arises. Corollaries 1(i) and (ii) in Section 2.4 ensure that ξ_1 and ξ_2 exist and are unique.

Equation (1) can be interpreted as follows: the expected present value for holding job 1 is discounted by the interest rate ρ and the job separation rate δ_1. It includes the earned income w_1h_1, the value of leisure time $K(1-h_1)$, and the expected present value of unemployment V_0 in the event of job separation. The last two items in (1) are the option values for on-the-job search. The first one refers to the expected gain relative to the current position when a single job offer arriving with a rate of λ_1 is accepted. A new single job is accepted when the combination of (w_1', h_1') induces a greater expected present value than the current one with (w_1, h_1). The expected gain is derived by integrating $V_1(w_1'h_1') - V_1(w_1h_1)$ over the acceptable set of (w_1', h_1') that ensures $V_1(w_1'h_1') \geq V_1(w_1h_1)$ as is mentioned in (4) below.

$$\int_{(w_1',h_1'):V_1(w_1'h_1')\geq V_1(w_1h_1)}\left[V_1(w_1'h_1') - V_1(w_1h_1)\right]dJ_1(w_1',h_1') \tag{4}$$

$$= \int_0^1\int_{\xi_1}^{\infty}\left[V_1(w_1'h_1') - V_1(w_1h_1)\right]dG_1(w_1')dM_1(h_1'). \tag{5}$$

When we derive the optimal working hours in Section 2.4, we need to differentiate this option value with respect to h_1. It is difficult to differentiate the double integral in (4) as the domain does not have an obvious upper and lower limit. Nevertheless, (4) can be simplified to (5) using Corollary 1(i), as $V_1(w_1'h_1')$ increases monotonically with w_1' for any given value of h_1'. Conditional on h_1', any wage greater than the conditional reservation wage defined as ξ_1 in (2) is considered an acceptable set of w_1' in the inner integral of (5). It is then integrated over the feasible set of the offered hours $h_1' \in (0,1)$ to form the outer integral. The iterated integral in (5) can be differentiated using the Leibniz integral rule in a straightforward manner.

The option value for searching a second job, i.e., the last item in (1), is derived similarly using Corollary 1(ii). Remarkably, the feasible set of the working time for the second job h_2' is not (0,1). When people are holding job 1 with a working time of h_1, they cannot accept a second job offer with a working time exceeding $1-h_1$. However, 99% of the combined working hours from two jobs are less than 120 (< 168) in our dataset and hence the working time for the second job is not binding, i.e., $h_1 + h_2' < 1$. We assume in the following that the support of h_2' has a constant upper end c where $c < \min(1-h_1)$ to ease analytic computation.

The Bellman equation for multiple job holders is

$$\begin{aligned}&(\rho+\delta_1+\delta_2)V_2(w_1h_1,w_2h_2)\\ &= w_1h_1 + w_2h_2 + K(1-h_1-h_2) + \delta_2V_1(w_1h_1) + \delta_1V_1(w_2h_2),\end{aligned} \tag{6}$$

It includes the wage income from both jobs $w_1h_1 + w_2h_2$ and the utility derived from leisure $K(1-h_1-h_2)$. When multiple job holders separate from job 2, they obtain an expected present value from job 1 only, i.e., $V_1(w_1h_1)$. When they separate from job 1, they obtain an expected present value from job 2 only, i.e., $V_1(w_2h_2)$. There is no option value from further job search when two jobs are held.

The Bellman equation for unemployed people is

$$\rho V_0 = K(1) + \lambda_0\int_0^1\int_{\xi_0(h_1')}^{\infty}\left[V_1(w_1'h_1') - V_0\right]dG_0(w_1')dM_0(h_1'). \tag{7}$$

The conditional reservation wage $\xi_0(h_1')$ in (7) is computed from the following equation:

$$V_1\left(\xi_0\left(h_1'\right)h_1'\right)=V_0. \tag{8}$$

This conditional reservation wage is a function depending on the offered wage h_1', which is simplified as ξ_0. Unemployed people obtain utility by spending all their time on leisure, i.e., $K(1)$. It includes the option value from a post-unemployment offer arriving at a rate of λ_0.

2.4 **Work-leisure balance**

Tradeoffs between work and leisure are discussed in Corollaries 1 to 4 below. Their proofs are found in Appendix. To start with, we introduce additional notations for the sake of simplicity. We let $\overline{F}(x)=1-F(x)$ for any cdf $F(x)$. Hence, $\overline{G}_1\left(\xi_1\right)=1-G_1\left(\xi_1\right)=\Pr\left(w_1'>\xi_1\right)$ is the conditional probability that a single job offer is accepted when its offered hours equal to h_1'. The (unconditional) probability that a single job offer is accepted is derived by integrating $\overline{G}_1\left(\xi_1\right)$ over the distribution of h_1', which is denoted as P_1 where

$$P_1=\int_0^1\overline{G}_1\left(\xi_1\right)dM_1(h_1'). \tag{9}$$

The (unconditional) acceptance probability for a second job offer and a post-unemployment job offer is respectively

$$P_2=\int_0^c\overline{G}_2\left(\xi_2\right)dM_2(h_2'),\text{ and} \tag{10}$$

$$P_0=\int_0^1\overline{G}_0\left(\xi_0\right)dM_0(h_1'). \tag{11}$$

We denote $E_{h_2'}(\cdot)$ as the expectation operator that integrates out h_2' over the job acceptance probability P_2. For instance, the expected value of a function $F(h_2')$ over P_2 is defined as

$$E_{h_2'}\left(F\left(h_2'\right)\right)=\int_0^c F\left(h_2'\right)\overline{G}_2\left(\xi_2\right)dM_2\left(h_2'\right). \tag{12}$$

2.4.1 **Current wage**

Corollary 1. *(i) The expected present value $V_1(w_1h_1)$ increases with wage w_1 for any given working time h_1. (ii) The expected present value $V_2(w_1h_1, w_2h_2)$ increases with wage w_1 and w_2 for any given working time h_1 and h_2. (iii) The option value for a single job search decreases with the current wage w_1. (iv) The option value for a second job search increases with the current wage w_1.*

Corollary 1(i) and (ii) state that a higher current wage increases the payoff of working at any given working hours. These corollaries ensure that there is a unique solution for ξ_1 in equation (2) at any given w_1, h_1 and h_1'. Similarly, they ensure the uniqueness of ξ_2 and ξ_0 in (3) and (8). These corollaries simplify the expression of the option values in (1) and (7).

Corollary 1(i) leads to Corollary 1(iii) because a higher current wage implies a higher current expected value, which reduces the potential gain from searching another single job. Corollary 1(iv) is less obvious when workers are searching for a second job. On the one hand, the current wage w_1 has a negative effect on the potential gain $V_2\left(w_1h_1,\ w_2'h_2'\right)-V_1\left(w_1h_1\right)$, as

$V_1(w_1h_1)$ is larger due to a larger w_1. On the other hand, workers keep their current job in a multiple job spell and so a higher current wage in job 1 increases the expected value for a future multiple job spell, i.e., $V_2\left(w_1h_1, w_2' h_2'\right)$ is larger due to a larger w_1. Corollary 1(iv) states that the positive effect dominates the negative effect. Corollary 1(iii) and (iv) have the following implications. Workers earning a high wage currently are less likely to quit their current job, as the option value is smaller. However, these workers are more likely to accept a second job, as the option value is larger. It is consistent with Auray et al. (2021) who find that workers having higher wages are more likely to take a second job.

2.4.2 **Optimal working and leisure time in a single job spell**

The optimal working time for a single job spell with a given wage w_1 is defined as h_1^*, which is solved from the following optimization problem:

$$\max_{h_1} V_1\left(w_1h_1\right). \tag{13}$$

By taking the first order condition of $V_1(w_1h_1)$ with respect to h_1, we derive the following corollary regarding the properties of h_1^*. Proofs are found in Appendix.

Corollary 2. *(i) The optimal working time h_1^* that maximizes $V_1(w_1h_1)$ for any given wage rate w_1 can be solved from*

$$w_1 - K'\left(1-h_1^*\right) + \frac{\lambda_2 E_{h_2'}\left[w_1 - K'\left(1-h_1^*-h_2'\right)\right]}{\rho+\delta_1+\delta_2} = 0. \tag{14}$$

(ii) The optimal h_1^* is unique. (iii) When there is no second job offer (i.e., $\lambda_2 = 0$) and $K(1-h) = a(1-h)(1-\log(1-h))$, the unique optimal working time defined as h_1^0 is

$$h_1^0 = 1 - \exp\left(-\frac{w_1}{a}\right), \tag{15}$$

with $h_1^0 > h_1^*$. (iv) The uncompensated wage elasticity of optimal leisure time $\left(l^* = 1-h_1^*\right)$ in a single job spell defined as

$$\epsilon_{l,w_1} = \frac{w_1}{l^*}\frac{\partial l^*}{\partial w_1} \tag{16}$$

is negative. When $K(1-h) = a(1-h)(1-\log(1-h))$, ϵ_{l,w_1} is equal to

$$\epsilon_{l,w_1} = -\frac{w_1}{aB}, \tag{17}$$

where $B \geq 1$ and is simplified as

$$B = \frac{\rho+\delta_1+\delta_2+\lambda_2 E_{h_2'}\left[\left(1-h_1^*\right)/\left(1-h_1^*-h'_2\right)\right]}{\rho+\delta_1+\delta_2+\lambda_2 P_2}. \tag{18}$$

Interpretation of (14) is straightforward. The marginal gain of increasing working time in job 1 is the wage rate w_1 and the marginal loss is the forgone leisure $K'(1-h_1)$. Since workers keep job 1 after a second job is accepted, the marginal gain and loss include $E_{h_2'}\left[w_1 - K'\left(1-h_1-h_2'\right)\right]$, which is the expected gain from wage w_1 and the expected loss of leisure $K'\left(1-h_1^*-h_2'\right)$ upon the acceptance of the second job with offered hours h_2'. The expected net gain is computed by integrating out the acceptance probability P_2 and is represented by the expectation operator $E_{h_2'}$ defined in (12).

The optimal working time for job 1, denoted as h_1^*, is unique. When there is no second job offer ($\lambda_2 = 0$), our model is reduced to a static model (Conway and Kimmel, 1998) and the optimal working time denoted as h_1^0 solves $w_1 - K'\left(1-h_1^0\right) = 0$. Optimality requires longer working hours when the second job is unavailable, i.e., $h_1^0 > h_1^*$. Consider the special case where $K(1 - h) = a(1 - h)(1 - \log(1 - h))$, the optimal working time is given by (15), which is an increasing function of the wage rate w_1. If w_1 is infinity, it is optimal to work for all available time ($h_1^0 = 1$); if w_1 is zero, it is optimal not to work ($h_1^0 = 0$).

Corollary 2(iv) states that the uncompensated wage elasticity of optimal leisure time (ϵ_{l,w_1}) is negative, which is the consequence of using a quasilinear utility function, where wages have no income effect on labor supply decisions but only a substitution effect. This theoretical result parallels the modest empirical consensus that the uncompensated wage elasticity of labor supply is positive. For reviews and meta-analyses, see McCelland and Mok (2012) and Bargain and Peichl (2016).

Using $K(1 - h) = a(1 - h)(1 - \log(1 - h))$, ϵ_{l,w_1} simplifies to (17). If there is no second job offer ($\lambda_2 = 0$), B in (18) equals to one and the elasticity becomes $\epsilon_{l,w_1} = -w_1/a$. This elasticity depends on the wage rate w_1 scaling down by a, the minimum dollar value of leisure. a can be conveniently interpreted as the preference for leisure. If $a > w_1$, leisure is more valuable than work and $\left|\epsilon_{l,w_1}\right| < 1$; leisure is inelastic and people prefer to have a fixed amount of leisure time. If $a < w_1$, leisure is less valuable and $\left|\epsilon_{l,w_1}\right| > 1$; leisure is elastic, and people are willing to sacrifice more leisure time in exchange for a higher wage income. By assuming that a is individual specific and depends on a vector of covariates, we can discover which factors and to which extent they determine the desired work-leisure balance. If obtaining a second job is possible ($\lambda_2 > 0$), the value of leisure is scaled up by B (> 1) and is equal to aB (> a). Leisure becomes less elastic. In other words, the wage elasticity of leisure time is overstated when the possibility of a second job offer is ignored. We reported in Section 4.2.5 that the estimated elasticity is inflated by more than 50% from -0.26 to -0.41 in a model ignoring multiple job holdings. This theoretical implication is comparable to Ham (1982) and Kahn and Lang (1999), who reported that the wage elasticity of labor supply is biased upward when actual hours instead of desired hours are considered.

2.4.3 **Optimal working and leisure time in a multiple job spell**

We have derived the optimal working time for a single job holder in the last section. Nevertheless, workers might not be able to find a single job that offers h_1^* due to hours constraints. We consider a general case in which the accepted hours h_1 are allowed to be different from the desired hours h_1^* in this Section. After accepting h_1, the workers continue to search for a second job. 98.7% of the workers in our dataset have no change in h_1 by the time they accept the second job. We reason that certain kinds of contractual restrictions forbid them from doing so. Workers can only attain their desired work-leisure balance by choosing their optimal working time h_2^{**} for the second job subject to the given h_1 in the following optimization problem:

$$\max_{h_2} V_2\left(w_1 h_1, w_2 h_2\right). \tag{19}$$

By taking the first order condition of (19), the properties of h_2^{**} are derived in Corollary 3. Proofs are found in Appendix.

Corollary 3. *(i) The optimal working time h_2^{**} for a second job (job 2) that maximizes $V_2(w_1h_1, w_2h_2)$ for any given w_1, h_1 and w_2 can be solved from*

$$w_2 - K'\left(1-h_1-h_2^{**}\right)+\delta_1\nabla_{h_2}V_1\left(w_2h_2^{**}\right)=0. \tag{20}$$

(ii) h_2^{**} is unique. (iii) The wage elasticity for optimal leisure time $\left(l^{**}=1-h_1-h_2^{**}\right)$ at any given h_1 in a multiple job spell defined as

$$\epsilon_{l,w_2}=\frac{w_2}{l^{**}}\frac{\partial l^{**}}{\partial w_2} \tag{21}$$

is negative. When $K(1-h)=a(1-h)(1-\log(1-h))$, it is equal to

$$\epsilon_{l,w_2}=-\frac{w_2}{aE} \tag{22}$$

where E (>0) is simplified as

$$E=\frac{D+\delta_1\left[\frac{1-h_1-h_2^{**}}{1-h_2^{**}}\right]+\delta_1\lambda_2\left(\rho+\delta_1+\delta_2\right)^{-1}E_{h_2'}\left[\frac{1-h_1-h_2^{**}}{1-h_2^{**}-h'_2}\right]}{D+\delta_1+\delta_1\lambda_2\left(\rho+\delta_1+\delta_2\right)^{-1}P_2}, \tag{23}$$

where $D=\rho+\delta_1+\lambda_1P_1+\left(\rho+\delta_1\right)\left(\rho+\delta_1+\delta_2\right)^{-1}\lambda_2P_2$ and $E<B$. (iv) When the wage rates for a single and a second job are identical ($w_{1=}w_2$), the leisure time in a multiple job spell is more elastic than that in a single job spell, that is, $\left|\epsilon_{l,w_2}\right|>\left|\epsilon_{l,w_1}\right|$.

