# ( $)$ Central European Economic Journal 

ISSN: 2543-6821 (online)
Journal homepage: http://ceej.wne.uw.edu.pl

## Basil Dalamagas, John Leventides, Stefanos Tantos

## Equity-efficiency dilemma and tax harmonization

To cite this article
Dalamagas, B., Leventides, J., Tantos, S. (2022). Equity-efficiency dilemma and tax harmonization. Central European Economic Journal, 9(56), 342-353.

DOI: 10.2478/ceej-2022-0020
↔ To link to this article: https://doi.org/10.2478/ceej-2022-0020

Basil Dalamagas<br>National and Kapodistrian University of Athens, Department of Economics, Sofokleous \& Aristidou 1, P.C.10559, Athens, Greece<br>corresponding author: dalamaga@econ.uoa.gr

## John Leventides

National and Kapodistrian University of Athens, Department of Economics, Sofokleous \& Aristidou 1, P.C.10559, Athens, Greece

## Stefanos Tantos

National and Kapodistrian University of Athens, Department of Economics, Sofokleous \& Aristidou 1, P.C.10559, Athens, Greece

# Equity-efficiency dilemma and tax harmonization 


#### Abstract

The present paper attempts to demonstrate that finding an appropriate trade-off between direct and indirect taxes can help smooth policy makers' way through reconciling the contradictory notions of equity and efficiency. Our theoretical and empirical analysis is based on the assumption that direct taxes discourage work effort, thus impinging on the incentives to supply labour, to save and to invest, and finally, to grow, whereas indirect taxes discourage consumption and bear more heavily on the poor. Central to our discussion is the argument that carefully designed adjustments in the tax mix can reduce distortions in the consumption-leisure decision, thus leading to an optimal allocation of resources between the equity and efficiency objectives. To derive a competitive equilibrium setting, a social welfare function is maximized and the first-order conditions are manipulated to trace out the optimal direct-indirect tax rates that pave the way for the equity-efficiency goals to be reconciled with each other.


## Keywords

equity-efficiency trade off, direct-indirect tax rates, growth, income distribution, tax harmonization |

JEL Codes
H21, H3O, E62

## 1. Introduction

The main task of policy makers is to create a favourable economic environment for achieving high rates of growth at a low inflation level. A long-term growing GDP ensures that sufficient tax revenue would be raised to finance the provision of social services and attain optimal allocation of resources and welfare maximization. This in turn would contribute towards achieving the four desired fiscal goals: stabilization, growth, fair distribution of income, and efficient allocation of resources.

Unfortunately, the above government targets do not seem easily attainable. For example, over the years we have had numerous debates about whether
equity (income distribution) should or should not take priority over efficiency (growth). Researchers appear to have well-founded arguments over which of the two policy plans to support or to reject.

The reason for the dispute is that growth without a properly designed policy of reducing income inequality may lead any country to a state of sociopolitical turmoil, whereas an income redistribution policy with low or zero growth tends to undermine the resilience of any social plan, given the scarcity of available resources. The heated controversy surrounding the equityefficiency trade-off, ever since the adoption of fiscal policy measures, has added a new dimension to politics via the emergence of parties that align with the notion of equity (socialists) or efficiency (liberals).

For a detailed presentation and description of the equity-efficiency trade-off, as well as for an extensive argument in support of (or against) either of these principles, there is a good bibliography that deserves recognition for its inclusiveness and organization. See, for example, Saez and Stantcheva (2018), Chiappori and Mazzocco (2017), Gayle and Shephard (2019), Farhi and Gabaix (2020), Stevans (2012), Forbes (2000), Dawson (1998), Deininger and Squire (1996), Perotti (1993), Barro (1999), Saez (2001), Tillmann (2005), Slemrod (1990), Bird (1992), Alesina and Rodrik (1994), Okun (2015).

A general conclusion that can be drawn from a review of these articles is that most of them argue over whether equity and efficiency are rivals or complementary factors in the design and implementation of economic policy. A number of researchers contend that equity and efficiency cannot be achieved simultaneously, as a greater equity comes at the cost of a loss of efficiency: income redistribution results in changes in work effort, in savings, and in physical and human capital, thus leading to less efficiency. This argument, however, is challenged by the incomplete markets constructive framework and sociopolitical theories of an inverse relationship between inequality and growth. Finally, other researchers claim that equity and efficiency complement each other.

## 2. The scope of the present study

The aim of the present study is to move the discussion from the ongoing equity-efficiency tradeoff or conflict to the establishment of a well-founded relationship between the direct-indirect tax trade-off on the one hand and the equity-efficiency trade-off on the other, given that taxation is the main source of financing for the equity/efficiency policy goals. If each of these two categories of taxes could be assigned solely to one of the policy targets, then policy makers would be able to redirect resources to the most desired objective, simply by altering the size and the structure of the tax revenue, provided that the electorate would approve proposed fiscal policy changes.

There are strands of literature on the effects of income/consumption taxes on equity (income redistribution) and efficiency (incentives and growth). For example, progressive labour taxes are shown
to have a positive effect on income distribution (Journard et al., 2012). The corporate profits tax in open economies tends to fall on labour income and worsen income distribution, but in countries with a low mobility of capital, corporate taxes may discourage investment projects (Harberger, 1995). Consumption taxes tend to be shifted to consumers, with lowerincome households invariably paying a larger portion of their income on necessities. Therefore, indirect taxes are expected to result in income inequality (IMF, 2014).

Theoretical discussion showing so many noticeable and opposing viewpoints on the economic effects of direct/indirect taxes implies that an appropriate mix of these taxes should be devised to attain the equity/ efficiency goals (Musgrave, 1959) and maximize social welfare. Atkinson and Stiglitz (1976) claim that a fair income distribution can be achieved by income taxation alone without employing consumption taxes. However, Cremer et al. (2001) hold the opposite view that, in a tax mix of direct/indirect taxes, the benefits of commodity taxes are of a strong redistributive nature.

More of an interest should also be taken in the work of García-Peñalosa and Turnovsky (2011), who examine how changes in the combinations of direct/indirect taxes affect income distribution. For example, an increase in income taxes - far in excess of consumption taxes - that would introduce disincentives to work effort and reduce labour supply are associated with lower output but also with lower income inequalities.

