

HOW TO PLAY BOARD GAMES? A FRAMEWORK PROPOSAL FOR CLASSROOM SETTINGS

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Board games have been mainly used in school in order to develop students' cognitive skills such as recognizing oneself intellectual potential, developing different and original strategies in confronting problems, developing quick thinking and decision-making skills, and using reasoning and logic correctly and effectively. However, detailed guidance about how the teacher could manage the game-playing process and about how it will play a role in helping students achieve the targeted skills has not been provided in the studies on games, the curriculum, or the game guides. In this study, a framework for the teacher in managing the process of playing board games is proposed. This framework is based on the Theory of Didactical Situations and has been developed in the context of strategic board games that are mostly played against an opponent on a board and do not involve luck. Pentago was used to illustrate the proposed framework. At the end of the study, the use of this framework for other board games is also discussed.

Keywords: board games, game playing process, mathematics education, pentago game

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Introduction

Playing games is a very valuable activity for thought, culture, and education (Huizinga, 1955). Every game has a goal and predetermined rules. If the rules are broken, the game could not be played and would never have been won (Suits, 1978). For a game to be played, it is essential to winning within the rules of the game, and, to reject the non-game possibilities that will allow winning the game. While looking for effective ways to win, players engage in thinking and research process in which they analyze the structure of the game, their own, and the opponent's moves. This process not only allows the players to have a good time but also enables them to learn about the game and develop some skills. Players also experience an enculturation process such as obeying ethical rules and playing gentlemanly, and they develop themselves socially and psychologically (Huizinga, 1955).

There are various studies on how games can be used in educational processes (Hinebaugh, 2009). Many studies revealed the relationship between games and mathematics (Berlekamp, Conway & Guy, 2001; Olivastro, 1993; Silva, 2011; Smith, 2015). When mathematics learning and teaching processes are considered, the first games that come to mind are various board games called intelligence games, jigsaw puzzles, riddles, and puzzles. Some of these games (for example, chess, mancala, tic-tac-toe) are widely known. Some of them (e.g., corridor, abalone, pentago) have been popular for recent years and have been the subject of many competitions or tournaments around the world. Although none of these games directly aims at teaching mathematical concepts, most of them can be related to mathematical knowledge and skills in terms of their designs, rules, and strategies (Erdoğan, Eryılmaz Çevirgen & Atasay, 2017; Kurbal, 2015).

This relationship between board games and mathematics has also affected many curriculums and educational assessment approaches (Hinebaugh, 2009). For example, since the 2013-2014 academic year, the Intelligence Games Course (Ministry of National Education [MoNE], 2013) have taken place among the secondary school elective courses. In addition, intelligence games, as a context of problems, have been used in the examination such as the High-School Entrance System in Turkey (LGS), Kangaroo Mathematics, and Caribou Mathematics.

In addition, schools are willing to get various intelligence games for their courses or in order to allow students to play them in their free time at school (Hinebaugh, 2009). Unfortunately, it is observed that many of these games are only used for a few weeks. School administrators and teachers are often

satisfied with the emotional benefits of these games: “They are good, students are having fun.”, “Students could see that at least every lesson in the school was not boring.”, and students show similar reactions “We had a lot of fun during this lesson, I wish all lessons were like this!”. As can be seen, such emotional explanations do not mention the possible contribution of intelligence games to the development of students’ cognitive skills. However, in the process of playing the game, the players use cognitive skills such as perception, attention, memory, and logical reasoning. In light of this, developing quick thinking and decision-making skills, improving cognitive skills along with recognizing students’ intelligence potential can be thought of as prominent objectives of playing intelligence games (Hinebaugh, 2009). In order to achieve these goals, learning outcomes related to each game, learning outcomes of courses, teaching approaches, game types, and the game process should be carefully considered and planned. However, sufficient and detailed guidance on how the teacher will manage the game-playing process and how he/she will play a role in achieving the targeted skills of the students has not been provided. In this study, a framework to guide the teacher in supporting the development of students’ cognitive skills by effectively managing the process of playing board games is proposed. The framework has been developed in the context of strategic board games which are mostly played against an opponent and do not usually involve luck.