The marginal gain of increasing working hours in job 2 is the wage rate w_2 and the marginal loss is the forgone leisure $K'(1-h_1-h_2)$ in Corollary 3(i). If separated from job 1 at rate δ_1, workers' payoff changes to $V_1(w_2h_2)$. Therefore, the marginal gain includes $\delta_1\nabla_{h_2}\left[V_1\left(w_2h_2\right)\right]$, which is the marginal change in $V_1(w_2h_2)$ when h_2 changes. The optimal time h_2^{**} is unique. The value of leisure in a multiple job spell is scaled up by E in (22). Since $E<B$, leisure time is less valuable in multiple job spells (aE) than in single job spells (aB). It results in Corollary 3(iv): Job seekers are more willing to sacrifice their leisure time for a second job than for a single job when both jobs have the same wage. Conway and Kimmel (1998) found a similar conclusion, stating that the desired labor supply is more responsive to wage changes when the possibility of multiple jobholding is considered. Renna and Oaxaca (2006) found that the labor supply elasticity for a second job is 1.8, which is three times that for a single job.

2.4.4 **Conditional reservation wages**

The conditional reservation wages defined in (2) and (3) depend on the offered working hours in the following manners:

Corollary 4. *(i) Conditional reservation wage for single job offers ξ_1 drops with the offered working time h_1' when h_1' is shorter than its optimal value ($h_1'<h_1^*$); ξ_1 increases with h_1' otherwise*

($h_1' > h_1^*$). (ii) Conditional reservation wage for second job offers ξ_2 drops with the offered working time h_2' when $h_2' < h_2^{**}$; ξ_2 increases with h_2' when $h_2' > h_2^{**}$.

Corollary 4 implies that job seekers require a higher reservation wage to compensate the loss due to unattractive working hours, In particular, the conditional reservation wage has a U-shaped against the offered hours. Workers require the least reservation wage when the offered hours are equal to the optimal value. This result is consistent with the findings of Altonji and Paxson (1988), and Bloemen (2008).

We describes the dataset and the empirical models we utilize to estimate the structural parameters of the above-presented model in the next section.

3 Data and empirical models

3.1 Data

We extracted data from the National Longitudinal Surveys managed by the U.S. Bureau of Labor Statistics. This survey followed a cohort of American youth born between 1980 and 1984. Respondents aged 12-16 years were first interviewed in 1997. Follow-up surveys were done yearly until 2011 and biyearly afterward. We considered observations with age ≥ 25, as teenage workers presumably have stronger job frictions that would affect their labor supply elasticity and value of leisure. Our sample includes 7,573 individuals, with the oldest respondent aged 35. The dataset contains information about respondents' employment and unemployment spells over the sample period. Starting and ending calendar week are available for each of these spells. We construct the entire career history of each respondent using this information.

The way we managed multiple jobs needs further discussion. Suppose an individual works for job A from week 1 to week 6 while holding job B between weeks 4 and 10, this working spell is split into three parts. The first part is a single job spell for job A that takes place from weeks 1 to 4, and job A is defined as job 1. The second part is a multiple job spell that includes the spell for the current job A (job 1) and the spell for the second job B (job 2) between weeks 4 and 6. This multiple job spell ends with the termination of job A (job 1). The last spell is a single job spell between weeks 6 and 10 for job B, which is defined as job 1 as it is the only job held during this time interval.

24,675 working spells are constructed in our sample consequently. 20,959 (85%) of them are single job spells, and 3,716 (15%) are multiple job spells. Among the single job spells, 6,953 (33%) are terminated by unemployment, 4,938 (21%) are terminated by accepting a new single job, 2,820 (13%) are terminated by taking a second job, and 6,803 (32%) are right-censored at the last date of the interview in the sample. Among the multiple job spells, 2,052 (55%) are terminated by a separation in job 1 and 1,664 (45%) by a separation in job 2. As was mentioned in Section 2.2, 4% (151) of these multiple job spells are followed by another multiple job spells, as workers take another second job immediately after quitting either one of the multiple jobs. Such an extension will be discussed in Section 4.2.4. Lastly, there are 6,953 unemployment spells.

Information about wages, working hours, and individual characteristics for each working spell were updated in the dataset at each interview date. We collected this information when there is a change in job status. For example, when an individual accepts a job at calendar time

t, the wage and working hours for the old and the new job are recorded at time t. Individual characteristics include time-invariant information such as gender and race, and time-variant information such as age, education, marital status, number of children, net worth, work experience, number of previous jobs held, and types of industry. We classify industries into three groups: public sector, professional services (legal, accounting, architecture, scientific and technical, financial, insurance and real estate) and others. We convert nominal wages into real wages using 1997 as the base year to eliminate the effect of inflation. The real interest rates (ρ) over the period are collected from the World Bank database.

Table 1 provides descriptive statistics. Average working hours for single job spells are 36.6 per week, while that for multiple job spells are 62.1 per week. It is consistent with the findings of Kostyshyna and Lalé (2022), as they found that two-thirds of the multiple job spells are used to increase working hours. Average employment duration for single job spells (106 weeks) is longer than multiple job spells (34 weeks), suggesting that multiple job holdings are short-term in nature. Similar patterns were observed in Kimmel and Conway (2001). A recent study in Hahn, Hyatt, and Janicki (2021) found that job movers are more likely to increase their working hours than job stayers and the growth rates range from 0.5% to 1.4%. We have similar findings as the average hours for new single job offers (39.1 per week) are longer than the current single jobs (36.6 per week) by around 7%.

Table 1 Descriptive statistics.

Variables	Mean	SD	min	max
Real wage (cent) in current job	452	261	1	1858
Real wage (cent) in new single job offer	517	267	9	1636
Real wage (cent) in new second job offer	434	239	39	1623
Real wage (cent) in post unemployment job offer	409	212	67	1644
Weekly working hours in current job (one job)	36.6	11.7	0	168
Weekly working hours in current jobs (two jobs)	62.1	19.5	0	168
Weekly working hours in new single job	39.1	8.77	1	168
Weekly working hours in new second job	29.8	13.7	1	140
Weekly working hours in post unemployment job	35.0	11.0	1	144
Weekly leisure hours	127	16.0	0	168
Duration (week) for employment spell (one job)	106	133	1	1346
Duration (week) for employment spell (two jobs)	34.2	47.1	1	630
Duration (week) for unemployment spell	49.5	65.6	1	631
Previous work experience (week)	391	197	1	1646
Number of jobs held prior to change in job status	7.25	4.67	1	36
Age	29.2	3.13	25	35
Female (dummy)	0.50	0.50	0	1
White (dummy)	0.51	0.50	0	1
Black (dummy)	0.28	0.45	0	1
Married (dummy)	0.32	0.47	0	1
Parenthood (dummy)	0.72	0.45	0	4
Net worth (thousand dollars)	61	136	−300	600
Non high school qualification or lower (dummy)	0.15	0.35	0	1
Industry - public sector (dummy)	0.22	0.41	0	1
Industry- professional services (dummy)	0.21	0.41	0	1
Real interest rate (percent)	1.80	1.90	0.50	6.02

3.2 Empirical models

The different components in the model are estimated separately. Let z be a covariate vector comprising the current working time (h), current wage (w) and a constant. Let $\tilde{z} = (z', h')'$ including the working time for a new job offer (h'). Any function $F(\cdot)$ conditional on z is defined as $F(\cdot|z)$, e.g., the conditional offered wage distribution $G_j(w')$ is denoted as $G_j(w'|z)$.

3.2.1 Conditional reservation wages

The observed wage distribution is truncated at the conditional reservation wage $\xi_j(\tilde{z})$. Instead of solving $\xi_j(\tilde{z})$ using (2), (3) and (8), the reservation wage is commonly estimated by the minimum wage observed in the dataset, as no one would accept a job offer if the wage were lower than the reservation wage (Flinn and Heckman, 1982; Lancaster, 1990). This strategy results in one single reservation wage that applies to all individuals, which is inappropriate in our model as $\xi_j(\tilde{z})$ is the reservation wage condition on $\tilde{z}$. Although one might stratify the entire sample by different values of $\tilde{z}$ and find the corresponding minimum in each sub-sample, its practicability is questionable when $\tilde{z}$ contains numerous continuous variables. Similar issues were discussed in Gørgens (2002). Here, we approximate the conditional minimum wages by their q–centile for a sufficiently small q and estimate them with the quantile regression model:

$$\xi_j(\tilde{z}) = \inf\left\{w'_j : G_j\left(w'_j \mid z\right) \geq q\right\} = \tilde{z}'\beta_{\xi,j}, \tag{24}$$

for j = 0,1,2. We let $\beta_\xi = \left(\beta_{\xi_0}, \beta_{\xi_1}, \beta_{\xi_2}\right)'$. (24) can be estimated by the least absolute deviation estimators (Koenker and Hallock, 2001).

The estimated conditional minimum wages are consistent estimates of the actual conditional minimum wages when $q \to 0$. If the chosen q is too large, the estimated conditional reservation wages would be biased upward, and a fraction of observed wages would be smaller than the estimated reservation wages. Conversely, if q is too small, the estimates would be sensitive to measurement errors. That the observed minimum hourly wage in our sample is one cent only (see Table 1) is probably driven by measurement errors. Unbiased estimation and robustness require a balance.

Table 2 shows the estimated conditional reservation wages using some selected values of q. Since $\xi(\tilde{z})$ depends on $\tilde{z}$, different observations have different values of $\xi(\tilde{z})$. We report the average value $\bar{\xi}$ in Table 2. Also reported is the fraction of observed accepted wages w' that are smaller than the estimated $\xi(\tilde{z})$. We denote it as $\Pr(w' < \xi)$ in Table 2. The estimated

Table 2 Estimated conditional reservation wages (cent). The proportion of observed accepted wages w' smaller than the estimated ξ (percent). Estimated offer acceptance rates for single job offers, second job offers, and post-unemployment offers.

q	**0.2**	**0.5**	**1**	**2**	**3**	**5**
Estimated conditional reservation wages, $\bar{\xi}$	57	82	113	158	191	231
$\Pr(w' < \xi)$ - in percent	0.6	0.7	1.0	1.6	2.3	3.4
Acceptance probability of single job offers, $\bar{P}_1$	0.89	0.87	0.78	0.61	0.41	0.20
Acceptance probability of second job offers, $\bar{P}_2$	0.86	0.84	0.76	0.60	0.53	0.24
Acceptance probability of post-unemployment offers, $\bar{P}_0$	0.93	0.90	0.85	0.72	0.54	0.20

reservation wages have critical impact on the job offer acceptance probability $P_j(z)$ (j = 1,2,0) defined in (9) - (11), as the smaller the reservation wage is, the more likely a worker accepts an offer. Different observations have different $P_j(z)$'s as they depend on z. We report the average values denoted as $\overline{P}_j$ in Table 2.

We use q = 2 as the benchmark scenario when we present the main results in Section 4.1. Sensitivity checks using other values of q will also be provided there. With q = 2, $\overline{\xi}$ is 158 cents, around one-third of the average offered wage (445 cents). There are 1.6% of the observed accepted wages w' smaller than the estimated $\xi(\tilde{z})$, and we replace the latter by the former in such cases. The estimated offer acceptance probabilities range from 60% to 72%. Unemployed job seekers are less selective in the job market, as they accept 72% of the offers in the job market on average, while people holding a job already are more selective, as they consider only 60% of the available offers. Beyond the average values, there is a large variety of individual job acceptance rates (see Table 4). For instance, the estimated $P_1(z)$ ranges from 6% to 99%, meaning that some individuals accept only the highest wages in the wage ladder while some accept almost all offers in the market.

3.2.2 **Offered wage distributions**

After estimating the conditional reservation wage $\xi(\tilde{z})$, we estimate offered wage distribution $G_j(w'|z)$ (j = 0,1,2), which is truncated by $\xi(\tilde{z})$. We assume that $G_j(w'|z)$ has a normal distribution, with mean denoted as $\mu_{w,j}$ and standard deviation as $\sigma_{w,j}$. The truncated normal density function (Amemiya, 1973; Heckman, 1976) is

$$g_j\left(w'|w'\rangle\xi;z\right)=\frac{\phi\left(\left(w'-\mu_{w,j}\right)/\sigma_{w,j}\right)/\sigma_{w,j}}{1-\Phi(\xi)},\quad w>\xi. \tag{25}$$

We assume that $\sigma_{w,j}$ is a constant, while $\mu_{w,j}=z\beta'_{\mu_{w,j}}$. We let $\beta_\mu=\left(\beta_{\mu_{w,0}},\beta_{\mu_{w,1}},\beta_{\mu_{w,2}}\right)'$ and $\sigma=\left(\sigma_{w,0},\sigma_{w,1},\sigma_{w,2}\right)'$. Model (25) is estimated by maximum likelihood method where $L\left(\beta'_\mu,\sigma'|w_i,\xi_i,z_i\right)=\prod_{i=1}^{N}\prod_{j=0}^{2}g_j\left(w'_i|w'_i,\xi_i,z_i\right)$. The estimated $G_j(w'|z)$ is used to construct the empirical $P_j(z)$ in (27). As an alternative to (25), we fit another truncated model by assuming that $G_j(w'|z)$ has log-normal distribution. Since the two competing models are non-nested, we use the model selection test suggested by Vuong (1989). We let $L_{i,0}$ and $L_{i,1}$ be the ith contributions to the likelihood function under the normal and log-normal specification, respectively. Defining $y_i = \ln L_{i,0} - \ln L_{i,1}$ and $\overline{y}$ be the sample mean of y_i, the test statistic is

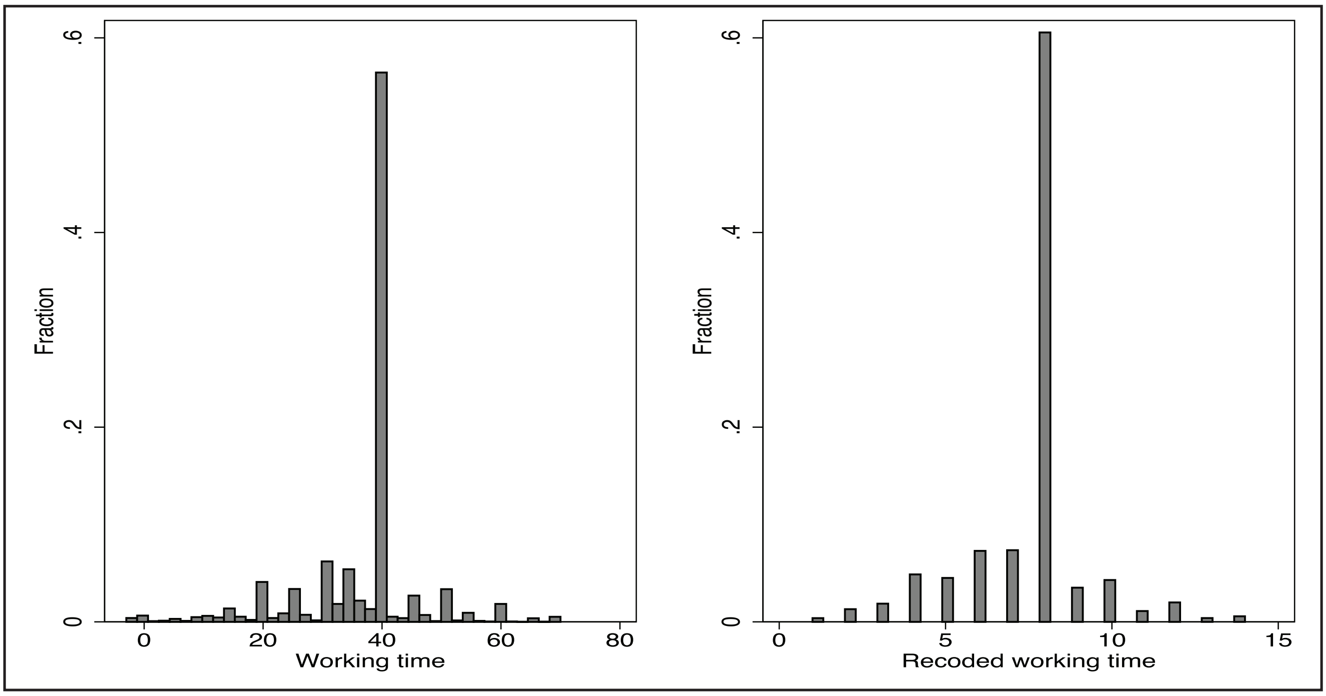
$$V=\frac{n^{-1/2}\sum_{i=1}^{n}y_i}{\sqrt{n^{-1}\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2}}\xrightarrow{D}N\left[0,1\right].$$

The computed value of V is 62.9, which is highly significant. The Vuong's test strongly favors the normal specification against the log-normal specification.