A great contribution to the literature on the effects of direct/indirect taxes and their combinations on equity and/or efficiency has also been made by Chari et al. (2020), Heathcote et al. (2020), Stiglitz (2018), Aghion et al. (2016), Uchida and Ono (2021), Beffy et al. (2019), Jantti et al. (2015), Keane et al. (2016), Stiglitz (2015), Chang et al. (2021), and Reis (2020).

The present study aims at contributing to the design of an optimal direct/indirect tax rate combination that would be perfectly in tune with the ideal of incorporating two opposing fiscal targets (equity-efficiency) into a single and manageable objective. To this end, a social welfare function is developed to amalgamate the widely used index of equity - that is, consumption or income - with the conventional index of efficiency - that is, the leisure or labour force supply. The utility function is then maximized with respect to the direct and indirect tax
rate and the first-order conditions are manipulated to provide the optimal tax mix that can promote simultaneously both policy objectives. In carrying out the estimation process for a sample of six developed countries, a number of assumptions is adopted:

1) According to the preceding analysis, direct taxes - that is, progressive personal income taxes and proportional corporate taxes - are taken to discourage the creation of human and physical capital and hence to limit the prospects for growth, while ensuring a fair distribution of income. Conversely, indirect taxes, such as a VAT with a uniform proportional tax rate, favor income inequalities, since the indirect tax burden as a percentage of income rises as income falls.
2) It becomes evident that the present study sidesteps the question of whether the equity of efficiency principle is the most preferred choice, by focusing our attention on the inner equilibrium obtained from the first-order conditions. It is well known that the ratio of the marginal utilities derived from maximizing the utility function with respect to direct and indirect tax rates represents the marginal rate of substitution (MRS) in consumption between the above tax rates. By the same token, the ratio of the cost shares in terms of distortions that are caused by the introduction of taxation to equity or efficiency represents the marginal rate of transformation in production (MRT) between direct and indirect taxes. If there are no distorting factors arising from the level or the structure of direct-indirect tax rates, the manipulation of the first-order conditions leads to Pareto optimality, where $\mathrm{MRS}=\mathrm{MRT}$ and no government intervention is required.
3) In the prevailing imperfect competition framework, however, private agents and fiscal authorities tend to escape the Pareto optimality rules, so that a scope is provided for policy makers to restore public confidence in the management of the economy. The present study extends the analysis beyond the conventional Pareto's optimality conditions by postulating noncompetitive markets, which are characterized by the existence of monopoly power and distortions through price setting. Private agents are not taken to act as independent units and to interact via the price system, whereas externalities and public goods give rise to divergencies between the private valuation and the social valuation of a wide array of services provided by private and public agents. What should be stressed at this point is
that the present study does not give any attention either to the description and the analysis of the nature of the distortions or to the establishment of the causes of their existence. Instead, we will concentrate on the lack of equality, MRS $\neq \mathrm{MRT}$, due to the existence of distortions, and the design of an optimal mix of direct/indirect tax rates that would help redress the balance, MRS=MRT.
4) Lastly, the present study further extends the traditional approach of evaluating the impact of taxation on equity (efficiency) in a way that restricts the analysis to include the effects of income taxes alone. The notion of mixing taxes requires that combinations of both direct and indirect tax-rate changes should be tested before finding a solution that is reasonable and acceptable to all agents. The underlying assumption is that consumers' behaviour is significantly affected by adjustments in indirect tax rates, which are shown to "force wedges" between post-tax and pre-tax prices.

The rest of the paper is organized as follows: In section 3, we outline some of the important considerations that have been largely ignored by the conventional analysis, by using econometric techniques, simulations, and mathematical tools to underline the practical implications of incorporating our theoretical work into the sphere of applied fiscal policy management. The empirical investigation of the model is presented in section 4, and section 5 concludes the discussion, laying out directions for further work.

## 3. The model

### 3.1. The linear optimization model

The utility maximization model that will be built in the present study to determine the most suitable combination of direct/indirect taxes for attaining the optimal allocation of resources between equity and efficiency will be presented in two working forms, the linear and the logarithmic. The linear model has the property of putting the analysis into a form that is understandable to the reader by considering the circumstances under which distortions are generated and then eliminated. The logarithmic model facilitates developing computational techniques which lead to manageable solutions.

Starting with the linear model, consider an economy consisting of $H$ households indexed $h=1 \ldots H$. Each household has a utility function

$$
\begin{equation*}
U^{h}=U^{h}\left[c^{h}\left(t_{i}, y^{h}\right), l^{h}\left(t_{y}, w\right)\right] \tag{1}
\end{equation*}
$$

where $c^{h}$ is the consumption of household $h$ of the vector of private goods and $l^{h}$ is the household's $h$ supply of labour. Private consumption is taken to be a function of disposable income, $y$, and the indirect tax rate, $t_{i}$, whereas labour supply depends on an exogenously determined wage rate, $w$, and the direct tax rate, $t_{y}$.

To characterize the set of first-best or Paretoefficient allocations, each household chooses $c^{h}$ and $l^{h}, h=1 \ldots H$, to maximize their utility level, constrained by the requirement that all the other households, 2 to H , obtain given utility levels, and by the condition that the government will raise sufficient revenue to finance the provision of public goods. Varying the given utility levels for households 2 to H traces out the set of Pareto-efficient allocations. The Lagrangian for this maximization problem is written

$$
\begin{gather*}
\mathscr{L}=\mu^{1} U^{1}\left[c^{1}\left(t_{i}, y^{1}\right), l^{1}\left(t_{y}, w\right)\right]+ \\
\sum_{h=2}^{H} \mu^{h}\left\{U^{h}\left[c^{h}\left(t_{y}, y^{h}\right), l^{h}\left(t_{y}, w\right)\right]-\bar{U}^{h}\right\}-  \tag{2}\\
-\lambda\left(t_{i} C+t_{y} w L-G\right)
\end{gather*}
$$

where $C=\sum^{H}{ }_{h=1} c^{h}, L=\sum^{H}{ }_{h=1} l^{h}$, and $\bar{U}^{h}$ is the fixed utility level that is achieved by $h=2 \ldots . H$. The term $\mu^{h}$ may be interpreted as the social welfare weight that is given to each household ( $\mu^{h}=1$ for $h=1$ ). Assuming that the specified utility levels can be reached simultaneously, the necessary conditions describing the optimal choice of consumption with respect to the indirect tax rate and the optimal choice of working time with respect to the direct tax rate are