Theoretical and Practical Background

The use of games in mathematics teaching dates back to the 1970s. One of the most influential studies on this subject was carried out by the French mathematics educator Guy Brousseau. Brousseau gave a special place to mathematical games in the Theory of Didactical Situations (Brousseau, 2002), he introduced and built his theory on the components of the game-playing process. The theoretical background of the proposed framework was based on Brousseau’s theory. The proposed framework has also a practical background, which was based on the observations of the behaviors of different participant groups, from primary school to university degree, while playing board games in contexts of the lectures, thesis, and projects authors have carried out for many years (Atasay, 2018; Erdoğan, Erdoğan, Garan & Güler, 2012; Erdoğan & Erdoğan, 2013; Erdoğan, Eryılmaz Çevirgen & Atasay, 2017).

According to Brousseau (2002), discovering the winning strategies of a game with a mathematical structure is only possible by discovering the

knowledge on which the game is built, and this knowledge can only be discovered in a learning environment meticulously planned by the teacher. Brousseau opposed the approaches that see the development of games' strategies as a natural result of the process of playing games and argued that without an educational environment design, there can be no transition from games to knowledge and that the acquisition of mathematical knowledge cannot be equated with playing games. According to Brousseau, the process from the game to the knowledge includes some phases: devolution, action, formulation, validation, and institutionalization. In the theory, the role of the teacher in these phases is limited by the design and management of a game environment that allows the student-game interaction to occur in a way to ensure the emergence of the targeted knowledge.

Brousseau's games, which are designed to explore certain mathematical concepts or skills, have general solution strategies and these strategies can be explained in a mathematical language. In board games, winning strategies cannot be built on a single mathematical concept, these strategies require a wide range of skills, and the strategies cannot always be expressed in a mathematical language (Erdoğan, Eryılmaz Çevirgen & Atasay, 2017). In this study, Brousseau's game phases and approach of game environment design were adapted in the context of the purposes, designs, and rules of board games.

The first phase, Brousseau called devolution, means the assignment of the task and the responsibility to the students. Devolution includes sub-goals such as understanding the game rules, its purpose, the cause-effect relationship, the necessity of making predictions and developing a winning strategy. In the context of board games, the step of *the first encounter with the game*, which includes the detailed examination of the material features and rules of the game by the students, has been considered as the first playing step of board games. In addition, it is important to realize the purpose of the game and to understand the moves and situations that should and should not be done to win the game, rather than learning the rules. For this purpose, the step of *understanding the cause-effect relationships in the game* has been added as the phase of devolution. Although it is known that developing a strategy in a game is not an easy and fast-growing process, it is thought that the phase of devolution is a critical stage that does not happen all at once, and that needs to be passed by detailing each stage of the process and spreading it over the process. For this reason, the determined steps are divided into sub-steps and detailed.

Other phases Brousseau referred to are the action phase in which the students play the game and interact with the game, the formulation phase

in which they express the hypotheses developed in the game, and the validation phase in which they try to confirm and refute these hypotheses with propositions. In parallel with these phases, *the strategy development* step was identified for the action phase. A step that is determined for the formulation phase is identified as *clarifying the strategy* because it does not seem possible to express a strategy without clarification as a set of moves. A step expressed as the validation phase is identified as *verifying the strategy*. Brousseau also includes the institutionalization phase, which involved the explanation of the mathematical knowledge underlying the strategies that emerge in the game. Since board games do not directly aim at introducing mathematical knowledge, this phase has been determined as the step of *evaluating the board game and the process*. Thus, the players are expected to explain and discuss their experiences and knowledge in the process.

A Framework for Game Playing Process

The proposed framework consists of six steps in which the role and responsibilities of the teacher are explained.