3.2.3 **Working time distributions**

Let $\tilde{h}$ be the non-normalized weekly working hours ranging from 0 to 24 × 7 = 168. The distribution of $\tilde{h}$ for single job offers is displayed in the left panel of Figure 1. In addition to a distinctive spike at $\tilde{h}=40$, working hours tend to be concentrated at certain discrete values and

Figure 1 Histograms for working hours in single job.



are roughly symmetric around the mode at $\tilde{h}=30$. It is common to use discrete distributions to estimate the discretized working hours (see Blundell and MaCurdy, 1999, for review), for examples, multinominal logit model (Van Soest, 1995; Keane and Moffitt, 1998; Creedy and Kalb, 2005), multinominal Probit model (Bingley and Walker, 1997), Poisson model (Jansen et al., 2003), and nonparametric models (Dickens and Lundberg, 1985; Hoynes, 1996; Bloemen, 2008). We fit a mixture Poisson distribution combined with a spike to the data. We recode the working hours into 14 categories defined by v, where $v=\min\left\{\text{ceil}\left(\tilde{h}/5\right),14\right\}$. Hence, $v = 1$ when the working hours fall into the range of $\tilde{h}\in(0,5)$; $v = 2$ for $\tilde{h}\in(5,10)$; and $v = 14$ for $\tilde{h}>65$. The histogram for the categorized working hours v is displayed in the right panel of Figure 1. We let $Poisson_j(v)=\Pr(V=v)=\gamma_j\exp(-\gamma_j)/v!$ be the Poisson density function and v_j be the residual density at $v = 8$ ($\tilde{h}=40$) for job $j = 0,1$ and 2. The mixed density function is

$$m_j(v\,|\,z)=\left(Poisson_j(v\,|\,z)+v_j(z)\mathbb{1}\left\{v=8\right\}\right)/\left(\sum_{x=1}^{14}Poisson_j(x\,|\,z)+v_j(z)\right) \tag{26}$$

for $j = 0,1,2$. We assume that $\gamma_j=\exp\left(z'\beta_{\gamma_j}\right)$ and $v_j(z)=\exp\left(z'\beta_{v_j}\right)$ with $\beta_\gamma=\left(\beta_{\gamma_0},\beta_{\gamma_1},\beta_{\gamma_2}\right)'$ and $\beta_v=\left(\beta_{v_0},\beta_{v_1},\beta_{v_2}\right)'$. Model (26) is estimated by maximum likelihood method, where $L\left(\beta'_\gamma,\beta'_v\,|\,v_i,z_i\right)=\prod_{i=1}^{N}\prod_{j=0}^{2}m_j\left(v_i\,|\,z_i\right)$. The estimated $m_j(v|z)$ is used to construct the empirical $P_j(z)$ in (27). Remarkably, we use the normalized working time $h=\tilde{h}/168\in[0,1]$ in the Bellman equations, but we use the non-normalized weekly working hours $\tilde{h}$ and the categorized working hours v to elaborate the estimation model, as using them is easier to present than using h.

3.2.4 **Hazard rates and sub-density functions for changing job status**

The job offer and separation rates are $\lambda_j(z)=\exp\left(z'\beta_{\lambda_j}\right)$ and $\delta_j(z)=\exp\left(z'\beta_{\delta_j}\right)$, respectively, with $\beta_\lambda=\left(\beta_{\lambda_0},\beta_{\lambda_1},\beta_{\lambda_2}\right)'$ and $\beta_\delta=\left(\beta_{\delta_1},\beta_{\delta_2}\right)'$. The hazard rate of getting a new job is the product of the job offer rate and the offer acceptance probability $P_j(z)$. Using the discretized density function $m_j(v|z)$, $P_j(z)$ defined in (9) - (11) becomes

$$P_j\left(z\right)=\sum_{v=0}^{14}\bar{G}_j\left(\xi_j\mid z\right)m_j\left(v\mid z\right)dv. \tag{27}$$

The hazard rate of accepting a new offer for job j at any time t is

$$\theta_j\left(z\right)=\lambda_j\left(z\right)\sum_{v=0}^{14}\bar{G}_j\left(\xi_j\mid z\right)m_j\left(v\mid z\right)dv. \tag{28}$$

Single job holders have three possible changes in job status: accepting a new job, accepting a second job, and job separation. Since these three possible changes are competing risks, the probability of these three transitions at time t (sub-density functions) are

$$f_{\lambda_j}\left(t;z\right)=\theta_j\left(z\right)\exp\left(-\left(\theta_1\left(z\right)+\theta_2\left(z\right)+\delta_1\left(z\right)\right)t\right)\quad \text{for}\quad j=1,2,\quad \text{and} \tag{29}$$

$$f_{\delta_1}\left(t;z\right)=\delta_1\left(z\right)\exp\left(-\left(\theta_1\left(z\right)+\theta_2\left(z\right)+\delta_1\left(z\right)\right)t\right). \tag{30}$$

The only risk for unemployed individuals is accepting a new offer. Its sub-density function is

$$f_{\lambda_0}\left(t;z\right)=\theta_0\left(z\right)\exp\left(-\theta_0\left(z\right)t\right). \tag{31}$$

For multiple job holders, the two possible changes are separations from either job. A job separation at time t implies that the employment duration for another job exceeds t. Assuming that the employment duration for jobs 1 and 2 are independent after controlling for all observable z, the sub-density functions are

$$f_{\delta_j}\left(t;z\right)=\delta_j\left(z\right)\exp\left(-\delta_j\left(z\right)t\right)\exp\left(-\delta_k\left(z\right)t\right),\quad \text{for}\quad j\neq k\in\left\{1,2\right\}. \tag{32}$$

The probability of separation in job j at time t is $\delta_j(z)$ exp $(-\delta_j(z)t)$ and the probability that the duration of job k exceeds t is exp $(-\delta_k(z)t)$. The independence assumption may be violated if there are unobserved factors governing the durations for jobs 1 and 2. For instance, workers having lower preference of leisure may work longer in both jobs j and k. To evaluate the validity of the independence assumption, we apply the Cox proportional hazard model (Cox, 1972) using the ongoing duration for job k as the dependent variable while the duration for job j together with the other covariates are regressors. The Cox model is chosen here as it is the canonical semi-parametric duration model without specifying the distribution of the dependent variable. Result shows that the duration for job j is insignificant with a p-value equals 0.48.

3.2.5 **Likelihood function**

The likelihood for all spells (indexed as $i = 1,\ldots, n$) is the product of the sub-density functions defined in (29) - (32). Let Δ_{λ_1}, Δ_{λ_2}, and Δ_{δ_1} be the risk indicators for obtaining a new single job, a second job, and job separation, respectively, in single job spells. Let Δ_{δ_1} and Δ_{δ_2} be the risk indicators of separation in jobs 1 and 2, respectively, in multiple job spells. Let Δ_{λ_0} be the risk indicator of taking a post unemployment offer in unemployment spells. The conditional likelihood function is

$$L\left(\beta_\lambda', \beta_\delta' \mid \beta_\xi', \beta_\mu', \sigma', \beta_\gamma', \beta_\nu'\right) = \prod_{i=1}^{N}\prod_{j=0}^{2}\prod_{m=1}^{2}\left(f_{\lambda_j}\left(t_i; z_i\right)\right)^{\Delta_{\lambda_j,i}}\left(f_{\delta_m}\left(t_i; z_i\right)\right)^{\Delta_{\delta_m,i}}. \tag{33}$$

After β_ξ, β_μ, σ, β_γ, and β_ν are estimated using (24), (25), and (26), β_λ and β_δ are estimated using (33) by the maximum likelihood method.

3.2.6 **The minimum value of leisure and optimal working hours**

We consider the case of $\kappa(1-h) = a(1-\log(1-h))$. The minimum value of leisure a_i is affected by z_i in the form of $a_i^{-1} = -z_i'\alpha$. The marginal effect of a covariate z_{ik} on a_i is

$$\frac{\partial a_i}{\partial z_{ik}} = a_i^2 \alpha_k \propto \alpha_k. \tag{34}$$

The marginal effect is individual specific and has the same sign as α_k. To estimate α, we rewrite the wage elasticity of optimal leisure time in (22) as semi-elasticity

$$\frac{\partial \log l_i^{**}}{\partial w_{2i}} = -\frac{1}{a_i E_i} = \frac{z_i'\alpha}{E_i}. \tag{35}$$

As discussed in the introduction, observed working hours h_{1i} in the first job could differ from the optimal h_{1i}^* derived in Corollary 2(i). We assume that workers use the second job to adjust their total working hours to their desired level at any given h_{1i}. Consequently, the observed working hours h_{2i} for the second job represent the optimal h_{2i}^{**} derived in Corollary 3(i) and the observed leisure time $l_i = 1 - h_{1i} - h_{2i}$ is the optimal leisure $l_i^{**} = 1 - h_{1i} - h_{2i}^{**}$ at any given h_{1i}. We can use the information on multiple job spells to identify the structural parameter a_i in (35) using the following reduced form semi-log regression equation:

$$\log l_i = \frac{z_i'\alpha}{\hat{E}_i} w_{2i} + e_i. \tag{36}$$

$\hat{E}_i$ is estimated from (23) using the estimated parameters obtained in the previous steps. e_i is the residual term and is assumed to be uncorrelated with w_{2i} and z_i. This assumption is verified in Section 4.2.6.

After α is estimated, we estimate the optimal working time for each single job spell h_{1i}^* defined in (14) of Corollary 2(i) by solving the following equation (see proof in Appendix):

$$\left(1-h_{1i}^*\right)\prod_{v=0}^{14}\left(1-h_{1i}^*-v\right)^{\hat{\lambda}_{2i}\hat{\bar{G}}_{2i}(\hat{\xi})\hat{m}_{2i}(v)/(\rho+\hat{\delta}_{1i}+\hat{\delta}_{2i})} = \exp\left(-\frac{w_{1i}}{\hat{a}_i}\left(1+\frac{\hat{\lambda}_{2i}\hat{P}_{2i}}{\rho+\hat{\delta}_{1i}+\hat{\delta}_{2i}}\right)\right) \tag{37}$$

The estimated optimal hours $\hat{h}_{1i}^*$ is then compared with the actual hours h_{1i} to estimate the extent of working hours mismatches in single job spells.

4 **Results and Discussions**

In this section we summarize the main results. Detailed reports are provided in supplements upon request.

4.1 Main results

4.1.1 Conditional reservation wages and truncated offer wage distributions

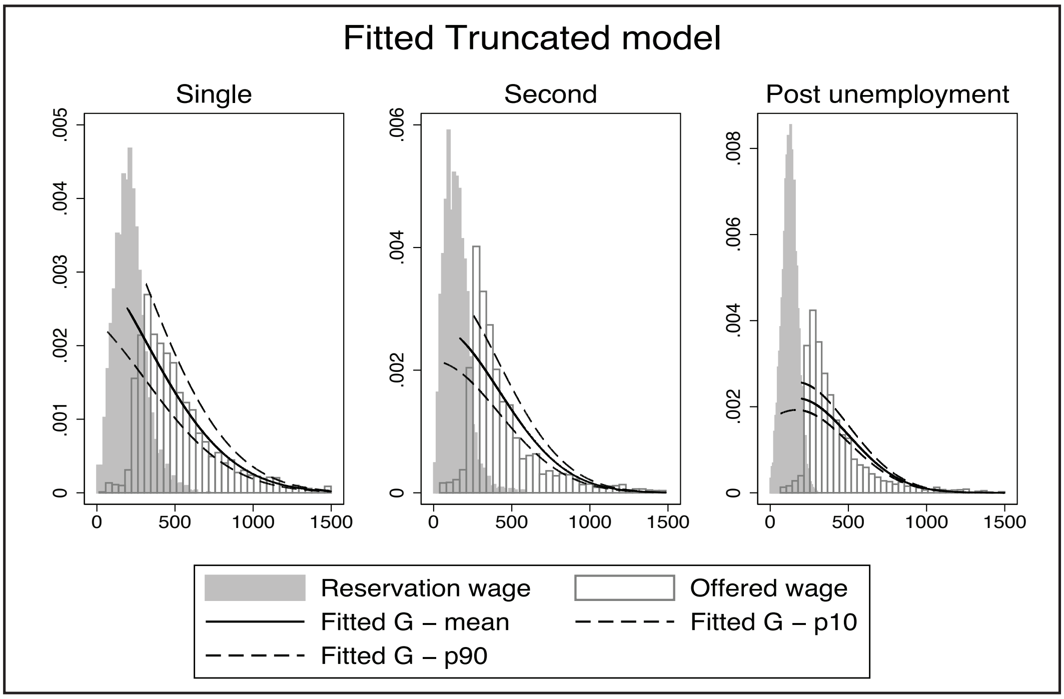
Table 3 shows the summary statistics for the estimated conditional reservation wages. To facilitate comparisons, we report the observed offered wages as well. The estimated conditional reservation wages are highest in single job offers and lowest in post-unemployment offers, whereas the second job offers are somewhere in between. The observed offered wages have the same patterns. We reason that unemployed people have zero wage income and have a lower opportunity cost to accept new offers relative to working people who perform on-the-job search. Conditional reservation wages for second job offers are lower than single job offers as the former are a device to adjust the working hours to the optimal level.

