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Bipolar Fuzzy Line Graph of a Bipolar Fuzzy Hypergraph

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Abstract: *This paper introduces the concept of a bipolar fuzzy line graph of a bipolar fuzzy hypergraph and some of the properties of the bipolar fuzzy line graph of a bipolar fuzzy hypergraph are also examined.*

Keywords: *Bipolar fuzzy set, line graph, hypergraph, fuzzy hypergraph.*

1. Introduction

In 1965, Z a d e h [10] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines, including medical and life sciences, computer networks and many other fields. In 1994, Z h a n g [11, 12] initiated the concept of bipolar fuzzy sets as generalization of the fuzzy sets. Bipolar fuzzy sets are an extension of the fuzzy sets whose membership degree range is $[-1, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, and the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter-property. This domain has recently motivated new research in several directions. In particular, fuzzy and possibilistic formalisms for bipolar information have been proposed.

In 1736, E u l e r first introduced the notion of graph theory. In 1975, R o s e n f e l d [7] introduced the concept of fuzzy graphs. Later on, B h a t t a c h a r y a

[4] gave some remarks on fuzzy graphs. Recently, A k r a m [1] has introduced the notions of bipolar fuzzy graphs. A k r a m and D u d e k [2] introduced the notion of the bipolar fuzzy line graph of a bipolar fuzzy graph.The hypergraph was introduced by B e r g e [3] and has been considered as a useful tool to analyze the structure of a system and to represent a partition and clustering. The notion of a hypergraph has been extended to fuzzy theory and the concept of fuzzy hypergraphs was proposed by K a u f f m a n [6]. The line graph of the hypergraphs is generalization of the line graph of simple graphs. The name line graph comes from a paper by Harary and Norman (1960) although both Whitney (1932) and Krausz (1943) used the construction before this.

2. Basic definitions

Definition 2.1 [5]. An undirected graph, or simply a *graph*, is a pair $G = (V, E)$ where V is a set of vertices or nodes and E is a collection of two-element sets.

Definition 2.2 [10]. A *fuzzy set A* can be defined mathematically by assigning to each possible individual in the universe of discourse X' a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree, to which this individual is similar or compatible with the concept represented by the fuzzy set. Formally, a fuzzy subset A of a set X' is a map μ_A: $X' \to [0, 1]$, called the membership function.

Definition 2.3 [7]. A *fuzzy graph* with X as the underlying set is a pair $G:(\sigma, \mu)$ where σ: $X \to [0, 1]$ is a fuzzy subset, $\mu : X \times X \to [0, 1]$ is a fuzzy relation on the fuzzy subset σ, such that $\mu(x, y) \le \min(\sigma(x), \sigma(y))$ for all $x, y \in X$.

Definition 2.4 [11]. Let X be a non-empty set. A *bipolar fuzzy set E* on X is an object having the form $E = \{(x, \mu_E{}^P(x), \mu_E{}^N(x)): x \in X\}$, where $\mu_E{}^P: X \to [0, 1]$ denotes a positive membership degree of the elements of X and $\mu_E{}^N: X \to [-1, 0]$ denotes a negative membership degree of the elements of X.

Definition 2.5 [1]. By a *bipolar fuzzy graph* we mean a pair $G = (A, B)$ where $A = (\mu_A{}^P, \mu_A{}^N)$ is a bipolar fuzzy set in V and $B = (\mu_B{}^P, \mu_B{}^N)$ is a bipolar relation on V, such that $\mu_B{}^P(\{x, y\}) \le \min(\mu_A{}^P(x), \mu_A{}^P(y))$ and $\mu_B{}^N(\{x, y\}) \ge \max(\mu_A{}^N(x), \mu_A{}^N(y))$ for all $\{x, y\} \in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively.

Definition 2.6 [5]. The *line graph $L(G^*)$ of a simple graph G^** is another graph $L(G^*)$ that represents the adjacencies between the edges of G^*. Given a graph G^*, its line graph $L(G^*)$ is a graph such that:

(i) each vertex of $L(G^*)$ represents an edge of G^*, and

(ii) two vertices of $L(G^*)$ are adjacent if and only if their corresponding edges share a common endpoint.

Definition 2.7 [2]. Let $L(G^*) = (Z, W)$ be a line graph of a simple graph $G^* = (V, E)$. Let $G = (A_1, B_1)$ be a bipolar fuzzy graph of G^*. We define a *bipolar fuzzy line graph* $L(G) = (A_2, B_2)$ *of a bipolar fuzzy graph* G as follows:

(i) A_2 and B_2 are bipolar fuzzy sets of Z and W, respectively;

(ii) $\mu_{A_2}{}^P(S_x) = \mu_{B_1}{}^P(x) = \mu_{B_1}{}^P(u_x v_x)$;

(iii) $\mu_{A_2}{}^N(S_x) = \mu_{B_1}{}^N(x) = \mu_{B_1}{}^N(u_x v_x)$;

(iv) $\mu_{B_2}{}^P(S_x S_y) = \min\left(\mu_{B_1}{}^P(x), \mu_{B_1}{}^P(y)\right)$;

(v) $\mu_{B_2}{}^N(S_x S_y) = \max\left(\mu_{B_1}{}^N(x), \mu_{B_1}{}^N(y)\right)$ for all $S_x, S_y \in Z$, $S_x S_y \in W$.