$$
\begin{array}{r}
\frac{\partial \mathscr{L}}{\partial t_{i}}=\mu^{h} \frac{\partial U^{h}}{\partial c^{h}} \frac{\partial c^{h}}{\partial t_{i}}-\lambda\left(C+t_{i} \frac{\partial C}{\partial t_{i}}\right)=0 \\
\frac{\partial \mathscr{L}}{\partial t_{y}}=\mu^{h} \frac{\partial U^{h}}{\partial l^{h}} \frac{\partial l^{h}}{\partial t_{y}}-\lambda\left(w L+t_{y} w \frac{\partial L}{\partial t_{y}}\right)=0 \tag{4}
\end{array}
$$

dividing (3) by (4) and rearranging terms gives

$$
\begin{equation*}
\frac{\frac{\partial U^{h}}{\partial c^{h}}}{\frac{\partial U^{h}}{\partial l^{h}}}=\frac{\frac{\left[c+t_{i}\left(\frac{\partial C}{\partial t_{i}}\right)\right]}{\frac{\partial c^{h}}{\partial t_{i}}}}{\frac{\left[w L+t_{y} w\left(\frac{\partial L}{\partial t_{y}}\right)\right]}{\frac{\partial l^{h}}{\partial t_{y}}}} \tag{5}
\end{equation*}
$$

The left-hand term in (5), i.e. the ratio of the marginal utilities $\left(\partial U^{h}\right) /\left(\partial c^{h}\right) /\left(\partial U^{h}\right) /\left(\partial l^{h}\right)=\left(\partial l^{h}\right) /\left(\partial c^{h}\right)$ represents the marginal rate of substitution in consumption between consumption and leisure (or labour), $M R S_{c, l}$. The term $M R S_{c, l}$ measures the extent to which the utility (benefit) of the household from the consumption of the good will increase if they are willing to sacrifice an extra unit of leisure or to work an extra unit of time in the production of this good (demand function).

It is presumed throughout that
(i) the response of consumption spending to changes in indirect tax rates is equalized across households and to the population as a whole, i.e.,

$$
\begin{equation*}
\frac{\partial C}{\partial t_{i}}=\frac{\partial c^{h}}{\partial t_{i}} \tag{6}
\end{equation*}
$$

and
(ii) the response of labour supply to changes in direct tax rates is equalized across households and to the population as a whole, i.e.,

$$
\begin{equation*}
\frac{\partial L}{\partial t_{y}}=\frac{\partial l^{h}}{\partial t_{y}} \tag{7}
\end{equation*}
$$

Rearranging the terms in (5) gives

$$
\begin{equation*}
\frac{\frac{\partial U^{h}}{\partial c^{h}}}{\frac{\partial U^{h}}{\partial l^{h}}} \frac{\frac{\partial c^{h}}{\partial t_{i}}}{\frac{\partial l^{h}}{\partial t_{y}}}=\frac{C+t_{i} \frac{\partial c^{h}}{\partial t_{i}}}{w L+t_{y} w\left(\frac{\partial l^{h}}{\partial t_{y}}\right)} \tag{8}
\end{equation*}
$$

Given that the entire disposable income is assumed to be consumed, i.e., $w L=C$, and following the simple mathematical formula

$$
\begin{equation*}
\frac{1+A}{1+B}=1+A-B \tag{9}
\end{equation*}
$$

where $A=t_{i}\left(\frac{\partial c^{h}}{\partial t_{i}}\right)$ and $B=t_{y} w\left(\frac{\partial l^{h}}{\partial t_{y}}\right)$, equation (9) takes the form

$$
\begin{align*}
& \frac{\frac{\partial U^{h}}{\partial c^{h}}}{\frac{\partial U^{h}}{\partial l^{h}}} \frac{\frac{\partial t_{i}}{\partial l^{h}}}{\frac{\partial t_{y}}{\partial}}=\frac{C+A}{C+B}=\frac{1+\frac{A}{C}}{1+\frac{B}{C}}=1+\frac{A}{C}- \\
& -\frac{B}{C}=\frac{C+A-B}{C}=\frac{C+t_{i}\left(\frac{\partial c^{h}}{\partial t_{i}}\right)-t_{y} w\left(\frac{\partial c^{h}}{\partial t_{y}}\right)}{C} \tag{10}
\end{align*}
$$

The second term of the left-hand side of (10) is $\left[\left(\partial c^{h}\right) /\left(\partial t_{i}\right)\right] /\left[\left(\partial l^{h}\right) /\left(\partial t_{y}\right)\right]=\left[\left(\partial c^{h}\right) /\left(\partial l^{h}\right)\right] \times\left[\left(\partial t_{y}\right) /\left(\partial t_{i}\right)\right]$ and the term $\partial c^{h} / \partial l^{h}$ represents the marginal rate of transformation in production between consumption and labour.

The term $M R T_{c, l}$ represents the hourly cost of employing an extra unit of labour in the production (and consumption) of the good (the marginal product of labour or the wage rate, w), i.e., the cost of transforming an extra unit of labour in the production (and consumption) of the good (supply function). Therefore, eq. (10) can be written

$$
\begin{equation*}
\frac{\frac{\partial U^{h}}{\partial c^{h}}}{\frac{\partial U^{h}}{\partial l^{h}}}=M R S_{C, l}=M R T_{c, l} \frac{\partial t_{i}}{\partial t_{y}} \frac{C+t_{i}\left(\frac{\partial c^{h}}{\partial t_{i}}\right)-t_{y} w\left(\frac{\partial l^{h}}{\partial t_{y}}\right)}{C} \tag{11}
\end{equation*}
$$

Thus, it becomes evident that, if taxation and other distorting factors are introduced into the analysis, Pareto efficiency can be achieved $\left(M R S_{c, l}=M R T_{c, l}\right)$ only when the term
$\frac{\partial t_{i}}{\partial t_{y}}\left[\frac{c+t_{i}\left(\frac{\partial c^{h}}{\partial t_{i}}\right)-t_{y} w\left(\frac{\partial l^{h}}{\partial t_{y}}\right)}{c}\right]$ is equal to 1 , or when

$$
\begin{equation*}
\frac{\partial t_{y}}{\partial t_{i}}=1+\frac{t_{i}\left(\frac{\partial c^{h}}{\partial t_{i}}\right)}{c}-\frac{t_{y} w\left(\frac{\partial l^{h}}{\partial t_{y}}\right)}{c} \tag{12}
\end{equation*}
$$