Figure 2 provides the histograms of the estimated reservation wages and the observed offered wages. The observed offered wages are skewed to the right, arguably due to truncations by the reservation wages from below. Since different observations have different reservation wages, the truncation points are also different for different observations. We plot the fitted truncated models for using three truncation points: the mean, 10th percentile, and 90th percentile of the reservation wages. The models fit the data quite well except that the estimated densities are underestimated at the mode for the post-unemployment offers. To evaluate whether

Table 3 Observed offered wages and estimated conditional reservation wages for different types of job, measured in cent $q = 2$.

	Observed offered wage			Estimated reservation wage		
Job types	**Single**	**Second**	**Post-unemployment**	**Single**	**Second**	**Post-unemployment**
Mean	517	434	408	218	154	125
Median	446	355	338	206	139	124
S.D.	268	239	212	121	113	53

Figure 2 Offered wages and reservation wages - Fitted truncated models and histograms.



G_1, and have identical distributions, we apply the cross-model comparison test suggested by Clogg et al. (1995). The chi-square test statistics with 13 degrees of freedom range from 55 to 78 and are rejected at 1% level, indicating that the three offered wage distributions are different.

4.1.2 **Working hours distributions**

Figure 3 reports the histograms for the categorized weekly working hours and the estimated mixed Poisson models. Compared with single jobs, the proportion of second jobs offering the standard 40 hours ($v = 8$) is much smaller, and there is a remarkable proportion having working hours shorter than 25 ($v = 5$). The working hours for post-unemployment offers are somewhere in between. The fitted models match the actual data closely.

4.1.3 **Job offer rates and separation rates**

Table 4 summarizes the estimated job offer rates λ_j, separation rates δ_j, and job acceptance rates P_j. Second jobs have lower offer rates (inflow) and higher separation rates (outflow) than single jobs. These explain why total number of multiple job spells is only one-sixth that of single job spells. Higher separation rates also explain why the average duration for second jobs (34 weeks) is shorter than single jobs (106 weeks). Post-unemployment offer has higher acceptance rates than single and second jobs, implying that unemployed people are less selective in accepting a job offer than job seekers holding job(s).

Figure 3 Fitted distributions for working time.

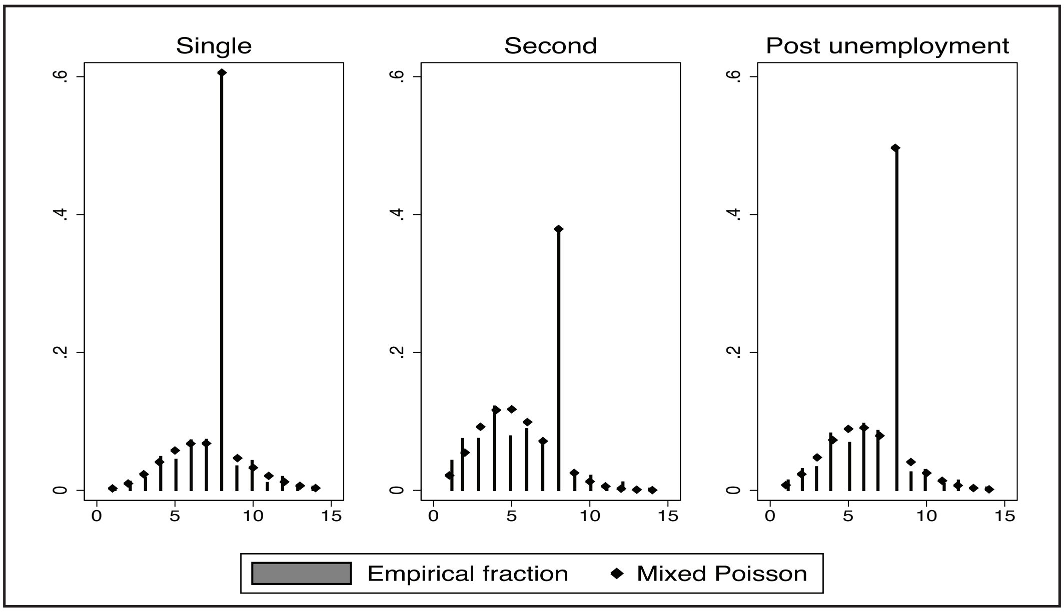


Table 4 Estimated job offer rates λ_j, separation rates δ_j, and acceptance rates P_j in percent.

	Job offer rates, λ_j			**Job separation rates, δ_j**		**Job acceptance rates, P_j**		
Job types	**Single**	**Second**	**Post-unemp**	**Job 1**	**Job 2**	**Single**	**Second**	**Post-unemp**
Mean	0.78	0.55	2.18	1.24	1.71	61.0	59.8	72.4
1 percentile	0.18	0.10	1.16	0.14	0.40	5.58	7.02	21.6
99 percentile	7.36	5.86	7.62	14.1	5.31	98.9	98.9	99.7

4.1.4 **Working hours mismatches**

Results are reported in Table 5. The estimated average minimum value of leisure ($16.8) is around 3.8 times the observed average offered real wage ($4.45). We compare this figure with a few previous studies that estimated similar objects. Bockstael et al. (1987) and Larson and Shaikh (2001) estimated the shadow values of time using static structural models with hours constraints, which are 3.5 to 7 times the wage. Using a reduced form Tobit model with hours constraints, Feather and Shaw (1999) found that the reservation wages are around 106% to 112% of the accepted wages. Allowing for intertemporal substitution in a structural model, Phaneuf et al. (2000) and Lloyd-Smith et al. (2019) found that the values of time are around 90 percent of the wages. It is reminded that the above studies used different theoretical and empirical approaches, which make comparisons not straightforward. In particular, the Tobit model concerns the extensive margin (work or not work) rather than the intensive margin (number of working hours) of labor supply. It is well-known that the extensive margin is more elastic than the intensive margin (see, e.g., Keane and Rogerson, 2012). A higher elasticity of labor supply is accompanied by a lower value of leisure according to Corollary 2(iii), which plausibly explains why the estimated value in Feather and Shaw (1999) is relatively small. For similar reasons, models allowing for intertemporal substitutions tend to provide smaller values of leisure, as the intertemporal elasticities of labor supply are larger than the non-intertemporal elasticities. For instance, Chetty et al. (2011) found that the former are about 3 times larger than the latter. The implied value of leisure would then be 3 times smaller than our estimated values.

The estimated values of B in Table 5 are greater than or equal to one, which is consistent with Corollary 2(iii), but the sizes are relatively small (with mean 1.02). The values of E are smaller than B as predicted by Corollary (3)(iv), and the difference is relatively substantial (with mean 0.67). These suggest that the possibility of multiple job searches adds only a little to the values of leisure in single job spells (measured by aB), but it reduces the values of leisure for multiple job spells (measured by aE) by around one-third.

Agreeing with Corollary (2)(iv) and Corollary (3)(iii), the estimated wage elasticities of leisure time, ϵ_{l,w_1} and ϵ_{l,w_2}, are negative. The average values are -0.26 for single job spells and -0.40 for multiple job spells. Leisure times are around 50% more elastic when multiple jobs are held. Since most of the previous studies estimated the elasticities of labor supply, we transformed them into elasticities of leisure so that direct comparisons are possible. Transformations as such were discussed in Mankiw et al. (1985) and Aguiar et al. (2021):

$$\epsilon_{h,w} = \frac{w}{h}\frac{\partial h}{\partial w} = -\frac{w}{l}\frac{\partial l}{\partial w}\frac{l}{h} = -\frac{l}{h}\epsilon_{l,w} \tag{38}$$

Table 5 Wage rate w_1 (cent), estimated value of leisure a (cent), adjustment factor B and E, wage elasticity of optimal leisure time ϵ_{l,w_1} and ϵ_{l,w_2}, actual working hours h_1 (weekly hour), estimated optimal working hours h_1^* (weekly hour), and the difference between h_1 and h_1^* (weekly hour).

	w_1	a	B	E	ϵ_{l,w_1}	ϵ_{l,w_2}	h_1	h_1^*	$h_1 - h_1^*$
Mean	445	1680	1.02	0.67	−0.26	−0.40	39.0	46.5	−7.50
S.D.	239	410	0.01	0.11	0.12	0.22	8.39	17.4	17.7
1 percentile	126	1101	1.00	0.40	−0.71	−1.27	12	14.4	−57.9
99 percentile	1345	3152	1.08	0.92	−0.08	−0.10	65	97.1	27.7

Leisure l is about four times labor supply h in the dataset of Mankiw et al (1985). In our data sample, the average leisure for a single job spell is 127 hours a week while the average working hours is 39 hours a week. Hence, we use l/h = 3.3 to do similar calculations. In Chetty et al. (2011) the estimated wage elasticity in a model without considering hours constraints is 0.33. The implied leisure elasticity is then -0.10. For models considering hours constraints: the implied leisure elasticities for Zabel (1993) range from -0.02 to -0.17; for Conway and Kimmel (1998) it is -0.32; for Feather and Shaw (2000) they range from -0.15 to -0.21; and for Renna and Oaxaca (2006) they range from -0.12 to -0.15. Our estimated average leisure elasticity for a single job (-0.26) is closed to the lowest end of these previous findings. The above results concern single job spells. For multiple job spells, the implied elasticities of leisure for Shishko and Rostker (1976) range from -0.55 to -0.78; for Conway and Kimmel (1998) they range from -0.01 to -0.14; for Renna and Oaxaca (2006) it is -0.54. Our estimated average elasticity for multiple job spells is -0.40, which is somewhere in between the previous findings.

Next, we discuss the estimated working hours mismatch, defined as $\Delta h = h_1 - h_1^*$. The estimated optimal weekly working hours h_1^* have a mean value of 46.5 hours, which are longer than the observed working hours h_1 averaging at 39.0. Workers underwork by 7.5 hours on average. Figure 4 shows the histogram of the estimated values for Δh. A thicker tail on the left represents the underworking cases.

Table 6 reports the disaggregated figures in detail. For our benchmark model using q = 2, 37% of the observations work longer than optimal, and 63% work shorter. The average working hours for the overworked employees are 41.5, which are longer than their desired 33.1 hours by 8.5 hours. The underworked employees work an average of 37.5 hours, which are shorter than their desired 54.4 hours by 16.9 hours. These suggest that working hours mismatches are common phenomena and the discrepancies are quite substantial. If a week has five working days, the average overworked hours are 1.7 (= 8.5/5) per day, whereas the average underworked hours would be 3.4 (= 16.9/5) per day. As robustness checks, Table 6 shows the estimated results using other values of q, which differ from the benchmark case only marginally. According to Bloemen (2008), workers declared dissatisfied with their working hours only when the

Figure 4 Histogram for the dierences between actual and optimal working hours, Δh.

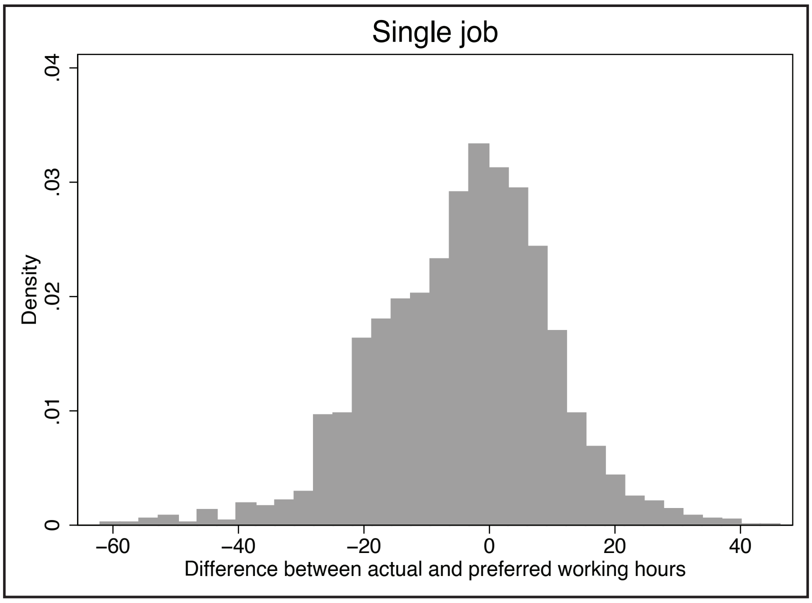


Table 6 Estimated values of leisure a (cent), actual working hours h (weekly hours), estimated optimal working hours h_1^* (weekly hour), and their difference Δh. Different values of q used in quantile regression.

					overworked group			underworked group		
	a	h_1^*	Δh	$\Pr(h_1 > h_1^*)$	h_1	h_1^*	Δh	h_1	h_1^*	Δh
$q = 0.2$	1595	46.6	−7.6	0.36	41.7	32.4	9.3	37.5	54.7	−17.2
$q = 0.5$	1655	46.6	−7.6	0.36	41.7	32.9	8.8	37.5	54.6	−17.1
$q = 1$	1655	46.8	−7.8	0.36	41.7	32.6	9.1	37.5	54.7	−17.2
$q = 2$	1680	46.5	−7.5	0.37	41.5	33.1	8.5	37.5	54.4	−16.9
$q = 3$	1681	47.5	−8.7	0.34	41.2	33.5	7.8	37.5	54.8	−17.2
$q = 5$	1693	48.0	−9.2	0.34	41.1	33.8	7.3	37.5	55.2	−17.7

Table 7 Estimated parameter for the value of leisure (measured in e^{-6}), the estimated partial effect $a_i^2\alpha_k$ - mean, minimum and maximum (measured in cent).

Variables	α_k	The partial effect $a_i^2\alpha_k$		
		mean	min	max
Constant	−857**	−2594	−30665	−793
Previous work experience (weeks)	0.06	0.19	0.06	2.31
Number of previous jobs held	−8.90**	−27.0	−318	−8.23
Age (years)	10.5**	31.8	9.69	375
Female (dummy)	−27.4*	−83.1	−980	−25.3
White (dummy)	17.7	53.6	16.3	632
Black (dummy)	−69.5**	−210	−2486	−64.3
Married (dummy)	4.76	14.4	4.40	170
Parenthood (dummy)	−17.2*	−52.2	−615	−15.9
Net worth (thousands dollar)	0.30**	0.92	0.28	10.8
Non high school qualification or lower (dummy)	−146**	−440	−5202	−134
Industry - public sector (dummy)	115**	348	106	4124
Industry- professional services (dummy)	63.2**	191	58.4	2258

difference is greater than certain threshold values, averaged at 16 hours. Based on this estimated value, the proportion of workers in our sample would have declared overworked reduces from 37% to 5%, while that declared underworked reduces from 63% to 26%, leaving 69% declared satisfied with their working hours. Survey data would have understated the degree of work-leisure mismatch substantially.