Definition 2.8 [3]. A *hypergraph* H^* is an ordered pair $H = (X, E)$ where $X = \{x_1, x_2, \ldots, x_n\}$ is the set of vertices (or nodes) and $E = \{E_1, E_2, \ldots, E_m\}$ with E_i contained in X for $i = 1, 2, \ldots, n$ is the set of hyperedges.

Definition 2.9 [9]. For a hypergraph H^*, the *line graph of the hypergraph* $L(H^*)$ is defined as follows:

(i) The vertex set of H^*, $V_L(H^*) = E_H$ (hyperedges of H). In accordance with the definition of a hypergraph, $V_L(H^*)$ is a set and E_H is a family. In this situation the above equality means that if $E_H = \{E_i: 1 \leq i \leq m\}$, then $V_L(H) = \{E_1, E_2, \ldots, E_m\}$ is an m-element set. In other words, the multiple edges of H give rise to different vertices of $L(H)$.

(ii) Vertices E_i and E_j are adjacent in $L(H)$ if and only if $E_i \cap E_j \neq \emptyset$.

Definition 2.10 [8]. A *bipolar fuzzy hypergraph* is defined to be a pair $H = (X, \xi)$ where X is a finite set and ξ be a finite family of the bipolar fuzzy subsets E on X such that $X = \bigcup_{E \in \xi} \operatorname{supp}(E)$, E is said to be hyper edge of H.

3. Bipolar fuzzy line graph of a bipolar fuzzy hypergraph

Definition 3.1. Let $L(H^*) = (S, T)$ be a line graph of a simple hypergraph. $H^* = (X, E)$. Let $H = (X, \xi)$ be bipolar fuzzy hypergraphs of H^*. We define a bipolar fuzzy line graph $L(H) = (A_1, B_1)$ where A_1 is the vertex set of $L(H)$ and B_1 is the edge set of $L(H)$ as follows:

(i) A_1 and B_1 are bipolar fuzzy sets of S and T, respectively;

(ii) $\mu_{A_1}{}^P(E_i) = \min_{x \in E_i}\left(\mu_{E_i}{}^P(x)\right)$;

(iii) $\mu_{A_1}{}^N(E_i) = \max_{x \in E_i}\left(\mu_{E_i}{}^N(x)\right)$, $E_i \in \xi$;

(iv) $\mu_{\boldsymbol{B}_1}{}^P(E_j E_k) = \min_i\left(\min\left(\mu_{E_j}{}^P(x_i),\ \mu_{E_k}{}^P(x_i)\right)\right)$ and

$\mu_{\boldsymbol{B}_1}{}^N(E_j E_k) = \max_i\left(\max\left(\mu_{E_j}{}^N(x_i), \mu_{E_k}{}^N(x_i)\right)\right)$ where $x_i \in E_j \cap E_k$, $j, k = 1, 2, \ldots, n$.

Example 1. Consider $H^* = (X, E)$; $X = \{x_1, x_2, x_3, x_4, x_5\}$ and $E = \{E_1, E_2, E_3\}$ where $E_1 = \{x_1, x_2, x_3\}$; $E_2 = \{x_2, x_3, x_4\}$; $E_3 = \{x_3, x_4, x_5\}$; Define $H = (X, \xi)$ as $X = \{x_1, x_2, x_3, x_4, x_5\}$; $\xi = \{E_1, E_2, E_3\}$ where $E_1 = \{(x_1, 0.4, -0.3), (x_2, 0.7, -0.8), (x_3, 0.2, -0.1)\}$, $E_2 = \{(x_2, 0.9, -1), (x_3, 0.8, -0.4), (x_4, 0.5, -0.4)\}$, $E_3 = \{(x_3, 0.4, -0.5), (x_4, 0.3, -0.6), (x_5, 0.2, -0.3)\}$.

The line graph of H can be obtained as, $L(H) = (A_1, B_1)$ where $A_1 = \{(E_1, 0.2, -0.1), (E_2, 0.5, -0.4), (E_3, 0.2, -0.3)\}$ is the vertex set and $B_1 = \{(E_1E_2, 0.2, -0.1), (E_2E_3, 0.3, -0.4), (E_1E_3, 0.2, -0.1)\}$ is the edge set of the bipolar fuzzy line graph.