In eq. (12), we note that:

1) The term $\partial c^{h} / \partial t_{i}$ demonstrates the effect of a unit change in indirect tax rate on consumption. Since this relationship is expected to be negative, the term $\partial c^{h} / \partial t_{i}$ may be interpreted as the marginal loss of consumption (utility) resulting from a unit increase in indirect tax rate.
2) The term $w\left(\left(\partial l^{h}\right) /\left(\partial t_{y}\right)\right)$ represents the effect of a unit change in direct tax rate on labour supply
(with the wage rate being treated as numeraire). Assuming that this relationship may bear a negative (positive) sign, the term $w\left(\left(\partial l^{h}\right) /\left(\partial t_{y}\right)\right)$ is taken to measure the loss (gain) in terms of hours worked (or income) that arises from a unit increase in the income tax rate.
3) In practice, the equilibrium condition $M R S_{c, l}$ $=M R T_{c, l}$ can rarely be satisfied. This occurs only if the effects of changes in indirect tax rates on consumption, as well as the effects of direct tax rate changes on labour supply, are exactly balanced out. In this case, which is an exception to the rule, $\left(\partial l^{h}\right) /\left(\partial t_{y}\right)=-\left(\partial c^{h}\right) /\left(\partial t_{i}\right)$ and $t_{i}=t_{y}$, so that $\left(\partial t_{i}\right) /\left(\partial t_{y}\right)=1$.
4) When these two kinds of effects differ in magnitude, and given that reactions of both consumers and workers to taxation are not directly controlled by the government, it is only the fiscal instruments $t_{y}$ and $t_{i}$ that can be used by policy makers to ensure equilibrium between $M R S_{c, l}$ and $M R T_{c, l}$.

The extent to which direct and indirect tax rates can be re-combined to equalize $M R S_{c, l}$ and $M R T_{c, l}$ can be estimated from the solution of (12) for $t_{y}$, in order to generate an equilibrium reaction function. Solving (12) for $t_{y}$ results in a complicated formula, as shown in Appendix 1, which however cannot be easily manipulated by policy makers to achieve fiscal objectives. A readily manageable form of the reaction function is given below by eq. (13a),

$$
\begin{equation*}
t_{y}=\rho e^{-\frac{t_{i}}{C}\left(\frac{\partial l^{h}}{\partial t_{y}}\right)}+\frac{\frac{\partial c^{h}}{\partial t_{i}}}{\frac{\partial l^{h}}{\partial t_{y}}} t_{i}+C \frac{\left(\frac{\partial l^{h}}{\partial t_{y}}-\frac{\partial c^{h}}{\partial t_{i}}\right)}{\left(\frac{\partial l^{h}}{\partial t_{y}}\right)^{2}} \tag{13a}
\end{equation*}
$$

where $\rho$ stands for a constant. To simplify the analysis, we assume that labour supply is a linear function of the income tax rate, the wage rate, and a set of other explanatory variables $(X)$, i.e.,

$$
l^{h}=b_{0}+b_{1} t_{y}+b_{2} w+b_{3} X
$$

while consumption spending is a linear function of the indirect tax rate, the wage income, $w l^{h}$, and a set of its own explanatory variables $(Z)$, i.e.,

$$
c^{h}=a_{0}+a_{1} t_{i}+a_{2} w l^{h}+a_{3} Z
$$

The partial derivatives of the above functions with respect to tax rates are the constant coefficients $b_{1}=\left(\partial l^{h}\right) /\left(\partial t_{y}\right)$, and $a_{1}=\left(\partial c^{h}\right) /\left(\partial t_{i}\right)$. Therefore, eq. (13a) can be re-written

$$
\begin{equation*}
t_{y}=\rho e^{-\frac{b_{1}}{C} t_{i}}+\frac{a_{1}}{b_{1}} t_{i}+\frac{C\left(b_{1}-a_{1}\right)}{\left(b_{1}\right)^{2}} \tag{13b}
\end{equation*}
$$

where the constant, $\rho$, describes the initial conditions.
Following a similar procedure in solving (13a), a reaction function in which $t_{i}$ is expressed in term of $t_{y}$ may also be formulated.

Eq. (13b) measures the required change in the direct tax rate following a one-percentage point increase (decrease) in the indirect tax rate. These tax-rate changes are introduced by fiscal authorities in order to eliminate distortions in demand and/ or in labour supply. However, eq. (13b), even in its simplified form, cannot be easily subjected to closer study and our discussion will fail to emphasize some crucial aspects of fiscal policy. These aspects are most relevant to elaborating the practical implications of employing tax-policy instruments, in order to handle distortions in the economy. This occurs because some technical features of the economic system that are strictly necessary for later analysis are obscured by the quite complicated relationship between direct and indirect tax rates. In Section 3.2, we provide an analytical foundation for tax-policy intervention in the economy, in order to cope with distortions in consumer's (and/or worker's) behaviour, by using a non-linear utility function. Such a function allows us to express the variables of interest in terms of elasticities rather than in terms of changes in absolute values.

### 3.2. The logarithmic model

Keeping all the assumptions of Section 3.1 in the modified framework, the utility function takes the form $U^{h}=U^{h}\left[\ln c^{h}\left(t_{i}\right), \ln l^{h}(t)\right]$ while the Lagrangian for the maximization problem can be written

$$
\begin{equation*}
\mathscr{L}=U^{h}\left[\operatorname{lnc}^{h}\left(t_{i}\right), \ln l^{h}\left(t_{y}\right)\right]-\ln \left[t_{i} C\left(t_{i}\right)\right]-\ln \left[t_{y} w L\left(t_{y}\right)\right] \tag{14}
\end{equation*}
$$

where $\ln c^{h}=a_{0}+a_{1} \ln t+a_{2} \ln (w L)+a_{3} \ln (C P I)$, and $\ln l^{h}=$ $b_{0}+b_{1} \ln \left(t_{y}\right)+b_{2} \ln w$