Step 1: *Encounter with the game.* This first step consists of five sub-steps.

a) *Identifying the material.* Players are expected to carefully examine the game material (game board, game pieces, etc.). At this step, the teacher should allow the students to examine the game material without making any explanations about the game. Students are expected to examine and discover information such as the shape and size of the game board and how many are pieces. Thus, perceiving these features like a variable of the game and making inferences will enable the game rules to be understood and to be justified.

b) *Talking about the material and the game rules.* In this process, students are expected to realize the possible games that can be played with the game material. Students can develop and discuss their ideas about the game without knowing the game rules and how to play it. For this process to be more effective and for the players to be identified the material better, the teacher can ask to players questions such as, “What kind of game can be played with this material?”, “What the purpose and rules of this game can be?” These examinations made without getting acquainted with the game rules will not only trigger the students’ sense of curiosity but also will enable them to make sense easier of the game rules.

c) *Playing with arbitrary rules.* Players can play a few games with their own rules in line with the ideas they have developed and reviewed without learning the rules. This process is important for students to see what they can do with the predicted rules and the potential of the material, and to realize whether their ideas are practicable, without trying to be the winner.

d) *Examining the game rules.* In order to properly play the game, the game rules must first be understood. Understanding all the game rules is critical for the players' moves and the strategies they will develop, as well as the progress of the playing process without causing arguments and tensions among the players. Firstly, players may be asked to read the game rules from the guide. In order to make the game rules accessible to the players, the teacher can highlight some points, try to control whether the game rules are understood correctly or not by asking questions to the students, and explain the incomprehensible parts of the rules to the players who need clarification, the rules which are not understood. In this process, players should not be expected to fully understand the rules. In many board games, the players completely understand the game rules after a few plays.

e) *Playing with game rules.* In this process, being the winner or loser should not be important for players. What should be expected from the players is to play the game a few times by following the game rules without thinking detailly about winning and losing. Players should keep playing only to understand the rules, and how the rules are applied with game material. Encouraging students to develop strategies or to put forward hypotheses on winning methods should not be rushed. It should be considered that the game rules that are not well understood will cause problems for the teacher in the following games.

The step of encounter with the game covers the process from the first meeting with the game to the observation that playing with rules is unproblematic. It is important to give enough time to all players to experience this process in their rhythm and not to try to accelerate this process.

Step 2: Understanding the cause-effect relationships in the game.

In all strategic board games, winning or losing situations occur depending on the game rules and the moves of the players (Thompson, 2015). Every move made following the rules creates a certain result, and the next move is built on this result. Players need to understand this cause-effect relationship and take responsibility for their moves. This step includes four sub-steps.

a) *Understanding the purpose of the game.* Understanding the game rules that took place in the previous step does not always mean understanding the purpose of the game. Of course, the goal in every strategy game is to win, but in strategic board games, players need to understand what the rules are they should respect and what they need to do to win. Understanding that winning the game by chance or as a result of the opponent's carelessness cannot be thought of as winning the game and understanding that winning the game is only possible with conscious strategy development is related to understanding the purpose of the game.

b) *Simplifying the game.* In some games, it is possible to play a simpler version of the game by decreasing the size of the game board, reducing the number of pieces, limiting some moves that can be made. Playing the game in such a simplified way can help the players better notice the cause-effect relationships. At this point, the teacher can make suggestions on how to simplify the game or consider playing a simplified version of the game in the first stage for games with complex or detailed rules.

c) *Examining particular cases.* In order to understand that each move is a factor affecting the outcome, activities such as analyzing a given situation in the game, discussing how it can be played in this situation, talking about which moves will provide advantages or disadvantages, and trying to determine which player will win in such a situation can be done. Thus, it is expected to realize that each planned move is one of the many moves that can be made and that it is a question of making a choice.

d) *Investigating about winning and losing cases.* In strategic board games, a player's winning or losing is the result of her/his moves. Although theoretically speaking, this cause-effect relationship in the games is very clear, it is not always easy to understand, especially for younger students which mostly explain by psychic/psychological arguments (you're lucky, how did you see it, that's a bit much...). If a player is constantly losing or winning, after a while, inquiries such as "Why am I losing?" or "How did I win now?" can be expected. Players need to realize that winning or losing is not a coincidence but for certain reasons. If players do not ask these questions themselves, the teacher should encourage them by asking appropriate questions.