Table 7 shows the estimated α_k and the estimated partial effect of covariates z_{ik} on the value of leisure a_i defined as $a_i^2\alpha_k$ in (34). Since $a_i^2\alpha_k$ is individual specific, its mean value, minimum and maximum values are reported. To make interpretations of the above results easier, we establish the relationship between our estimates and the wage elasticity of labor supply. As mentioned in equation (38), the wage elasticity of working hours $\epsilon_{h,w}$ has an opposite sign as the elasticity of leisure $\epsilon_{l,w}$. The more elastic (more positive) the labor supply is, the more elastic (more negative) the demand for leisure is. Note that $\epsilon_{l,w} = -w/aB$ from (17) and the partial effect of z_k on a has the same sign as α_k from (34). Put them together, a positive α_k increases the value of leisure a and leads to a less negative $\epsilon_{l,w}$ and a less positive $\epsilon_{l,h}$, so that labor supply is less elastic. The rationale is as follows: a higher value of leisure increases the opportunity cost of work, which makes an individual less willing to substitute work for leisure when wage increases. This rationale was applied in Connolly (2008) who used weather condition to measure the utility of leisure, while González-Chapela (2007) used the price for various leisure-related goods and services to measure the disutility for work. Using this rationale one can compare our results with the estimated elasticities of labor supply in the literature directly.

Table 7 shows that age and net worth have important roles in the value of leisure. When age increases from 25 to 35, the value of leisure increases by \$3.20 (=\$0.318 × 10) on average, which is approximately 20% of the average value of a (\$16.8). Workers with one more thousand net worth value leisure \$0.92 (5.4%) more. These results are consistent with Attanasio et al. (2018) who found that younger workers are more elastic in labor supply, as young workers have fewer financial responsibilities and more alternative usages of their time. Also, Domeij and Floden (2006) found that workers having financial constraints are two times more elastic in labor supply.

Education has the strongest effect among the dummy variables: workers without high school and degree qualifications have a lower value of leisure by \$4.40 (26%). Types of industry also matter: workers in the public sector and professional services have a higher value of leisure by \$3.48 (20.7%) and \$1.91 (11.3%), respectively, than other industries. These results agree with Bils et al. (2012) as workers having different kinds of human capital value their leisure differently. Also, Kudoh and Sasaki (2011) found that industries requiring different levels of professional training have different search frictions that affect the costs of working. Generally speaking, workers who have better education and/or receive professional training have a higher opportunity cost for leisure, which makes them less elastic in labor supply.

Females value leisure \$0.83 (4.9 %) less than males meaning that females are more elastic in labor supply. It is intuitive as female labors are usually regarded as more substitutable for domestic work than male labors (e.g., Blundell and MaCurdy, 1999), although female elasticities dropped by half over the two decades (Heim, 2007). Workers having at least one child value their leisure \$0.52 (3.1%) less. It is in some way in line with Blundell et al. (1998) who found that the elasticity of labor supply for parents is the largest when their child is aged below four years old, as the young parents in our sample arguably have relatively small kids. The remaining results in Table 7 show that Black workers have lower value of leisure by \$2.10 (12.5%) than white worker and Hispanics. The effect of previous job turnovers is weak, as the value of leisure drops by \$0.27 (1.6%) only when workers have one more previous job. The effects of previous employment duration and marital status are insignificant.

4.2 **Extensions**

4.2.1 **Non-substitutable hours**

We introduce some new variables in this section. The weekly time endowment is denoted as $\tilde{e}_i$, which is allocated to work $\tilde{h}_i$ and leisure $\tilde{l}_i$, such that $\tilde{e}_i = \tilde{h}_i + \tilde{l}_i$. The discussions so far assumed that $\tilde{e}_i$ is 168 hours a week and we did not distinguish the different usages of leisure. Particularly, we assumed that the marginal utility derived from sleeping is the same as the marginal utility derived from home production, transportation, and recreational activities. Suppose now that people are committed to spend $\tilde{n}_i$ hours on certain activities (e.g., sleeping) that cannot be

Table 8 Estimated value of leisure a (cent), wage elasticity of optimal leisure time ϵ_{l,w_1} and ϵ_{l,w_2}, actual working hours h_1 (weekly hour), estimated optimal hours h_1^* (weekly hour), working hours mismatches $\Delta h = h_1 - h_1^*$ (weekly hour). q = 2. Dierent values of $\tilde{e}_i$.

								overwork	underwork
	a	ϵ_{l,w_1}	ϵ_{l,w_2}	h_1	h_1^*	Δh	$\Pr(h_1 > h_1^*)$	Δh	Δh
$\tilde{e}_i = 168$	1680	−0.26	−0.40	39.0	46.5	−7.5	0.37	8.5	−16.9
$\tilde{e}_i = 140$	1508	−0.28	−0.51	39.0	42.1	−3.1	0.47	9.0	−13.7
$\tilde{e}_i = 126$	1485	−0.32	−0.59	39.0	38.3	0.6	0.53	9.9	−10.1

substituted for work $\tilde{h}_i$ or leisure $\tilde{l}_i$, the total allocatable time would be shorter. The question is how far our results are affected by introducing $\tilde{n}_i$.

We consider a simple case, in which $\tilde{n}_i$ is not a decision variable in the optimization problem. $\tilde{n}_i$ is exogenously given and is a fixed value for each individual i independent of the job status. Workers' problem is to maximize their payoff by allocating $\tilde{h}_i$ and $\tilde{l}_i$ subject to the new time endowment $\tilde{e}_i = 168 - \tilde{n}_i$. The model can easily incorporate such a change by defining the new normalized working hours as $h_i = \tilde{h}_i / (168 - \tilde{n}_i)$ and the new normalized leisure hours as $l_i = \tilde{l}_i / (168 - \tilde{n}_i)$. The time constraint implies $h_i + l_i = 1$ as before. The theoretical results established in Corollaries 1-4 apply equally to this extension.

This extension would affect the empirical results. The reason is explained in the following. Our dataset provides us with the individuals' working hours only, but not their leisure time. Leisure time is computed as the residual hours out of working, i.e., $\tilde{l}_i = \tilde{e}_i - \tilde{h}_i$ in the empirical application. If we reduce the value of $\tilde{e}_i$ from 168 to $168 - \tilde{n}_i$, the normalized working hours h_i would be divided by a smaller denominator which makes h_i larger. Likewise, the normalized leisure time $l_i = 1 - h_i$ would become smaller. For any given w_i, Δw_i, and Δl_i, the elasticity of leisure defined as $\epsilon_{l,w} = w_i / l_i \times \Delta l_i / \Delta w_i$ would have larger absolute value when a smaller l_i is used. As can be seen from $\epsilon_{l,w} = -w_i / a_i E_i$ in (22), a larger absolute value of elasticity would lead to a smaller estimated value of leisure a_i. To sum up, leisure is more elastic and the value of leisure is smaller when the value of $\tilde{n}_i$ increases. Effects on the optimal working hours h_i^* defined in (14) and the working hours mismatches computed by $\Delta h = h_i - h_i^*$ are however unclear.

We have to fix $\tilde{n}_i$ to perform the empirical analysis. Since the actual leisure time and the usages of leisure are not provided in the dataset, such a choice is inevitably arbitrary. As robustness checks, we set two values of $\tilde{n}_i$. If people need 4 hours a day for the non-substitutable activities, $\tilde{n}_i = 4 \times 7 = 28$, and the new time endowment becomes $\tilde{e}_i = 168 - 28 = 140$. If people need 6 hours for these activities, $\tilde{n}_i = 6 \times 7 = 42$, and the new time endowment becomes $\tilde{e}_i = 168 - 42 = 126$. Since the results in Table 2 to 4 are not affected by this extension, we provide the new results of some key figures in Table 5 and 6 below. To ease comparisons, the original results using $\tilde{e}_i = 168$ are also reported.

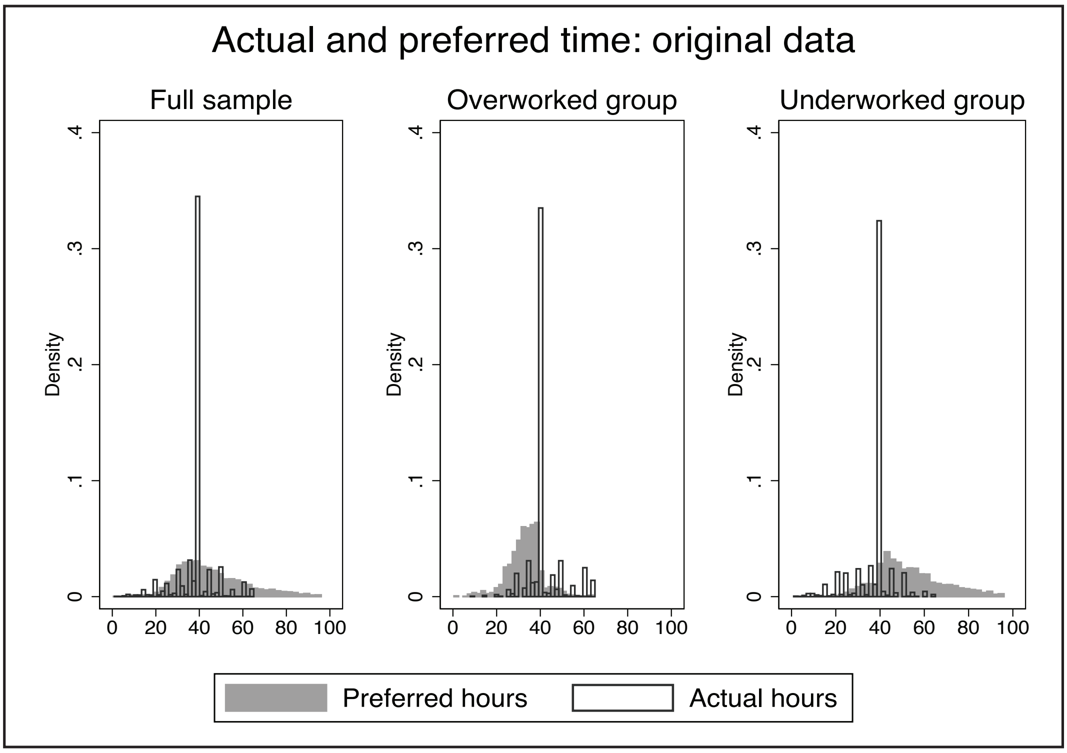
Consistent with our predictions, leisure is more elastic (changing from -0.26 to -0.32 for single job holdings and from -0.40 to -0.59 for multiple job holdings) while the value of leisure drops (from \$16.8 to \$14.9), when people have a smaller time endowment $\tilde{e}_i$ due to their commitments in non-substitutable activities. Regarding the working hours mismatches, the estimated optimal working hours h_1^* drop from 46.5 hours a week to 38.3 hours a week, which makes the probability of overwork increases from 37% to 53%. The average mismatch Δh changes from underworking for 7.5 hours a week to overworking 0.6 hours a week. For the overworked group, the average number of overworked hours increases from 8.5 to 9.9 a week. For the underworked group, the average number of underworked hours drops from 16.9 to 10.1. The main impact of shortening the endowment $\tilde{e}_i$ is mitigating the extent of underwork. To conclude, when workers are committed to more non-substitutable activities, their value for leisure would be smaller and their demand for leisure would be more elastic due to a smaller endowment for allocatable time. Consequently, the optimal working hours would be smaller and the chance of underwork would be smaller.

4.2.2 **Policy analysis**

We examine to what extent a policy enhancing working hours flexibility would alleviate working hours mismatches in this section. Policies promoting flexible hours have become popular in the last decades in European countries (see Lewis, 2003; Plantenga, 2009; Messenger, 2018, for reviews). These policies give employees the right to bargain with their employers so that they can have more flexibility in determining their working schedules. It was found that a quarter of workers had access to flexible schedules across 30 European countries by 2015 (Chung and Van der Lippe, 2020). Females with family responsibility were most benefited by these policies (Song and Gao, 2020). There are evidence that these policies increase workers' leisure satisfaction and reduce their turnover intention (Kröll and Nüesch, 2019). A survey regarding the increasing practices of flexible work arrangements in the U.S. is found in Katz and Krueger (2019).

We start with a closer look on the nature of mismatches by studying the differences between actual and optimal hours in Figure 5. In our model hours distribution is exogenously determined by the employees and is featured by spikes at certain fixed value of hours (see the left panel of Figure 5). Most noticeably, the spike at the standard 40 hours occupies around 35%

Figure 5 Histograms for the preferred hours and actual hours, original data. *SD* = 0.



of the entire distribution. The desired hours in contrast are smoothly distributed. The middle panel of Figure 5 shows the distributions for the overworked group. It is apparent that most of the overworked workers prefer to work shorter than 40 hours while a lot of them have to work for 40 hours or longer. For the underworked group in the right panel of Figure 5, most of them prefer to work longer than 40 hours while a lot of them have to work for 40 hours or less. Mismatches occur as workers do not have enough choices to match their desired hours.

It is natural to ask to what extent the problem of hours mismatches is alleviated when the offered hours have more variety. We introduce flexibility in the offered hours distribution by adding random noises to the working hours. The random noises have a normal distribution with zero mean and we consider different values of the standard deviation defined as *SD* (measured in weekly hour). If such flexibility were available, the underworked group would choose longer hours, while the overworked group would choose shorter hours. Since shorter hours would never be considered by the underworked group, adding flexibility to reduce the hours is irrelevant to the underworked workers. We add only the positive random noises to the underworked group. Similarly, longer hours would never be considered by the overworked group, we add only the negative random noises to the overworked group. The simulated results using $SD = 4$ are provided in Figure 6. The simulated working hours in the left panel of Figure 6 do not have a distinct spike at 40 hours anymore, and they are more smoothly distributed than the original data. For the overworked group in the middle panel, the size of the overlapping region between the actual and the preferred hours increases substantially, meaning that the differences between the actual and the preferred hours are smaller. Similar observations are found for the underworked group in the right panel.

We report the numerical results in Table 9 using different values of *SD*, and $SD = 0$ refers to original data. Although the probability of overwork $\Pr\left(h_1 > h_1^*\right)$ does not change much with

Figure 6 Histograms for the preferred hours and actual hours, simulated data. $SD = 4$.