The necessary condition describing the choice of the indirect tax rate is

$$
\begin{gather*}
\frac{\partial \mathscr{L}}{\partial t_{i}}=\frac{\partial U^{h}}{\partial \ln c^{h}} \frac{\partial \ln c^{h}}{\partial \ln t_{i}}-\left(1+\frac{\partial \ln C}{\partial \ln t_{i}}\right)=0, \text { or } \\
a_{1} \frac{\partial U^{h}}{\partial \ln c^{h}}=1+a_{1} \tag{15a}
\end{gather*}
$$

For the choice of the level of the direct tax rate, optimizing with respect to $\operatorname{lnt}_{\mathrm{y}}$ gives

$$
\begin{gather*}
\frac{\partial \mathscr{L}}{\partial t_{y}}=\frac{\partial U^{h}}{\partial \ln l^{h}} \frac{\partial \ln l^{h}}{\partial \ln t_{y}}-\left(1+\frac{\partial \ln (L)}{\partial \ln t_{y}}\right)=0, \text { or } \\
b_{1} \frac{\partial U^{h}}{\partial \ln l^{h}}=1+b_{1} \tag{15b}
\end{gather*}
$$

given that the wage rate is treated as an exogenous variable dividing (15b) by (15a) gives

$$
\begin{equation*}
\frac{b_{1}\left(\frac{\partial U^{h}}{\partial \ln l^{h}}\right)}{a_{1}\left(\frac{\partial U^{h}}{\partial \ln c^{h}}\right)}=\frac{1+b_{1}}{1+a_{1}} \tag{16}
\end{equation*}
$$

It is well known from the preceding discussion that

$$
\begin{gathered}
\text { (i) } \frac{\frac{\partial U^{h}}{\partial \ln l^{h}}}{\frac{\partial U^{h}}{\partial \ln c^{h}}}\left(=\frac{\partial \ln c^{h}}{\partial \ln l^{h}}\right)=M R S_{c^{h}, l^{h}} \\
\text { (ii) } \frac{a_{1}}{b_{1}}=\frac{\frac{\partial \ln C}{\partial \ln t_{i}}}{\frac{\partial \ln L}{\partial \ln t_{y}}}=\frac{\partial \ln C}{\partial \ln L} \frac{\partial \ln t_{y}}{\partial \ln t_{i}}=M R T_{c, l}\left(\frac{\partial \ln t_{y}}{\partial \ln t_{i}}\right)
\end{gathered}
$$

Therefore, we can recast eq. (16) to make it consistent with eq. (11) of the linear model:

$$
\begin{gather*}
M R S_{c^{h}, l^{h}}=M R T_{c, l}\left[\frac{\partial \ln t_{y}}{\partial \ln t_{i}}\left(\frac{1+b_{1}}{1+a_{1}}\right)\right]=  \tag{17}\\
M R T_{c, l}\left[\frac{\partial \ln t_{y}}{\partial \ln t_{i}}\left(1+b_{1}-a_{1}\right)\right]
\end{gather*}
$$

Eq. (17) implies that market equilibrium exists (MRS =MRT) only if

$$
\begin{gather*}
\frac{\partial \ln t_{y}}{\partial \ln t_{i}}\left(1+b_{1}-a_{1}\right)=1, \text { that is, if } \\
\frac{\partial \ln t_{i}}{\partial \ln t_{y}}=1+b_{1}-a_{1} \tag{18}
\end{gather*}
$$

The interpretation of (18) is quite similar to that of the linear model (see eq.11), that is,
a) Pareto efficiency results only if

- the direct-tax elasticity of labour supply and the indirect-tax elasticity of consumption goods exactly offset each other, and
- the percentage changes in both direct and indirect tax rates are exactly the same in size.
b) When the direct-tax elasticity of labour supply differs from the indirect-tax elasticity of consumption, then the only way to achieve Pareto efficiency is to change the structure of the tax system (tax mix) by placing greater emphasis on direct or indirect taxation.

In the usual case of asymmetric responsiveness of labour supply and consumption to changes in (in) direct tax rates, restructuring of the tax system is required to redress the balance. Policy makers have to reschedule the ratio of the proportional changes in the two categories of tax rates in a way that eliminates the distortions in demand and labour market. To establish a reasonable relationship between indirect and direct tax rates, eq. (18) is solved for $t_{i}$ to generate the following reaction function:

$$
\begin{aligned}
\int d \ln t_{i} & =\left(1+b_{1}-a_{1}\right) \int d \ln t_{y}, \text { or } \\
\ln t_{i} & =\left(1+b_{1}-a_{1}\right) \ln t_{y}+k
\end{aligned}
$$

where $k$ is a constant that captures the initial conditions in the economy. From the last equation, we receive

$$
\begin{equation*}
t_{i}=k t_{y}^{1+b_{1}-a_{1}} \tag{19}
\end{equation*}
$$

A reverse relationship between the above two tax rates can also be found by dividing (15a) by (15b) and then by replicating the procedure that led to eq. (19):

$$
\begin{equation*}
t_{y}=k t_{i}^{1+a_{1}-b_{1}} \tag{20}
\end{equation*}
$$

In order to measure the response of the indirect tax rate to changes in the direct tax rate in a practicable and manageable way that would help fiscal authorities to properly re-design the tax structure, we must turn to the empirical investigation of our model by using data from the national accounts of the countries considered.

## 4. Empirical evidence

The motive of our analysis is to provide practical policy recommendations. This implies that existing tax rules must be capable of being adjusted to whatever an optimal tax mix would dictate for improving social welfare. All the data series used in estimating the parameters of the above relationships have been taken from Ameco Database (Eurostat) and OECD Statistics.

To present estimates of optimal (in)direct tax rates, the first step is to maximize a social welfare function and manipulate the resulting first-order conditions with a view towards establishing a Pareto-efficient tax structure. The procedure used for this was discussed in Section 3. From now on, the focus of interest will be eq. (20), which describes an infinite number of combinations of optimal direct and indirect tax rates. It measures the extent to which the tax rates should change after distorting events in consumption or in labour markets come to the fore.

Eq. (20) may be interpreted as providing a map of indifference curves that present the preferences of policy makers for direct (indirect) tax rates over indirect (direct) tax rates, as a means of eliminating distortions coming from the labour market and/or from the market for consumer goods. It can be shown that, in eq. (20), the direct tax rate is a convex function of the indirect tax rate, if $1+a_{1}-b_{1}>1$, or $a>b_{1}$. In contrast, the indifference curves are concave to the origin if $1+a_{1}-b_{1}<1$, or $a_{1}<b_{1}$.