At this step, it is suggested that the teacher should allow the students to play enough to understand the cause-effect relationships, observe the win or loss situations that occur around these games, and question why a player is in a "difficult" position against his opponent's move or why she/he "gained an advantage" after her/his move. In addition, it is important for the teacher to think about the process of understanding the cause and effect

relationships in each game within the context of the purpose and rules of that game, to avoid making general statements without context about these relationships. Moreover, the teacher should ask questions based on concrete observations and specific situations.

Step 3: Strategy development. Winning in strategic board games is only possible by developing strategy. Three different play behaviors can be observed in this strategy development process: playing based on her/his own moves; playing based on the opponent's moves; playing based on both her/his own and opponent's moves. Players who play only for their own moves develop some strategies without considering the moves opponent will make (or hoping that the opponent will not make those moves) and try to win the game by employing these strategies. Players who play only based on the opponent's moves play only to prevent the opponent's moves and do not think of developing a strategy. Experienced players, that is, those who play based on both their own and the opponent's moves, try to develop their strategies by considering the opponent's possible moves. Developing effective strategies, players who adopt the first or second play behavior are expected to realize the lack of their approach over time and show the third play behavior. The strategy development process can also be summarized under certain sub-steps.

a) *Playing based on experience and inference.* Players get a separate experience of winning or losing from each game, and they try to reflect this experience in their next game. Because no rational player expects to win the game by persistently applying the moves that she/he sees as losing the game. Inferences from previous game experiences mostly create an answer to the question "I just did it like this, but it happened like this, so how about if I do it like this now?" The answer to this question allows the moves to be reviewed and to be achieved the inference up to strategy. Making inferences from previous experiences creates a "game memory" in the player and allows her/his to predict the possible outcomes of each move. At this step, the teacher can encourage students to replay the game when necessary (starting from the beginning or any point) or invite them to play against different players until they are sure that the players have sufficient experience of the game.

b) *Predicting a few moves ahead.* After reaching certain experiences and inferences, to develop a strategy the players are expected to play with the question constantly in mind, "When which move is made, which move should be made?" The players play with this mindset and quickly calculate the best possible moves in their minds is an indicator of predictive play and

is one of the important steps in strategy development. Similarly, understanding the purpose of the opponent by examining her/his moves, anticipating, and preventing the strategy she/he is trying to establish is an important part of the predictive play. Most beginner or junior players may lack this foresight. Such a player may focus on setting up her/his own strategy, forgetting that play the game against an opponent, and thinking that the opponent will make the moves she/he expects. This often results in that player losing the game. Foresight development has different dimensions. One of them is related to the playing style of the players. When setting up a game, players are expected to determine whether they are playing offensively or defensively. After the purpose of the moves, whether to attack or defend, is determined, foresight becomes a part of an activity. Answers to the questions of what the next move should be and why should be done can be sought. At the same time, solutions can be developed to prevent possible moves. At this step, again, the teacher can make the students realize the importance of seeing a few moves ahead with appropriate questions, otherwise, they will realize that they will only win the game by chance.

c) *Build the strategy as a set of moves.* Theoretically, the goal of each player should be to develop a chain of moves that will allow to win the game every time. However, this is almost impossible in strategic board games. This situation keeps the desire of the player to play the games continuously and to develop newer and more effective strategies. Developing strategies are necessary to gain an advantage over the opponent, since creating a winning sequence of moves is not possible every time. How soon these strategies will emerge and how effective they will be is closely related to how well the above-mentioned processes and steps are implemented. Players who are constantly trying to prevent the opponent's moves can prevent being defeated in a game, or rather, being defeated early. However, defeat will be inevitable as soon as playing defensively precludes developing a winning strategy. Defensive moves can only turn into moves that will win the game when considered together with offensive moves and winning strategies. Some players at this step may consider playing the same or symmetrical moves of the opponent as a strategy. Although such a playing strategy may seem beneficial at first, it can be understood that it is not enough to win the game when it is realized that the first player made the last move. It is predicted that these experiences will force the players to think as a set of moves in a cause-effect sequence and each player will start to develop their own strategies. The creation of strategies as a set of moves is generally a long-term process.