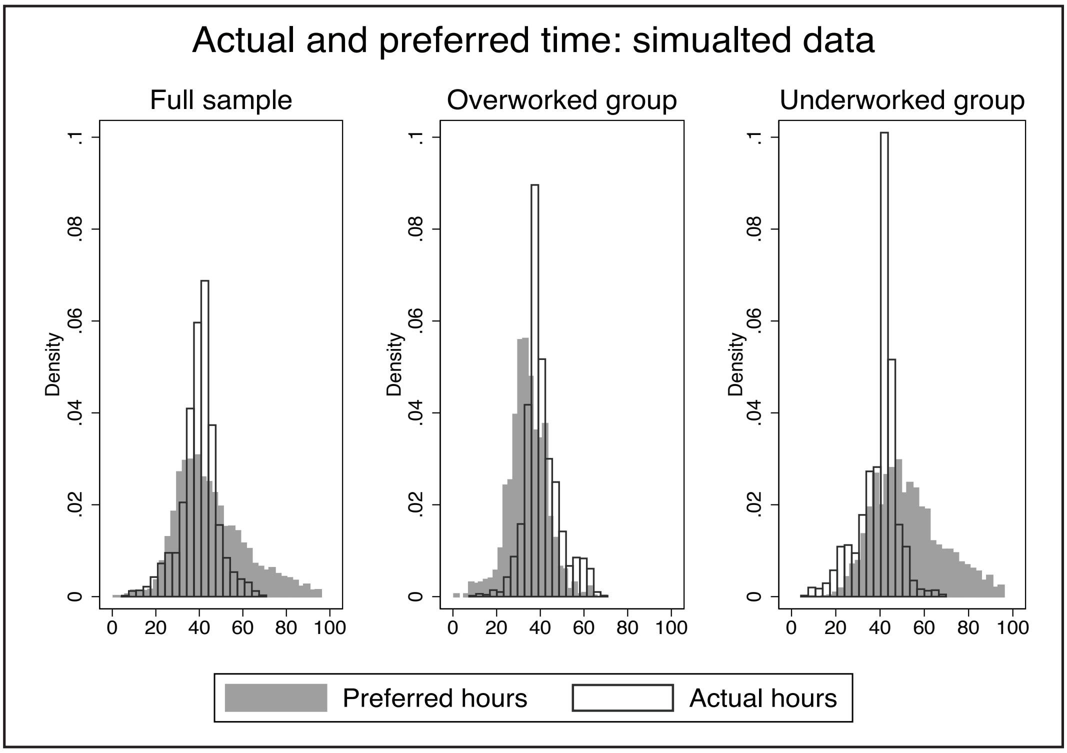


Table 9 Actual working hours h_1 (weekly hour), estimated optimal hours h_1^* (weekly hour), $\Delta h = h_1 - h_1^*$ (weekly hour). $q = 2$. Different SD.

					overwork group			**underwork group**		
	h_1	h_1^*	Δh	$\Pr(h_1 > h_1^*)$	h_1	h_1^*	Δh	h_1	h_1^*	Δh
$SD = 0$	39.0	46.5	−7.5	0.37	41.5	33.1	8.5	37.5	54.4	−16.9
$SD = 2$	39.8	46.5	−7.1	0.37	40.5	33.3	7.2	38.8	54.3	−15.6
$SD = 4$	39.8	46.5	−6.7	0.38	40.5	34.0	6.4	39.4	54.0	−14.5
$SD = 8$	40.7	46.5	−5.8	0.38	42.8	36.2	6.6	39.3	52.8	−13.5

different SD, the average hours mismatch Δh drops obviously with a larger value of SD. For instance, when $SD = 8$ hours a week, the average mismatch drops by one-fourth from underworking 7.5 hours to 5.8 hours. For the overworked group, the overworked hours drop from 8.5 to 6.6. Taking 5 working days a week, the average daily overworked hours drop from 1.7 hours to 1.3 hours. For the underworked group, the underworked hours drop from 16.9 to 13.5, which is a drop from 3.4 hours per day to 2.7 hours.

From the policy's point of view, one would want to know the average size of the noises added to the working hours distribution. It is found that the random noises have an average absolute size of 6.5 hours when $SD = 8$. In other words, a policy allowing employees to adjust their working hours by ±6.5 hours per week (or ±1.3 hours per day), the average mismatch would be reduced by almost one-fourth from 7.5 hours to 5.8 hours.

4.2.3 **A model with more than two jobs in multiple job spells**

We consider an extension where workers can hold more than two jobs in multiple job spells. We fix $J = 3$ for illustration. The term 'multiple job spell' is reserved for those holding two jobs in a working spell, while we introduce the term 'triple job spell' for those holding three jobs. In our dataset, single job holders never accept two new jobs at the same time and therefore it is not possible to switch from a single job spell to a triple job spell. Also, a switch from unemployment to a triple job spell is not found in the data. The only way a worker holds the third job (job 3) is a switch from a multiple job spell to a triple job spell. For this reason, we only need to revise the Bellman equations for the multiple job holders in (6). Let λ_3 be the job offer rate and δ_3 be the job separation rate for job 3. Let w_3' be the offered wage for job 3 with conditional distribution G_3 and h_3' be the offered hours for job 3 with distribution M_3. The expected present value for a triple job spell is defined as $V_3(w_1h_1, w_2h_2, w_3h_3)$.

Equation (6) is revised as

$$\begin{aligned}
&\left(\rho+\delta_1+\delta_2\right)V_2\left(w_1h_1,w_2h_2\right)\\
&=w_1h_1+w_2h_2+K\left(1-h_1-h_2\right)+\delta_2V_1\left(w_1h_1\right)+\delta_1V_1\left(w_2h_2\right)\\
&\quad+\lambda_3\int_0^{1-h_1-h_2}\int_{\xi_3}^{\infty}\left[V_3\left(w_1h_1,w_2h_2,w_3'h_3'\right)-V_2\left(w_1h_1,w_2h_2\right)\right]dG_3\left(w_3'\right)dM_3\left(h_3'\right).
\end{aligned} \tag{39}$$

The conditional reservation wage ξ_3 is the solution of

$$V_3\left(w_1h_1,w_2h_2,\xi_3h_3'\right)=V_2\left(w_1h_1,w_2h_2\right) \tag{40}$$

Comparing (6) with (39), the added item is the option value created by the search for job 3. Workers holding three jobs have three possible changes in their job status, i.e., separate from job 1 and hold jobs 2 and 3, or separate from job 2 and hold jobs 1 and 3, or separate from job 3 and hold jobs 1 and 2. The Bellman equation for a triple job spell is

$$\begin{aligned}&\left(\rho+\delta_1+\delta_2+\delta_3\right)V_3\left(w_1h_1,w_2h_2,w_3h_3\right)\\&=w_1h_1+w_2h_2+w_3h_3+K\left(1-h_1-h_2-h_3\right)\\&\quad+\delta_1V_2\left(w_2h_2,w_3h_3\right)+\delta_2V_2\left(w_1h_1,w_3h_3\right)+\delta_3V_2\left(w_1h_1,w_2h_2\right).\end{aligned}\tag{41}$$

Since $V_2(w_1h_1, w_2h_2)$ in (39) involves the option value, the derivations for Corollary 1-4 would be analytically complicated, although it does not provide any additional useful insight. Moreover, triple job spells represent only 1.5% of our dataset, we do not think it would create a significant difference in our estimates. Setting $J = 2$ is a reasonable tradeoff.

4.2.4 **A model with two consecutive multiple job spells**

It is possible that multiple job holders take a second job by the same time they quit one of the two existing jobs. The Bellman equation for multiple job holders in (6) would be extended to

$$\begin{aligned}&\left(\rho+\delta_1+\delta_2\right)V_2\left(w_1h_1,w_2h_2\right)\\&=w_1h_1+w_2h_2+K\left(1-h_1-h_2\right)+\delta_2V_1\left(w_1h_1\right)+\delta_1V_1\left(w_2h_2\right)\\&\quad+\lambda_4\int_0^{1-h_1}\int_{\xi_{14}}^{\infty}\left[V_2\left(w_1h_1,\,w_4'h_4'\right)-V_2\left(w_1h_1,w_2h_2\right)\right]dG_4\left(w_4'\right)dM_4\left(h_4'\right)\\&\quad+\lambda_4\int_0^{1-h_2}\int_{\xi_{24}}^{\infty}\left[V_2\left(w_2h_2,\,w_4'h_4'\right)-V_2\left(w_1h_1,w_2h_2\right)\right]dG_4\left(w_4'\right)dM_4\left(h_4'\right).\end{aligned}\tag{42}$$

Let λ_4 be the job arrival rate for a new second job (we call it job 4) when multiple job holders quit either of the existing jobs in the current multiple job spell. If these workers quit the existing job 2, they keep the existing job 1 and hold job 4 in the new multiple job spell. The option value in the second last item of (42) is the expected gain from the existing multiple job spell $V_2(w_1h_1,w_2h_2)$ to the new multiple job spell $V_2\left(w_1h_1, w_4'h_4'\right)$, where the new second job 4 has wage w_4' and hour h_4'. If these workers quit the existing job 1, the new multiple job spell is $V_2\left(w_2h_2, w_4'h_4'\right)$, as they keep the existing job 2 and hold the new job 4. Since this case represents only 4% of the multiple job spells, we ignore them in our main results.

4.2.5 **A model ignoring multiple job searching**

We discuss the biases created in our estimates when multiple job searching is ignored. In the framework of our model, the effect of ignoring multiple job holdings can be replicated by fixing the job offer rate for the second job as zero, i.e., $\lambda_2 = 0$. Also, h_1 would be zero, as there is no existing job 1 and the multiple job spell becomes a single job spell. as there is no existing job 1 and the multiple job spell becomes a single job spell. The structural parameter E in (23) would be identical to one. We repeat our estimations by setting $E \equiv 1$ in our estimating equation (36). The estimated average value of a drops by 35% from \$1680 to \$1099, while the demand for leisure computed in (22) is 58% more elastic changing from -0.26 to -0.41. The value of leisure is biased downward while the leisure elasticity is biased upward in a model ignoring multiple job holdings.

4.2.6 Unobservable heterogeneity

We consider the situation where unobserved heterogeneity θ_i may affect workers' value of leisure and their demand for leisure time. The error them e_i in the semi-log regression equation in (36) can be correlated with the covariates z_i and the wage w_i in this case. And we rewrite (36) as

$$\log l_i = \frac{z_i'\alpha}{\hat{E}_i} w_{2i} + \frac{\theta_i \gamma}{\hat{E}_i} w_{2i} + \tilde{e}_i, \tag{43}$$

By controlling θ_i, the error term $\tilde{e}_i$ is uncorrelated with the regressors. Suppose that θ_i is a fixed effect for each worker, θ_i can be captured by the lagged leisure time defined as $\log(Ll_i)$. Specifically, the lagged leisure time in the kth spell for a worker is the leisure time in his $(k-1)$ th spell. A worker having a larger preference of leisure due to the unobserved θ_i has longer leisure time in all of his working spells. We run (43) by replacing θ_i with $\log(Ll_i)$. Results show that $\log(Ll_i)$ is not significant with a p-value of 0.15, and the estimates for α do not have major changes. We conclude that unobserved heterogeneity has a limited role in our empirical results.

5 Conclusions

Work-leisure balances are advantageous for individuals and society as a whole. This paper used a partial equilibrium job search model to explain the optimal work-leisure tradeoffs for single-job holders and multiple-jobs holders. Using a structural model, we derived several empirically testable implications regarding job search behaviors for both single and multiple jobs. We suggested identification strategies to estimate how individual characteristics determine the perceived values of leisure. By comparing the empirically computed optimal working hours with the actual hours, we estimated the patterns of work-leisure mismatch for single job holders.

Our theoretical model has several implications. First, optimal working hours that maximize individuals' utility are unique. Workers accept jobs with unattractive hours provided that the wages are high enough to compensate for the loss of utility. Second, possibilities of job search create option values. Chances for taking multiple jobs increase the value of leisure for single-job holders. Models ignoring multiple jobs understate the value of leisure and overstate the optimal leisure time. Third, multiple job holders have smaller values of leisure and are more elastic in the demand of leisure than single job holders. Workers are more willing to sacrifice their leisure time to accept multiple jobs than a single job. Fourth, when workers are committed to more non-substitutable activities (e.g., childcare), their value for leisure would be smaller and their demand for leisure would be more elastic due to a smaller endowment for allocatable time.

We estimated our model empirically using a panel dataset containing young adults' work history. Remarkable work-leisure mismatches were found, both overworked and underworked. 63% of the observations worked shorter than desired by an average of 16.9 hours a week, while the remaining 37% worked on average 8.5 hours longer than desired. Our results enriched survey studies based on workers' subjective assessments on their optimal hours, which tended to understate the degree of work-leisure mismatch. The estimated minimum dollar value of leisure was about four times the average hourly real wage. The value of leisure dropped by one-third on average when multiple jobs were held. When workers required 6 hours per day (or 42 hours per week) in non-substitutable activities, the value of leisure dropped from \$16.8 to \$14.9, while the demand for leisure increased by 25% to 50%.

Age, education, and industry are the most important factors in determining leisure values, while gender and having kids play secondary roles. In particular, female, parents, older employees with more education who work in public or professional industries value leisure more than the others and are less elastic in their demand for leisure.

We found in a counterfactual experiment that policies promoting flexible working hours (e.g., allowing employees to adjust the working hours by ± 1.3 hours per day) alleviate the problem of working hours mismatches, as the discrepancy between the actual and desired hours would be reduced by one-fourth from 7.5 hours a week to 5.8 hours on average.

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Appendix

We define the option value for single job offers in (1) as $\psi_1(w_1h_1)$. Let $\overline{F}(y)=1-F(y)$ for any cdf $F(y)$, $\psi_1(w_1h_1)$ becomes

$$\psi_1(w_1h_1)=-\int_0^1\left(\int_{\xi_1}^{\infty}[V_1(w'_1h'_1)-V_1(w_1h_1)d\overline{G}_1(w'_1)\right)dM_1(h'_1).$$

The inner integral in $\psi_1(w_1h_1)$ can be simplified using integration by parts: $\int_{\xi_1}^{\infty}udv=uv|_{\xi_1}^{\infty}-\int_{\xi_1}^{\infty}vdu$. Setting $u=V_1(w'_1h'_1)-V_1(w_1h_1)$ and $v=\overline{G}_1(w'_1)$, $uv|_{\xi_1}^{\infty}=0$ as $\overline{G}_1(\infty)=0$ and $V_1(\xi_1h'_1)=V_1(w_1h_1)$ from (2). Moreover, $du/dw'_1=\nabla_{w'_1}[V_1(w'_1h'_1)]$ as $\nabla_{w'_1}[V_1(w_1h_1)]=0$. $\psi_1(w_1h_1)$ is simplified as

$$\psi_1(w_1h_1)=\int_0^1\left(\int_{\xi_1}^{\infty}\overline{G}_1(w'_1)\nabla_{w'_1}[V_1(w'_1h'_1)]dw'_1\right)dM_1(h'_1). \tag{44}$$

Similarly, we define the option value for second job offers in (1) as $\psi_2(w_1h_1)$, which is

$$\psi_2(w_1h_1)=\int_0^c\left(\int_{\xi_2}^{\infty}\overline{G}_2(w'_2)\nabla_{w'_2}[V_2(w_1h_1,w'_2h'_2)]dw'_2\right)dM_2(h'_2), \tag{45}$$

Proof of Corollary 1: We take the partial derivatives of (1) and (6) with respect to w_1

$$\nabla_{w_1}[V_1(w_1h_1)]=\frac{h_1+\lambda_1\nabla_{w_1}[\psi_1(w_1h_1)]+\lambda_2\nabla_{w_1}[\psi_2(w_1h_1)]}{\rho+\delta_1}, \tag{46}$$

$$\nabla_{w_1}[V_2(w_1h_1,w_2h_2)]=\frac{h_1+\delta_2\nabla_{w_1}[V_1(w_1h_1)]}{\rho+\delta_1+\delta_2}. \tag{47}$$

We have to solve $\nabla_{w_1}[\psi_1(w_1h_1)]$ and $\nabla_{w_1}[\psi_2(w_1h_1)]$ in (46) and (47).