It is well understood that the design of a map per se, which includes an infinite number of indifference curves on the basis of (20), does not seem to comprehend the scale of the problem. It is clear that eq. (20) by itself cannot solve the problem of finding an equilibrium point, unless a constraint on the direct/indirect tax-rate structure is placed. A suitable constraint is considered to be the equality between the total tax revenue, $T$, and the sum of direct and indirect $\operatorname{taxes}\left(T_{y}+T_{i}\right)$, i.e. $T=T_{y}+T_{i}$ or their ratio to GDP,

$$
\begin{equation*}
\frac{G}{Y}=\frac{T}{Y}(=\bar{\tau})=\frac{T_{y}}{Y}+\frac{T_{i}}{Y}\left(=\bar{\tau}_{y}+\bar{\tau}_{i}\right) \tag{21}
\end{equation*}
$$

where $G$ is government spending.
The budget constraint (21) describes affordable direct-indirect tax-rate combinations.

From the infinite number of indifference curves which can be derived on the basis of eq. (20), we choose the indifference curve that corresponds to the pair of the average (in)direct tax rates, that is,

$$
\begin{equation*}
\bar{\tau}_{y}=k \bar{\tau}_{i}^{1+a_{1}-b_{1}} \tag{20a}
\end{equation*}
$$

The intersection of (20a) with the budget constraint (21) provides the optimal level of the (in)direct tax rates that eliminates any distortions originating in consumption and/or in labour markets.

The logarithmic functions which were used for our estimates, adopting a cross-section, time-series analysis, are

$$
\begin{gathered}
\ln C=a_{0}+a_{1} \ln t_{i}+a_{2} \ln Y+a_{3} \operatorname{lncom} \\
\ln L=b_{0}+b_{1} \ln t_{y}+b_{2} \ln w+b_{3} \ln U N+b_{4} \ln C P I
\end{gathered}
$$

where com is compensation payments, $w$ is the wage rate, $U N$ is the unemployment rate and $C P I$ is the consumer price index. The results are presented in Table 1.

Note that the constant term, $k$, on Table 1C stands for the initial conditions prevailing in the sample of the six countries considered. A widely accepted indicator of initial conditions is argued to be the ratio of average direct to average indirect tax rates, $\frac{\bar{\tau}_{y}}{\bar{\tau}_{i}}$.

The final step is to introduce the parameter values - as shown in Table 1C - into eq. (20a), in order to obtain a numerically defined value of the indifference curve that describes on the average the preferences of the policy makers over a feasible combination of direct and indirect tax rates.

Equating then (20a) with (21) we get

$$
\begin{equation*}
\bar{\tau}=k \bar{\tau}_{y}^{1+b_{1}-a_{1}}+\bar{\tau}_{y} \tag{22}
\end{equation*}
$$

Table 1. Estimates of consumption and labour supply functions

| A. Consumption function (ln C) |  |  |  |
| :--- | :--- | :--- | :--- |
| Variable | Coefficient | t-stat | Probability |
| constant | 4.24 | 10.30 | 0.00 |
| $\operatorname{Int}_{\mathrm{i}}\left(\mathrm{a}_{1}\right)$ | -0.49 | -13.10 | 0.00 |
| $\operatorname{Incom}$ | 0.82 | 40.20 | 0.00 |

Adj. $\mathrm{R}^{2}=0.99, \quad$ J-stat $=2.93 \mathrm{E}-17$
B. Labour supply function, working hours (In L)

| Variable | Coefficient | t-stat | Probability |
| :--- | :--- | :--- | :--- |
| constant | 4.52 | 6.80 | 0.00 |
| Int $_{y}\left(\mathrm{~b}_{1}\right)$ | 1.49 | 13.71 | 0.00 |
| Inw | 1.25 | 32.60 | 0.00 |
| InUN | -0.12 | -2.23 | 0.03 |
| InCPI | -0.79 | -6.33 | 0.00 |

Adj. $\mathrm{R}^{2}=0.94$, J-stat $=2.11 \mathrm{E}-16$
C. Summary of the estimates of the main coefficient value

| (1) | (2) | (3) | (4) |
| :--- | :--- | :--- | :--- |
| $\alpha_{1}=d \operatorname{lnC} / d I n t_{i}$ | $b_{1}=d \operatorname{lnL} / d I n t_{y}$ | $\mathbf{1 - a _ { 1 } + \boldsymbol { b } _ { 1 }}$ | $\boldsymbol{k}=\bar{\tau}_{y} / \bar{\tau}_{i}$ |
| -0.49 | 1.49 | 2.98 | 1.67 |

Source: Ameco database (Eurostat), OECD Statistics

Eq. (22) can be easily solved in terms of the (average) optimal direct tax rate. Substituting the latter into (21) gives the (average) optimal indirect tax rate.

The pair of (in)direct tax rates derived from (22) and (21) determine the point at which the government budget constraint intersects with the average indifference curve, and corresponds to the optimal combination of direct and indirect tax rates. Accordingly, an argument is made that the above mix of (in)direct tax rates eliminates any distortion originating in the labour market and/or in the market for consumer goods, and achieves equilibrium via equating the marginal rate of substitution with the marginal rate of transformation. Remember that, according to eq. (11), the equality $M R S=M R T$ is attained only when the right-hand term of this equation is equal to one. This condition in turn is met only when macroeconomic data can prove that eq. (22) is true.

Table 2 describes the details of calculating the optimal direct tax rate and presents the actual vis-a-vis the optimal (in)direct tax rates. A graphical presentation of the results is provided in Fig. 1.