The most common incorrect approach observed regarding the strategy development process is that the teacher expresses the strategy. This behavior is the biggest obstacle to the mental processes that players will experience and the intellectual satisfaction and sense of competence they will eventually achieve, just like putting the pieces of a puzzle together by someone else. Moreover, a strategy expressed by someone else before it has matured enough in the mind of the student will become unclear again in the future and will be implemented incompletely or incorrectly. In this sense, the teacher should stay away from interventions containing the information as much as possible in the process of creating strategies and make her/his guidance only based on the moves of the players.

Step 4: *Clarifying the strategy.* The strategies developed in strategic board games are initially uncertain, all moves are not clear and systematic, and players may not remember some moves of this strategy exactly. The players need to reconstruct their strategies by reworking them in the games and to be able to clarify them in a way that can be shared orally or written with someone else. The step of clarification of the strategies will take place gradually with the careful guidance of the teacher. The teacher must not teach the students strategies that will enable them to “play well” in this game, however, must provide effective guidance for each player to determine their own strategy. The development of the strategies should be done by the players, and the opportunity to share the strategies with all the players should be provided in a way that will enable the players to develop their critical thinking skills. Players should be allowed to explain their strategies individually or as a group in the classroom, show them by playing, and try these strategies with other players. If there is more than one strategy described in the classroom, their similarities and differences may be examined and discussed whether the strategies described are related or similar. The strategies described may be examined whether cover all the moves to be made from the initial move. If there is an opportunity to combine and associate strategies, students can be supported to do so.

Step 5: *Verifying the strategy.* Every strategy described needs verification to be considered a game-winning strategy. Players usually carry out their strategies in silence. However, in cases where many students play the same game or there is no award as a result of the game, such as in the intelligence games course, the validation of the strategies can be carried out with interaction and cooperation. The teacher can allow clearer strategies to be played and justified by all the players in the classroom. Those who

explain the strategies can support these strategies with their own arguments, and other players can produce arguments to support that the explained strategy is valid or to refute it where it is invalid. At this step, the strategies can be rearranged by the classroom as a result of the discussions, criticisms, and arguments on the strategies.

Step 6: *Evaluating the game and the process.* As a result of the steps presented above, certain strategies related to the game can be developed and verified. Of course, this process can take weeks, sometimes months, depending on the game played, the levels, and the experiences of the players. At the end of such a process, evaluating the game and the process through the experiences and perspectives of the players will add more value than the fact that a game has been played and finished. In this step, the players are asked questions about their evaluation of the game like that, “If you wanted to change the game, how would you change?”, may also be asked questions about the process like that “When you evaluate the whole process you have been through, what was the most important step that allowed you to develop a strategy?” Thus, a process can be experienced in which all players explain their ideas and give various suggestions.

A Game Analysis According to the Proposed Framework: The Pentago Example

Pentago was developed by Tomas and Michael Flodén in 2003 in Sweden and is a very popular game in the game community (Figure 1). Pentago is a very rich game in terms of moves that can be made and



Figure 1: The Pentago game

strategies that can be developed, which are directly related to combination and permutation calculations. In a thesis study in the field of applied mathematics, ways to investigate possible strategies of this game are presented, but it is not possible to model winning strategies (Büscher, 2011). Despite this complex mathematical structure of the game, the simplicity of its rules and purpose makes it easily understandable and playable for players of all ages. Here is how Pentago can be played by following the steps of the proposed framework.

Step 1: *Encounter with the game.* Pentago materials are distributed to player groups. Although Pentago is a two-player game, player groups can consist of more than two people. Since the players do not know the game rules and how many players play, they can develop an idea about playing the game with different numbers of players.

a) *Identifying the material.* The players are asked to examine the materials presented. The players are expected to determine that the board and material characteristics are that the game board has a 6x6 structure, consists of 4 sections of 3x3, nine slots in each section, 36 slots in total, each section can be separated from the others and rotated, and there are 36 marbles, 18 white and 18 black (Figure 2).