Diagrammatic representation of the above Example 1 is given in Fig. 1.

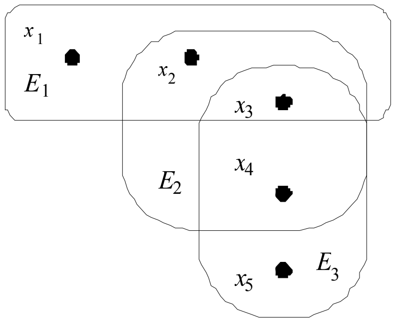

Fig. 1. Bipolar fuzzy hypergraph H

Bipolar fuzzy Line graph $L(H)$ of the above bipolar fuzzy Hypergraph H is shown in Fig. 2.

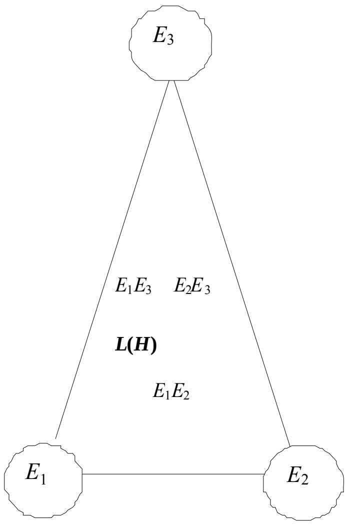

Fig. 2. Bipolar fuzzy line graph of H

Proposition 3.1. $L(H)$ is a bipolar fuzzy line graph corresponding to the bipolar fuzzy hypergraph H.

P r o o f: Obvious from Definition 3.1.

Proposition 3.2. If $L(H)$ is a bipolar fuzzy line graph of the bipolar fuzzy hypergraph H then $L(H^*)$ is the line graph of H^*.

P r o o f: Since $H = (X, \xi)$ is a bipolar fuzzy hypergraph and $L(H)$ is a bipolar fuzzy line graph, we have $\mu_{A_1}{}^P(E_i) = \min_{x \in E_i}\left(\mu_{E_i}{}^P(x)\right)$, $E_i \in \xi$; $\mu_{A_1}{}^N(E_i) = \max_{x \in E_i}\left(\mu_{E_i}{}^N(x)\right)$ and so $E_i \in S \Leftrightarrow x \in E_i$ and $E_i \in \xi$. Also,

$\mu_{\boldsymbol{B}_1}{}^P(E_jE_k) = \min_i\left(\min\left(\mu_{E_j}{}^P(x_i),\ \mu_{E_k}{}^P(x_i)\right)\right)$ and $\mu_{\boldsymbol{B}_1}{}^N(E_jE_k) =$ $= \max_i\left(\max\left(\mu_{E_j}{}^N(x_i),\ \mu_{E_k}{}^N(x_i)\right)\right)$ where $x_i \in E_j \cap E_k$, $j, k = 1, 2, \ldots, n$, for all $E_j, E_k \in S$ and so $T = \{E_jE_k : E_j \cap E_k \neq \emptyset, E_j, E_k \in \xi,\ j \neq k\}$. This completes the proof.

Proposition 3.3. $L(H)$ is a bipolar fuzzy line graph of some bipolar fuzzy hypergraph if and only if $\mu_{\boldsymbol{B}_1}{}^P(E_jE_k) = \min_{j,k}\left(\min\left(\mu_{A_1}{}^P(E_j), \mu_{A_1}{}^P(E_k)\right)\right)$ for all $E_jE_k \in T$ and $\mu_{\boldsymbol{B}_1}{}^N(E_jE_k) = \max_{j,k}\left(\max\left(\mu_{A_1}{}^N(E_j), \mu_{A_1}{}^N(E_k)\right)\right)$ for all $E_jE_k \in T$.

Proposition 3.4. $L(H)$ is a bipolar fuzzy line graph if and only if $L(H^*)$ is a line graph and $\mu_{\boldsymbol{B}_1}{}^P(E_jE_k) = \min_i\left(\min\left(\mu_{A_1}{}^P(E_j), \mu_{A_1}{}^P(E_k)\right)\right)$ for all $E_jE_k \in T$ and $\mu_{\boldsymbol{B}_1}{}^N(E_jE_k) = \max_i\left(\max\left(\mu_{A_1}{}^N(E_j),\ \mu_{A_1}{}^N(E_k)\right)\right)$ for all $E_jE_k \in T$.

4. Conclusion

The theory of hypergraphs is seen to be a very useful tool for the solution of integer optimization problems. In networks, the hypergraphs are used as nodes. Instead of using a hypergraph, the line graph of a hypergraph is useful in hyper-networks. The authors propose to extend further this work to intuitionistic fuzzy sets.

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