Figure 1. Optimal direct and indirect tax rates

Table 2. Optimal vis-à-vis actual tax rates

| A. $\boldsymbol{k} \overline{\boldsymbol{\tau}}_{y}{ }^{1+b_{1}-a_{1}+\overline{\boldsymbol{\tau}}_{y}=\overline{\boldsymbol{\tau}}}$ | optimal $\overline{\boldsymbol{\tau}}_{y}$ |  |  |
| :--- | :--- | :--- | :--- |
| $1.67 \tau_{y}{ }^{2.98}+\tau_{y}=0.506$ | 0.40 |  |  |
| B. actual <br> (average) $\overline{\boldsymbol{\tau}}_{y}$ | actual <br> (average) $\overline{\boldsymbol{\tau}}_{i}$ | optimal $\overline{\boldsymbol{\tau}}_{\boldsymbol{y}}$ | optimal $\overline{\boldsymbol{\tau}}_{\boldsymbol{i}}$ |
| 0.32 | 0.19 | 0.40 | 0.11 |

The familiar conflict between equity and efficiency is illustrated in Figure 1 and in Table 2. If all the conditions for equilibrium are satisfied, the economy is at any point on the map of indifference curves defined by eq. (20), $t_{y}=k t_{i}^{1+a_{1}-b_{1}}$. In Figure 1, social welfare is maximized at point A , where the slope of the indifference curve for welfare $\bar{\tau}_{y}=k \bar{\tau}_{i}^{1+a_{1}-b_{1}}$ intersects the slope of the budget constraint, $\bar{\tau}=\bar{\tau}_{y}+\bar{\tau}_{i}$. This is equivalent to saying that equilibrium in the economy is obtained when the marginal rate of substitution between consumption and leisure in consumption is equated with the marginal rate of transformation between consumption and leisure in production, as shown in eq. (11).

The resulting optimal pair of direct and indirect tax rates in equilibrium for the sample of the six countries considered are then used to construct Table 2. Finally, the above optimal values for (in)direct tax rates are compared to the corresponding actual
(average) tax rates to provide valuable information as to the desired restructuring of the tax system in the direction of removing distortions originating in the labour market and/or in consumer demand.

For the sample of the six countries considered, the optimal indirect tax rate ( 0.11 ) is lower than the actual indirect tax rate ( 0.19 ), optimal $\mathrm{t}_{\mathrm{i}}$ <actual $\mathrm{t}_{\mathrm{i}}$, whereas the optimal direct tax rate ( 0.40 ) is higher than the actual direct tax rate, optimal $\mathrm{t}_{\mathrm{y}}>$ actual $\mathrm{t}_{\mathrm{y}}$. This finding may be interpreted as follows:
(i) The direct-tax system must become more progressive in order to sustain the equity principle of a fair distribution of income.
(ii) The indirect tax system must be redesigned to place a tolerable tax burden on consumers and to be committed to the ideal of income equality.

The general conclusion that arises from the inspection of Table 2 is that the sample of six countries appears to assign a greater social welfare weight to equity aspects. Even though our findings seem to be in line with those of many other studies (see, for example, Sandmo, 1976; Forbes, 2000; Okun, 2015), it remains to be seen whether employing data from other countries or using alternative methodological procedures would reverse the observed tendency of the tax system to evaluate efficiency more highly than equity.

## 5. Concluding Remarks

This paper has presented a new characterization of the Mirrlees problem that is recast in terms of reaching an efficient trade-off of equity against efficiency and explores the practical insights that it provides.

Specifically, we define two kinds of distortions that capture the cost of providing utility and eliminate them. This in turn allows the principle of equality between the marginal rate of substitution, $M R S_{c, 1}$, and the marginal rate of transformation, $M R T_{c, l}$, to maximize social welfare. To ensure the best results, we introduce direct and indirect tax rate adjustments, which are capable of minimizing distortions in the labour market and/or in the commodities market. Moreover, our methodology analyzes existing tax schedules for a set of six developed countries, providing meaningful answers to the questions of whether the cost of additional inefficiency is (or is not) quite high to warrant improving income distribution.

By determining an appropriate combination of (in)direct tax rates and using it to simulate MRS-MRT equality paradigms for policy makers, the results strongly implied that the tax systems of the set of sample countries are giving efficiency quite a bit of weight and that this tendency should be reversed.

## Appendix 1

## Estimating the reaction function

Consider the reaction function

$$
\begin{equation*}
\frac{\partial t_{y}}{\partial t_{i}}=1+\frac{t_{i}}{C} \frac{\partial c^{h}}{\partial t_{i}}+\frac{t_{y}}{C} \frac{\partial l^{h}}{\partial t_{y}} \tag{A1}
\end{equation*}
$$

We have to agree to the following two crucial conditions as a prerequisite of solving eq. (A1):
(i) The term $\frac{1}{c} \frac{\partial c^{h}}{\partial t_{i}}$ is a function solely of $t_{i}$, that is, $\frac{1}{c} \frac{\partial c^{h}}{\partial t_{i}}=a_{1}\left(t_{i}\right)$
(ii) The term $\frac{1}{c} \frac{\partial l^{h}}{\partial t_{y}}$ is a function solely of $t_{i}$, that is, $\frac{1}{c} \frac{\partial l^{h}}{\partial t_{y}}=a_{2}\left(t_{i}\right)$

If these conditions are met, eq. (A1) can take the following form

$$
\begin{gather*}
\frac{\partial t_{y}}{\partial t_{i}}=a_{2}\left(t_{i}\right) t_{y}+a\left(t_{i}\right) \\
\text { where } a\left(t_{i}\right)=1+t_{i} a_{1}\left(t_{i}\right), \text { or } \\
\frac{\partial t_{y}}{\partial t_{i}}=a_{2}\left(t_{i}\right) t_{y}+a\left(t_{i}\right) \tag{A2}
\end{gather*}
$$

Equation (A2) is a first order linear differential equation, the solution of which is taken to express $t_{y}$ as a function of $t_{i}$, as shown in the following relationship,

$$
\begin{equation*}
t_{y}\left(t_{i}\right)=\frac{1}{I\left(t_{i}\right)} \int_{0}^{t_{i}} I\left(t_{i}\right) a\left(t_{i}\right) d t_{i}+t_{y}(0) \frac{1}{I\left(t_{i}\right)} \tag{A3}
\end{equation*}
$$

where $I\left(t_{i}\right)=e^{-\int_{0}^{t_{i}} a_{2}\left(t_{i}\right) d t_{i}}$.