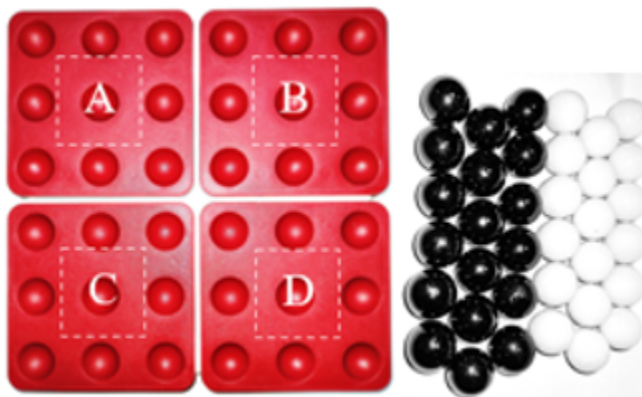


Figure 2: Sections of the Pentago board and marbles

b) *Talking about the material and the game rules.* At this step, players may think that the marbles should be placed in the slots on the board. They may find that the rotation move can be made by rotating the sections on the board. They are asked to guess how many people the game can be played with and to develop the game rules themselves. For example, they can

develop a four-player version of the game, thinking that each section belongs to one player. They may think that the marbles will be distributed equally to each player in color and number. They may also be asked to consider the details of the game rule, such as “What can be the purpose of the game in such a game and how can the game be won?” In the game where four people take turns, each player can place a marble in their own section or rotate their section in the desired direction. A rule can also be developed that the player who brings four marbles of the same color together wins the game.

c) *Playing with arbitrary rules.* The players are asked to play the game with the rules they developed in the previous step and to try whether their rules are applicable. Thus, they are made to realize that there are some potentials in the game as well as some things that cannot be done. For example, they may see that more than one person can win at the same time, according to the game rule they have developed, or that there is no move that can be made to prevent the opposing players from winning, or they may think that the game rules they have developed create a functional and fun game.

d) *Examining the game rules.* The players are asked to read the instructions containing the game rules. They are expected to understand that the game is played between two people and that each player has marbles of one color, the player must put a marble in an empty slot on the board and then rotate one of the four sections 90 degrees clockwise. They need to realize that they have to make moves that bring five marbles of their color side by side (horizontal, vertical, or diagonal).

e) *Playing with rules.* Understanding better the game rules and making the moves to be made correctly, the players are asked to play following the rules. At this step, it should be noted that the players only place the marbles of their own color on the board, one marble per move, and rotate the section clockwise once per move. Younger age groups can be asked or shown how to rotate 90 degrees.

Step 2: Understanding the cause-effect relationships in the game.

Pentago is a game with no chance factor, and what is considered chance is actually the result of carelessness. For example, a careless and unplanned move may cause a player’s marbles to form an order of five. Encountering similar situations is possible, especially during the first games. However, every move has a reason and an effect.

a) *Understanding the purpose of the game.* The purpose of the Pentago game is to bring together five marbles of their own color horizontally, vertically, or diagonally. The player must realize that the game’s aim is not just to line

up five marbles by chance, but to line up five marbles of her/his own color, and while doing this the player must avoid lining up the opponent's marbles. Players must make their moves so that five marbles of their own color come together in one direction.

b) Simplifying the game. Since there is a rotation factor in the Pentago game, reducing the board size through the game material may not be possible. The 4x4 size of the board can be formed from 2x2 sections, and the one who brings three marbles side by side can win the game. Instead, the number of marbles that can be placed side by side without making any changes on the board may be required to be four instead of five. Or the move to rotate the sections can be ignored. How these changes affect the flow of the game and how each move in the game has an effect can be examined. The goal here is to find out how moves should be made to win in the simplified situation.

c) Examining particular cases. A critical situation noticed during the games or a situation that the teacher will determine beforehand can be analyzed about what the players can do. The analysis of the situation given in Figure 3 is a good example of establishing a cause-effect relationship. In the given situation, the turn of playing is black. Is there a move Black can win? If black cannot make such a move, can white win, and how? Since it is black's turn, it is necessary to think and analyze simultaneously that there are cases of putting a marble and rotating it. When a black marble is placed on the number 1 and a 90-degree rotation is made in section B, a vertical of the five can be formed. Instead, if a black marble is placed on the number 2 and section A is rotated 90 degrees, a black five will be formed again (this time diagonally). In this situation actually, the game ends in a draw, as white fives will be formed simultaneously in the direction of the blue line. However, the only thing the white player can do to win is to align her/his marbles along