For this purpose, we apply the implicit function theorem to (2),

$$\nabla_{w_1}[\xi_1]\nabla_{\xi_1}[V_1(\xi_1h'_1)]=\nabla_{w_1}[V_1(w_1h_1)]. \tag{48}$$

Also, we apply the Leibniz's integral rule

$$\begin{aligned}&\nabla_{w_1}\left(\int_{\xi_1}^{\infty}\overline{G}_1(w'_1)\nabla_{w'_1}[V_1(w'_1h'_1)]\right)dw'_1\\ &=-\nabla_{w_1}[\xi_1]\overline{G}_1(\xi_1)\nabla_{\xi_1}[V_1(\xi_1h'_1)]+\int_{\xi_1}^{\infty}\frac{\partial}{\partial w_1}\left(\overline{G}_1(w'_1)\nabla_{w'_1}[V_1(w'_1h'_1)]\right)dw'_1\\ &=-\frac{\nabla_{w_1}[V_1(w_1h_1)]}{\nabla_{\xi_1}[V_1(\xi_1h'_1)]}\overline{G}_1(\xi_1)\nabla_{\xi_1}[V_1(\xi_1h'_1)]=-\nabla_{w_1}[V_1(w_1h_1)]\overline{G}_1(\xi_1).\end{aligned} \tag{49}$$

The second item in the second line is zero as $\overline{G}_1(w'_1)\nabla_{w'_1}[V_1(w'_1h'_1)]$ does not contain w_1. The third line comes from (48).

Now, $\nabla_{w_1}[\psi_1(w_1h_1)]$ can be computed by taking the partial derivative of (44) with respect to w_1 and using (49):

$$\begin{aligned}\nabla_{w_1}[\psi_1(w_1h_1)]&=\int_0^1\nabla_{w_1}\left(\int_{\xi_1}^{\infty}\overline{G}_1(w'_1)\nabla_{w'_1}[V_1(w'_1h'_1)]dw'_1\right)dM_1(h'_1)\\ &=-\int_0^1\nabla_{w_1}[V_1(w_1h_1)]\overline{G}_1(\xi_1)dM_1(h'_1)=-\nabla_{w_1}[V_1(w_1h_1)]P_1,\end{aligned} \tag{50}$$

where $P_1=\int_0^1\overline{G}_1(\xi_1)dM_1(h'_1)$ is defined in (9).

We obtain $\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]$ in the same way. Applying the implicit function theorem to (3),

$$\nabla_{w_1}\left[\xi_2\right]\nabla_{\xi_2}\left[V_2\left(w_1h_1,\xi_2h_2'\right)\right]=\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]-\nabla_{w_1}\left[V_2\left(w_1h_1,\xi_2h_2'\right)\right]. \tag{51}$$

Taking the partial derivatives of (45) with respect to w_1, and using Leibniz's rule,

$$\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]=-\int_0^c\left(\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]-\nabla_{w_1}\left[V_2\left(w_1h_1,\xi_2h_2'\right)\right]\right)\overline{G}_2\left(\xi_2\right)dM_2\left(h_2'\right) \tag{52}$$

Replacing $\nabla_{w_1}\left[V_2\left(w_1h_1,\xi_2h_2'\right)\right]$ by (47), (52) becomes

$$\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]=\frac{h_1-\left(\rho+\delta_1\right)\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]}{\rho+\delta_1+\delta_2}P_2. \tag{53}$$

where $P_2=\int_0^c\overline{G}_2\left(\xi_2\right)dM_2\left(h_2'\right)$ is defined in (10).

Now we can derive $\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]$ in (46) by substituting $\nabla_{w_1}\left[\psi_1\left(w_1h_1\right)\right]$ and $\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]$ in (50) and (53) into it.

$$\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]=\frac{h_1-\lambda_1P_1\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]+\lambda_2P_2\left(h_1-\left(\rho+\delta_1\right)\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]\right)\left(\rho+\delta_1+\delta_2\right)^{-1}}{\rho+\delta_1}.$$

By collecting the terms of $\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]$, it can be simplified as

$$\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]=h_1\left(1+\frac{\lambda_2P_2}{\rho+\delta_1+\delta_2}\right)\times\left(\rho+\delta_1+\lambda_1P_1+\frac{\left(\rho+\delta_1\right)\lambda_2P_2}{\rho+\delta_1+\delta_2}\right)^{-1}. \tag{54}$$

Since all items in (54) are positive, $\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]$ is positive. $V_1\left(w_1h_1\right)$ increases with w_1 as stated in Corollary 1(i). Corollary 1(ii) comes immediately from (47) and Corollary 1(i). Namely, $V_2\left(w_1h_1,w_2h_2\right)$ increases with w_1. Corollary 1(iii) comes from (50) and Corollary 1(i). Namely the option value for single job offer decreases with w_1.

For Corollary 1(iv), we put (46) into (53).

$$\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]=\frac{h_1P_2}{\rho+\delta_1+\delta_2}-\frac{\left(\rho+\delta_1\right)P_2}{\rho+\delta_1+\delta_2}\times\frac{h_1+\lambda_1\nabla_{w_1}\left[\psi_1\left(w_1h_1\right)\right]+\lambda_2\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]}{\rho+\delta_1}.$$

After collecting the terms of $\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]$,

$$\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]=-\frac{\lambda_1P_2}{\rho+\delta_1+\delta_2+\lambda_2P_2}\nabla_{w_1}\left[\psi_1\left(w_1h_1\right)\right]. \tag{55}$$

Using Corollary 1(iii), $\nabla_{w_1}\left[\psi_2\left(w_1h_1\right)\right]$ in (55) is positive. It proves Corollary 1(iv). Namely, the option value for second job offer increases with w_1. It completes the proof.

The following formula is useful for latter proofs: Substitute (55) into (46) and put it into (47). By rearranging (51) and due to Corollary 1(i) and (ii)

$$\nabla_{w_1}\left[\xi_2\right]=\frac{-\lambda_1P_1\nabla_{w_1}\left[V_1\left(w_1h_1\right)\right]}{\left(\rho+\delta_1+\delta_2+\lambda_2P_2\right)\nabla_{\xi_2}\left[V_2\left(w_1h_1,\xi_2h_2'\right)\right]}<0. \tag{56}$$

Proof of Corollary 2: The optimization problem is $\max_{h_1}V_1\left(w_1h_1\right)$. The FOC is obtained by taking the partial derivative of $V_1\left(w_1h_1\right)$ in (1) with respect to h_1,

$$\left(\rho+\delta_1\right)\nabla_{h_1}\left[V_1\left(w_1h_1\right)\right]=w_1-K'\left(1-h_1\right)+\lambda_1\nabla_{h_1}\left[\psi_1\right]+\lambda_2\nabla_{h_1}\left[\psi_2\right]=0. \tag{57}$$

Solving the optimal h_1^* requires $\nabla_{h_1}[\psi_1]$ and $\nabla_{h_1}[\psi_2]$.

Applying the implicit function theorem to (2),

$$\nabla_{h_1}[\xi_1] = \frac{\nabla_{h_1}[V_1(w_1 h_1)]}{\nabla_{\xi_1}[V_1(\xi_1 h'_1)]}. \tag{58}$$

Taking the partial derivative of (44) with respect to h_1, using Leibniz's rule and (58),

$$\nabla_{h_1}[\psi_1] = -\nabla_{h_1}[V_1(w_1 h_1)]P_1. \tag{59}$$

$\nabla_{h_1}[\psi_1]$ is solved in term of $\nabla_{h_1}[V_1(w_1 h_1)]$.

Next, we derive $\nabla_{h_1}[\psi_2]$. Taking partial derivative of (6) with respect to h_1,

$$\nabla_{h_1}[V_2(w_1 h_1, w'_2 h'_2)] = \frac{w_1 - K'(1-h_1-h'_2) + \delta_2 \nabla_{h_1}[V_1(w_1 h_1)]}{\rho+\delta_1+\delta_2}. \tag{60}$$

Applying the implicit function theorem to (3),

$$\nabla_{h_1}[\xi_2]\nabla_{\xi_2}[V_2(w_1 h_1, \xi_2 h'_2)] = \nabla_{h_1}[V_1(w_1 h_1)] - \nabla_{h_1}[V_2(w_1 h_1, \xi_2 h'_2)]. \tag{61}$$

Putting (60) into (61) and simplifying it

$$\nabla_{h_1}[\xi_2] = \frac{(\rho+\delta_1)\nabla_{h_1}V_1(w_1 h_1) - w_1 + K'(1-h_1-h'_2)}{(\rho+\delta_1+\delta_2)\nabla_{\xi_2}[V_2(w_1 h_1, \xi_2 h'_2)]}. \tag{62}$$

We use the Leibniz's rule to derive

$$\begin{aligned}
&\nabla_{h_1}\left(\int_{\xi_2}^{\infty}\bar{G}_2(w'_2)\nabla_{w'_2}[V_2(w_1 h_1, w'_2 h'_2)]dw'_2\right) \\
&= -\nabla_{h_1}[\xi_2]\bar{G}_2(\xi_2)\nabla_{\xi_2}[V_2(w_1 h_1, \xi_2 h'_2)] + \int_{\xi_2}^{\infty}\frac{\partial}{\partial h_1}\left(\bar{G}_2(w'_2)\nabla_{w'_2}[V_2(w_1 h_1, w'_2 h'_2)]\right)dw'_2 \\
&= -\left(\nabla_{h_1}[V_1(w_1 h_1)] - \nabla_{h_1}[V_2(w_1 h_1, \xi_2 h'_2)]\right)\bar{G}_2(\xi_2) \\
&= -\frac{(\rho+\delta_1)\nabla_{h_1}[V_1(w_1 h_1)] - w_1 + K'(1-h_1-h'_2)}{\rho+\delta_1+\delta_2}\bar{G}_2(\xi_2).
\end{aligned} \tag{63}$$

The second item in the second line disappears as $\nabla_{w'_2}[V_2(w_1 h_1, w'_2 h'_2)] = (h'_2 + \delta_1[V_1(w'_2 h'_2)])$ $(\rho+\delta_1+\delta_2)^{-1}$ (see (47)) does not contain h_1. The third line comes from (61) . The last line substitutes $\nabla_{h_1}[V_2(w_1 h_1, \xi_2 h'_2)]$ by (60).

We are now ready to derive $\nabla_{h_1}[\psi_2]$. Taking the partial derivative of (45) with respect to h_1, we have from (63)

$$\begin{aligned}
\nabla_{h_1}[\psi_2] &= -\int_0^c \frac{(\rho+\delta_1)\nabla_{h_1}[V_1(w_1 h_1)]}{\rho+\delta_1+\delta_2}\bar{G}_2(\xi_2)dM_2(h'_2) + \int_0^c \frac{w_1 - K'(1-h_1-h'_2)}{\rho+\delta_1+\delta_2}\bar{G}_2(\xi_2)dM_2(h'_2) \\
&= \frac{-(\rho+\delta_1)\nabla_{h_1}[V_1(w_1 h_1)]P_2}{\rho+\delta_1+\delta_2} + \frac{E_{h'_2}[w_1 - K'(1-h_1-h'_2)]}{\rho+\delta_1+\delta_2}.
\end{aligned} \tag{64}$$

where $P_2 = \int_0^c \bar{G}_2(\xi_2)dM_2(h'_2)$ is defined in (10) and $E_{h'_2}[w_1 - K'(h_1 + h'_2)] = \int_0^c [w_1 - K'(1-h_1-h'_2)]$ $\bar{G}_2(w'_2)dM_2(h'_2)$ is defined in (12). Put $\nabla_{h_1}[V_1(w_1 h_1)]$ in (57) into (64) and collecting the terms of $\nabla_{h_1}[\psi_2]$,

$$\nabla_{h_1}\left[\psi_2\right]=\frac{-\lambda_1 P_2 \nabla_{h_1}\left[\psi_1\right]+E_{h_2'}\left[K'\left(1-h_1\right)-K'\left(1-h_1-h_2'\right)\right]}{\rho+\delta_1+\delta_2+\lambda_2 P_2}. \tag{65}$$

We obtain $\nabla_{h_1}\left[\psi_2\right]$ in terms of $\nabla_{h_1}\left[\psi_1\right]$. So we can now put $\nabla_{h_1}\left[\psi_1\right]$ in (59) and $\nabla_{h_1}\left[\psi_2\right]$ in (65) into the FOC in (57) and replace $\nabla_{h_1}\left[\psi_1\right]$ by (59). After collecting terms,

$$\nabla_{h_1}\left[V_1\left(w_1 h_1\right)\right]=\left(w_1-K'\left(1-h_1\right)+\frac{\lambda_2 E_{h_2'}\left[w_1-K'\left(1-h_1-h_2'\right)\right]}{\rho+\delta_1+\delta_2}\right)\times\left(\rho+\delta_1+\lambda_1 P_1+\frac{\left(\rho+\delta_1\right)\lambda_2 P_2}{\rho+\delta_1+\delta_2}\right)^{-1}.$$

The optimal h_1^* is solved by setting $\nabla_{h_1}\left[V_1\left(w_1 h_1\right)\right]=0$, i.e.,

$$w_1-K'\left(1-h_1\right)+\frac{\lambda_2 E_{h_2'}\left[w_1-K'\left(1-h_1-h_2'\right)\right]}{\rho+\delta_1+\delta_2}=0. \tag{66}$$

It proves Corollary 2(i).

Taking the second order derivative of $V_1\left(w_1 h_1\right)$ with respect to h_1 using (66)

$$\nabla_{h_1}^{(2)}\left[V_1\left(w_1 h_1\right)\right] \quad \propto \quad K''\left(1-h_1\right)+\frac{\lambda_2 \nabla_{h_1}\left[E_{h_2'}\left[w_1-K'\left(1-h_1-h_2'\right)\right]\right]}{\rho+\delta_1+\delta_2}<0. \tag{67}$$

Since $K''<0$ by assumption, $E_{h_2'}\left[w_1-K'\left(1-h_1-h_2'\right)\right]$ drops with h_1. Both items in (67) is negative and therefore $\nabla_{h_1}^{(2)}\left[V_1\left(w_1 h_1\right)\right]$ is negative. There is a unique optimal (maximizing) solution for $\nabla_{h_1}\left[V_1\left(w_1 h_1\right)\right]=0$. Corollary 2 (ii) is proved.

We consider a case where there is no second job. We put $\lambda_2=0$ into the FOC in (66). The unique solution h_1^0 is solved by

$$w_1-K'\left(1-h_1^0\right)=0. \tag{68}$$

For the case of $K\left(1-h\right)=a\left(1-h\right)\left(1-\log\left(1-h\right)\right)$, $K'\left(1-h\right)=-a\log\left(1-h\right)$ $\left(>0\right)$ and $K''\left(1-h\right)=-a/\left(1-h\right)$ $\left(<0\right)$. We have $h_1^0=1-e^{-w_1/a}$, which is ([optw0log]) in Corollary 2 (iii).