## References

Aghion, P., Akcigit, U., Cage, J. \& Kerr, W. (2016). Taxation, corruption and growth. European Economic Review, 86(C): 24-51. doi: 10.1016/j. euroecorev.2016.01.012

Alesina, A. \& D. Rodrik, D. (1994). Distributive politics and economic growth. The Quarterly Journal of Economics, 109(2):465-490. doi:10.2307/2118470

Atkinson, A.B. \& Stiglitz, J.E. (1976). The design of tax structure: Direct versus indirect taxation. Journal of Public Economics, 6(1-2):55-75. doi:10.1016/0047-2727(76)90041-4

Barro, R. J. (1999). Inequality, Growth and investment. NBER Working Papers 7038. Handle: RePEc:nbr:nberwo:7038

Beffy, M., Blundell, R., Bozio, A., Laroque, G. \& To, M. (2019). Labour supply and taxation with restricted choices. Journal of Econometrics, 211(1):16-46. doi.org/10.1016/j.jeconom.2018.12.004

Bird, R. M. (1992). Tax Policy and Economic Development. Baltimore: Johns Hopkins Press.

Cremer, H., Pestieau, P. \& and Rochet, J.-C.(2001). Direct versus indirect Taxation: The Design of the Tax Structure Revisited. International Economic Review, 42(3), 781-799. doi.org/10.1111/1468-2354.00133

Chang, Y. \& Park, Y. (2021). Optimal taxation with private insurance. The Review of Economic Studies, 88(6): 2766-2798.http://hdl.handle.net/10.1093/restud/rdab003

Chari V., Nicolini J.P., Pedro T. (2020). Optimal capital taxation revisited. Journal of Monetary Economics, 116: 147-165 doi: 10.1016/j.jmoneco.2019.09.015

Chiappori, P-A \& Mazzocco, M. (2017). Static and intertemporal household decisions. Journal of Economic Literature, 55(3): 985-1045. doi: 10.1257/jel. 20150715

Dawson, J. W. (1998). Institutions, investment, and growth: New cross-country and panel data evidence. Economic Inquiry, 36(4):603619. doi:10.1111/j.1465-7295.1998.tb01739.x

Deininger, K. \& Squire, L. (1996). A new data set measuring income inequality. The World Bank Economic Review, 10(3): 565-591. doi:10.1093/wber/10.3.565

Farhi, E. \& Gabaix, X. (2020). Optimal taxation with behavioral agents. American Economic Review, 110(1): 298-336. doi: 10.1257/aer. 20151079

Forbes, K. J. (2000). A Reassessment of the relationship between inequality and growth. American Economic Review, 90(4), 869-887. doi: 10.1257/ aer.90.4.869

García-Peñalosa, C. \& Turnovsky, S. (2011). Taxation and income distribution dynamics in a neoclassical growth model. Journal of Money, Credit and Banking, 43(8): 1543-1577. doi.org/10.1111/j.15384616.2011.00458.x

Gayle, G.-L. \& Shephard, A. (2019). Optimal taxation, marriage, home production, and family labor supply. Econometrica, 87(1): 291-326. doi.org/10.3982/ ECTA14528

Harberger, A. C. (1995). The ABCs of corporation tax incidence: Insights into the open economy case. In Tax Policy and Economic Growth, 51-73. Washington, D.C.: American Council for Capital Formation.

Heathcote, J. \& Violante, G. (2020). Optimal progressivity with age-dependent taxation. Journal of Public Economics, 189(1): 104074. doi.org/10.1016/j. jpubeco.2019.104074

IMF (2014). Fiscal Policy and Income Inequality. IMF Policy Paper, January 23, 2014. https://www.imf. org/external/np/pp/eng/2014/012314.pdf

Jantti, M., Pirttila, J. \& Selin, H. (2015). Estimating labour supply elastisticities based on cross-country micro data: A bridge between micro and macro estimates?. Journal of Public Economics, 127:87-99. doi. org/10.1016/j.jpubeco.2014.12.006

Joumard, I., Pisu, M. \& Bloch, D. (2012). Tackling income inequality: The role of taxes and
transfers. OECD Journal: Economic Studies, 2012(1): 37-70. doi:10.1787/eco_studies-2012-5k95xd6l65lt

Keane, M. \& Wasi, N. (2016). Labour supply: The roles of human capital and the extensive margin. The Economic Journal, 126(592): 578-617. doi.org/10.1111/ecoj. 12362

Musgrave, R.A. (1959). The Theory of Public Finance. McGraw Hill, New York.

Okun, A. M. (2015). Equality and Efficiency: The Big Tradeoff. A Brookings Classic, Brookings Institution Press.

Perotti, R. (1993). Political equilibrium, income distribution, and growth. The Review of Economic Studies, 60(4):755-776. doi:10.2307/2298098

Reis, C. (2020). Optimal taxation with unobservable investment in human capital. Oxford Economic Papers, 72(2): 501-516. doi.org/10.1093/oep/gpz038

Saez, E. (2001). Using elasticities to derive optimal income tax rates. The Review of Economic Studies, 68(1): 205-229. doi:10.1111/1467-937X. 00166

Saez, E. \& Stantcheva, S. (2018). A simpler theory of optimal capitaltaxation.Journal ofPublicEconomics,162(1): 120-142. doi:10.1016/j.jpubeco.2017.10.004

Sandmo, A. (1976). Optimal taxation: An introduction to the literature. Journal of Public Economics, 6(1-2): 37-54. doi.org/10.1016/0047-2727(76)90040-2

Slemrod, J. (1990). Optimal taxation and optimal tax systems. Journal of Economic Perspectives, 4(1), pp. 157-178. doi: 10.1257/jep.4.1.157

Stevans, L. K. (2012). Income inequality and economic incentives: Is there an equity-efficiency tradeoff?. Research in Economics, 66(2): 149-160. doi: 10.1016/j.rie.2011.10.003

Stiglitz, J. (2015). The origins of inequality, and policies to contain it. National Tax Journal, 68(2): 425448. doi.org/10.17310/ntj.2015.2.09

Stiglitz, J. (2018). Pareto efficient taxation and expenditures: Pre- and re-distribution. Journal of Public Economics, 162: 101-119. doi.org/10.1016/j. jpubeco.2018.01.006

Tillmann, G. (2005). The equity-efficiency tradeoff reconsidered. Social Choice and Welfare, 24(1): 63-81. doi:10.1093/oxfordjournals.oep.a041947

Uchida, Y. \& Ono, T. (2021). Political economy of taxation, debt ceilings, and growth. European Journal of Political Economy, 68(C):101996. doi.org/10.1016/j. ejpoleco.2020.101996