Figure 3: Analysis of move on the particular case

the blue line. Obviously, it should be noticed that it is possible for the black player to rotate the A, B, C, D sections in such a way as to prevent this line-up.

d) *Investigating about winning and losing cases.* At the end of their games, players may be asked to examine who won, how they won, why the loser lost, and what moves they should or should not make. At this step, all the situations may be examined that enable to win the game and how many

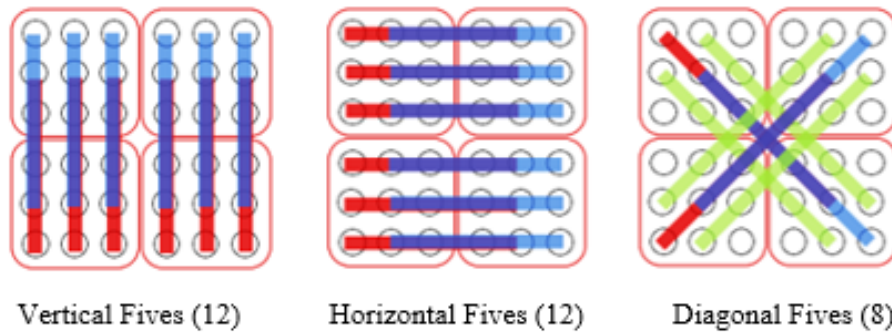


Figure 4: Calculation of possible fives

different situations there are. Figure 4 shows how many different rows of five can be made horizontally, vertically, diagonally, which will enable to win the Pentago game, by considering the ground as if it is stable, regardless of the rotation move.

The most advantageous places where marbles can be placed can be examined in the rankings of five. The places where the marbles are



Figure 5: Examination of marble in the center of the section

advantageous should be thought of considering the rotational situation. For example, the situation of marble in the middle of the sections can be examined first (Figure 5). A central slot is a place that can be considered independently of rotation. Because no matter how much the relevant section rotates, its place is in a constant position. In this case, considering all the situations that can be created with a marble in the central slot, it can be found that it contributes to 6 different fives, 2 horizontally, 2 vertically and 2 diagonally.

The situation of placing the marble in one of the corners can also be examined (Figure 6). Without rotating, the marble will contribute to 3 fives, 1 horizontally, 1 vertically, and 1 diagonally (Figure 6a). However, considering the rotation situation, as a result of 90-degree rotations, the marble placed in a corner may come to 3 different corners (Figure 6b, 6c, 6d). While the marble is in the corner, it takes place in the formation of 3 fives, when it is rotated 90-degree, it contributes to different 3 fives, and when it is rotated 180-degree, the number of fives increases to six (Figure 6c). Therefore, the advantage of the marble being placed in one corner of the section should not be considered independent of the rotation.

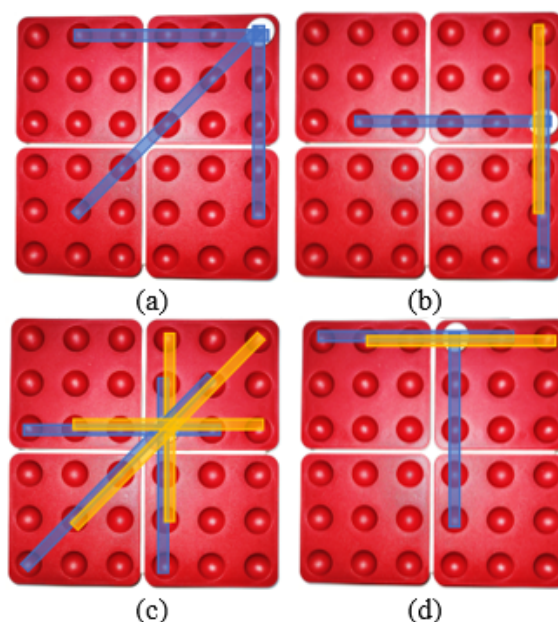


Figure 6: Examination of a marble placed in the corner

In this way, examining how the positions of the marbles provide an advantage will help players decide on the most effective move.