Next, we prove that $h_1^*<h_1^0$ in Corollary 2 (iii). Evaluating (66) at h_1^*,

$$w_1-K'\left(1-h_1^*\right)=-\frac{\lambda_2 E_{h_2'}\left[w_1-K'\left(1-h_1^*-h_2'\right)\right]}{\rho+\delta_1+\delta_2}. \tag{69}$$

Suppose $w_1-K'\left(1-h_1^*\right)<0$ then $E_{h_2'}\left[w_1-K'\left(1-h_1^*-h_2'\right)\right]$ must also be negative since $K'\left(1-h\right)$ is strictly increasing in h (as $K''<0$) and $K'\left(1-h_1^*-h_2'\right)>K'\left(1-h_1^*\right)>0$ for all $h_2'\geq 0$. However, it is not possible that the LHS of (66) is negative while the RHS is positive, unless both sides are zero. Such as case is also ruled out as the only way to make $E_{h_2'}\left[w_1-K'\left(1-h_1^*-h_2'\right)\right]$ identical to zero is that $h_2'\equiv 0$ (which is a trivial case). Therefore, $w_1-K'\left(1-h_1^*\right)<0$ leads to contradiction and it must be that

$$w_1-K'\left(1-h_1^*\right)>0, \tag{70}$$

$$E_{h_2'}\left[w_1-K'\left(1-h_1^*-h_2'\right)\right]<0. \tag{71}$$

Comparing (70) to (68), we have

$$K'\left(1-h_1^*\right)<K'\left(1-h_1^0\right).$$

As $K'(1-h)$ is strictly increasing in h, it must be that $h_1^* < h_1^0$. We finished the proof for Corollary 2(iii).

We proof Corollary 2(iv) as follows. Take the total differentiation of the FOC in (66) with respect to w_1 and evaluated at $h_1 = h_1^*$,

$$1 + K''(1-h_1^*)\frac{\partial h_1^*}{\partial w_1} + \frac{\lambda_2}{\rho+\delta_1+\delta_2}\int_0^c (1+K''(1-h_1^*-h_2')\frac{\partial h_1^*}{\partial w_1}\bar{G}_2(\xi_2)dM_2(h_2')$$
$$-\frac{\lambda_2}{\rho+\delta_1+\delta_2}\int_0^c \left(w_1 - K'\left(1-h_1^*-h_2'\right)\right)g_2(\xi_2)\nabla_{w_1}[\xi_2]dM_2(h_2') = 0, \tag{72}$$

where $g_2(\xi)$ is the pdf of $G_2(\xi)$. By collecting the terms of $\partial h_1/\partial w_1$, it is simplified as

$$\frac{\partial h_1^*}{\partial w_1} = -\frac{1+\lambda_2(\rho+\delta_1+\delta_2)^{-1}(P_2+H_2)}{K''(1-h_1^*)+\lambda_2(\rho+\delta_1+\delta_2)^{-1}E_{h_2'}\left[K''\left(1-h_1^*-h_2'\right)\right]} > 0, \tag{73}$$

where

$$H_2 = -\int_0^c \left(w_1 - K'\left(1-h_1^*-h_2'\right)\right)\nabla_{w_1}[\xi_2]g_2(\xi_2)dM_2(h_2') > 0, \tag{74}$$

H_2 in (74) is positive as $\nabla_{w_1}\left[\xi_{\lambda_2|v2}\right] > 0$ from Corollary 1(iv), and $E_{h_2'}\left[w_1 - K'\left(1-h_1^*-h_2'\right)\right] < 0$ from (71). Since $P_2 > 0$, $H_2 > 0$, $K''\left(1-h_1^*\right) < 0$ and $E_{h'_2}\left[w_1 - K'\left(1-h_1^*-h_2'\right)\right] < 0$, $\partial h_1^*/\partial w_1$ in (73) must be positive.

The wage elasticity of optimal leisure time is computed by putting (73) into the following equation where $\partial l^* / \partial w_1 = -\partial h_1^* / \partial w_1$,

$$\in_{l,w_1} = \frac{w_1}{l^*}\frac{\partial l^*}{\partial w_1} = \frac{w_1}{1-h_1^*} \times \frac{1+\lambda_2(\rho+\delta_1+\delta_2)^{-1}(P_2+H_2)}{K''(1-h_1^*)+\lambda_2(\rho+\delta_1+\delta_2)^{-1}E_{h_2'}\left[K''\left(1-h_1^*-h_2'\right)\right]} < 0. \tag{75}$$

It is negative due to (73).

Consider the special case where $K(1-h) = a(1-h)(1-\log(1-h))$. We substitute $K''(1-h) = -a/(1-h)$ into (75), which can be simplified as (16), i.e.,

$$\in_{l,w_1} = -\frac{w_1}{aB} \tag{76}$$

where

$$B = \frac{\rho+\delta_1+\delta_2+\lambda_2 E_{h_2'}\left[\left(1-h_1^*\right)/\left(1-h_1^*-h_2'\right)\right]}{\rho+\delta_1+\delta_2+\lambda_2(P_2+H_2)} \tag{77}$$

Substituting $\nabla_{w_1}[\xi_2]$ from (56) into (74) and simplifying it using (47) and (54), $H_2 = \lambda_1 P_1 C$, where

$$C = \int_0^c \frac{h_1^*}{h_2'}\frac{\left(w_1 + a\log\left(1-h_1^*-h_2'\right)\right)g_2(\xi_2)dM_2(h_2')}{\rho+\delta_1+\delta_2+\lambda_1 P_1+\lambda_2 P_2}.$$

B is simplified as

$$B \approx \frac{\rho+\delta_1+\delta_2+\lambda_2 E_{h_2'}\left[\left(1-h_1^*\right)/\left(1-h_1^*-h_2'\right)\right]}{\rho+\delta_1+\delta_2+\lambda_2 P_2}. \tag{78}$$

as $P_2+\lambda_1 P_1 C\approx P_2$. It is obvious that $B=1$ when $\lambda_2=0$. To prove $B>1$ when $\lambda_2>0$, it is equivalent to prove

$$E_{h_2'}\left[\left(1-h_1^*\right)/\left(1-h_1^*-h_2'\right)\right]>P_2 \tag{79}$$

We expand the above inequality using the definition of $E_{h_2'}$ in (12) and P_2 in (10). After rearranging, (79) is equivalent to

$$\int_0^c \frac{h_2'}{1-h_1^*-h_2'}\bar{G}_2\left(\xi_2\right)dM_2\left(h_2\right)>0. \tag{80}$$

It must hold as $h_1^*+h_2'<1$ due to time constraints. It completes the proof of Corollary 2(iv). □

Proof of Corollary 3: The FOC for the optimal hours h_2^{**} is given by taking the partial derivative of $V_2\left(w_1h_1,w_2h_2\right)$ in (6) with respect to h_2,

$$\left(\rho+\delta_1+\delta_2\right)\nabla_{h_2}\left[V_2\left(w_1h_1,w_2h_2\right)\right]=w_2-K'\left(1-h_1-h_2\right)+\delta_1\nabla_{h_2}V_1\left(w_2h_2\right)=0. \tag{81}$$

The optimal h_2^{**} is solved by setting $\nabla_{h_2}\left[V_2\left(w_1h_1,w_2h_2\right)\right]=0$. It proves Corollary (3)(i).

Taking the second order derivative of $V_2\left(w_1h_1,w_2h_2\right)$ with respect to h_2,

$$\nabla_{h_2}^{(2)}\left[V_2\left(w_1h_1,w_2h_2\right)\right]\propto K''\left(1-h_1-h_2\right)+\delta_1\nabla_{h_2}^{(2)}V_1\left(w_2h_2\right)<0. \tag{82}$$

It is negative as $K''<0$ and from (67) $\nabla_{h_2}^{(2)}V_1\left(w_2h_2\right)<0$. There is a unique solution for h_2^{**}. It proves Corollary (3)(ii).

Taking the partial derivative of (81) with respect to h_2^{**} and let $D=\rho+\delta_1+\lambda_1P_1+\left(\rho+\delta_1\right)\left(\rho+\delta_1+\delta_2\right)^{-1}\lambda_2P_2$,

$$\begin{aligned}&1+K''\left(1-h_1-h_2^{**}\right)\frac{\partial h_2^{**}}{\partial w_2}+\frac{\delta_1}{D}\left(1+K''\left(1-h_2^{**}\right)\frac{\partial h_2^{**}}{\partial w_2}\right)\\&+\frac{\delta_1}{D}\left(\frac{\lambda_2}{\rho+\delta_1+\delta_2}\right)\left(P_2+E_{h_2'}\left[K''\left(1-h_2^{**}-h_2'\right)\right]\frac{\partial h_2^{**}}{\partial w_2}+H_2\right)=0\end{aligned} \tag{83}$$

Collecting terms, it is simplified as

$$\begin{aligned}\frac{\partial h_2^{**}}{\partial w_2}=&-\left(D+\delta_1+\frac{\delta_1\lambda_2\left(P_2+H_2\right)}{\rho+\delta_1+\delta_2}\right)\\&\times\left(DK''\left(1-h_1-h_2^{**}\right)+\delta_1K''\left(1-h_2^{**}\right)+\frac{\lambda_2\delta_1}{\rho+\delta_1+\delta_2}E_{h_2'}\left[K''\left(1-h_2^{**}-h_2'\right)\right]\right)^{-1}\end{aligned} \tag{84}$$

Using $K''\left(1-h\right)=-a/\left(1-h\right)$, the wage elasticity of optimal leisure time by holding multiple jobs can be computed using (84) as

$$\in_{l,w_2}=-\frac{w_2}{l^{**}}\frac{\partial h_2^{**}}{\partial w_2}=-\frac{w_2}{aE}$$

where

$$E\approx\frac{D+\delta_1\dfrac{1-h_1-h_2^{**}}{1-h_2^{**}}+\delta_1\lambda_2\left(\rho+\delta_1+\delta_2\right)^{-1}E_{h_2'}\left[\dfrac{1-h_1-h_2^{**}}{1-h_2^{**}-h_2'}\right]}{D+\delta_1+\delta_1\lambda_2\left(\rho+\delta_1+\delta_2\right)^{-1}P_2}>0. \tag{85}$$

as H_2 is omitted for the above reason. Evaluating (85) at $h_1 = h_1^*$, using B from (78), and substituting D into it,

$$E = \frac{\rho+\delta_1+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2 + \delta_1 \frac{1-h_1^*-h_2^{**}}{1-h_2^{**}} + \frac{\delta_1\lambda_2}{\rho+\delta_1+\delta_2} E_{h_2'}\left[\frac{1-h_1^*-h_2^{**}}{1-h_2^{**}-h'_2)}\right]}{\rho+\delta_1+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2 + \delta_1 + \frac{\delta_1\lambda_2}{\rho+\delta_1+\delta_2} P_2}$$

$$= \frac{\left(\rho+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2\right) + \delta_1 \frac{1-h_1^*-h_2^{**}}{1-h_2^{**}} + \delta_1\left(1+\frac{\lambda_2}{\rho+\delta_1+\delta_2} E_{h_2'}\left[\frac{1-h_1^*-h_2^{**}}{1-h_2^{**}-h_2')}\right]\right)}{\left(\rho+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2\right) + \delta_1 + \delta_1 1 + \frac{\lambda_2}{\rho+\delta_1+\delta_2} E_{h_2'}\left[\frac{1-w_1^*-h_2^{**}}{1-h_1^*-h_2')}\right] \Big/ B}$$

$$< \frac{\left(\rho+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2\right) + \delta_1 + \delta_1\left(1+\frac{\lambda_2}{\rho+\delta_1+\delta_2} E_{h_2'}\left[\frac{1-h_1^*-h_2^{**}}{1-h_2^{**}-h_2')}\right]\right)}{\left(\rho+\lambda_1 P_1 + \frac{\rho+\delta_1}{\rho+\delta_1+\delta_2}\lambda_2 P_2\right) + \delta_1 + \delta_1\left(1+\frac{\lambda_2}{\rho+\delta_1+\delta_2} E_{h_2'}\left[\frac{1-w_1^*-h_2^{**}}{1-h_1^*-h_2')}\right]\right) \Big/ B}$$

The inequality comes from the fact that $1-h_1^*-h_2^{**} < 1-h_2^{**}$. $E < B$ as $B \geq 1$ and all items in E are positives. To see this,

$$E = \frac{E_1+E_2}{E_1+E_2/B} = \frac{E_1+E_2}{E_1 B + E_2} B < B. \tag{86}$$

It proves Corollary (3)(iii). When $w_1 = w_2 = w$, $\left|\in_{l,w_1}\right| = w/B < w/E = \left|\in_{l,w_2}\right|$. It proves Corollary (3)(iv). □

Proof of Corollary 4: Applying the implicit function theorem by differentiating (2) with respect to h_1',

$$\nabla_{h_1'}[V_1(w_1 h_1) = \nabla_{h_1'}\left[V_1(\xi_1 h_1') + \nabla_{\xi_1}\left[V_1(\xi_1 h_1')\frac{\partial \xi_1}{\partial h_1'}.\right.\right. \tag{87}$$

Since $\nabla_{h'_1}[V_1(w_1 h_1) = 0$, it becomes

$$\frac{\partial \xi_1}{\partial h'_1} = -\frac{\nabla_{h_1'}\left[V_1(\xi_1 h'_1)\right]}{\nabla_{\xi_1}\left[V_1(\xi_1 h'_1)\right]}. \tag{88}$$

From Corollary 1(i), $\nabla_{\xi_1}\left[V_1(\xi_1 h'_1)\right] > 0$, the sign of $\frac{\partial \xi_1}{\partial h'_1}$ is opposite to $\nabla_{h'_1}\left[V_1(\xi_1 h_1')\right]$. From Corollary 2(ii), $\nabla_{h'_1}\left[V_1(\xi_1 h_1')\right] > 0$ when $h'_1 < h_1^*$ and so $\frac{\partial \xi_1}{\partial h_1'} < 0$, the conditional reservation wage drops with h_1'. The opposite holds when $h_1' > h_1^*$. It proves Corollary 4(i). Corollary 4(ii) can be proved in a similar way using Corollary 1(ii) and Corollary 3(ii). □

Proof of equation (37):

Substituting $K(1-h) = a(1-h)(1-\log(1-h))$ into (14) and replace $E_{h_2'}$ using the discretized $m_2(v)$ in (26),

$$w_1 + a\log(1-h_1) + \frac{\lambda_2}{\rho+\lambda_1+\lambda_2}\left(\sum_v (w_1 + a\log(1-h_1-v))\bar{G}_2(\xi_2) m_2(v)\right) = 0.$$

Using $P_2 = \sum_v \bar{G}_2(\xi_2) m_2(v)$ and rearranging it

$$w_1 + \frac{\lambda_2 w_1}{\rho + \lambda_1 + \lambda_2} P_2 = -a \log(1 - h_1) - a \log\left(\prod_v (1 - h_1 - v)^{\frac{\lambda_2 \bar{G}_2(\xi_2) m_2(v)}{\rho + \lambda_1 + \lambda_2}} \right)$$

Hence, (37) is obtained from

$$\log\left((1 - h_1) \prod_v (1 - h_1 - v)^{\frac{\lambda_2 \bar{G}_2(\xi_2) m_2}{\rho + \lambda_1 + \lambda_2}} \right) = -\frac{w_1}{a}\left(1 + \frac{\lambda_2 P_2}{\rho + \lambda_1 + \lambda_2} \right).$$