Step 3: Strategy development. This step can be taken after the previous processes have been properly passed, but players should not rush to take this step.

a) *Playing based on experience and inference.* Players may be asked to note the moves made one by one so that they can notice the moves they have played and then analyze them. Note-taking can be done by writing the coordinate of the slot where the marble is placed and the code of the rotating section or can be done on step-by-step drawings. One of the noted games may be re-examined step-by-step and analyzed the reason, the result of the moves, and what kind of move. In this process, players may be asked to evaluate their moves as to whether they are defensive or offensive moves. Thus, they can realize why they made the moves they made. If a wrong move has been made at this step, players may be requested to continue by changing this move. Although the consequence of the moves made at the beginning seems to be more uncertain, inferences can be made about where the previous moves should be in the last moves. In this sense, examining the backward moves starting from the last move can be useful. Players may be asked to examine whether there is a situation where the losing player can win and any other moves they can make.

b) *Predicting a few moves ahead.* It is usual to analyze where to put the marble for the player's turn and what will happen on the game board after the rotation. The player has to think about what her/his opponent will do next. Let's take a look at the situations in which forecasting will occur, based on the previous example. Let the white player create the situation indicated in Figure 3 after placing the marble in section B and rotating it. In this case, the game will end in the defeat of the white, as she/he cannot foresee that her/his move will create the situation that will enable the black player to win. At this point, the prediction of a few moves beyond in the Pentago game requires an analysis of the situation that will occur with the opponent's placing and rotating, beyond she/he own moves of placing and rotating.

c) *Build the strategy as a set of moves.* Players at this step may explain their strategy mostly in the form of one or more moves. For example, they may have set a strategy such as "I place a marble in the center of the section and rotate this section, as the starting strategy." However, this cannot be expressed as a strategy because it only includes one or two moves. After this move, what is the best move to be made has to also be specified. If possible, a set of moves depending on the opponent's moves has to be determined.

In games such as *Pentago*, where certain winning strategies have not yet been determined, it is not possible to determine a strategy that will cover all moves from the beginning to the end of the game, but at least the moves that players can make in certain situations and positions can be determined. The player may be asked to integrate these moves, not only as a move, but also thinking before and after these situations as consisting of several moves such as “If the opponent does this, I will do this, and when the opponent does this, I will do this”.

Step 4: *Clarifying the strategy.* Until this step, the players must have constantly analyzed what moves were made during the games. At this step, the player can think about where to put the marble on the *Pentago* board and what kind of situation occurs after rotating. In addition, players can be supported to express verbally the experiences gained in the game and play the game repeatedly. For example, “I prefer not to put marbles in the middle of the sections because it invalidates the rotation.”; “I prefer to put a marble in the middle of the sections because it helps to form six different fives.” Likewise, players can share the moves with other players and inference from the criticisms made on the moves. The determined strategies can be written in a convenient place where everyone can see them. Thus, the strategies determined can be tried by others. After all the strategies have been tried, the type of strategies (attack, defense, starting situation, ending situation, etc.), their similarities, and relations with each other may be evaluated. Considering other shared strategies, the most appropriate strategy can be revealed to the whole class.

Step 5: *Verifying the strategy.* In this step, how a presented strategy works or why it doesn’t work should be explained with causes. At this point, as stated in the previous step, “I prefer not to put a marble in the middle of the sections because it invalidates the rotation”, the move can be tried with different players by creating different situations. Similarly, which player would be more advantageous can be examined if a player prefers to put a marble in the middle of the sections and the other player does not. The situations encountered are evaluated and which strategy is acceptable can be analyzed. At this step, some strategies may be accepted while others may be rejected. The parts of rejected strategies that are considered problematic may be edited and changed. On the other hand, what needs to be done to develop the accepted strategies can be examined. For example, besides placing marbles in the middle of the sections, whether it would be more advantageous to put marbles at the corners of the sections or in the middle

of their edges can be examined. Then, including the rotation, the “In what situation do you want the rotation to be ineffective?” question can be analyzed. This step should not be passed quickly, on the contrary, it will take a long time. The validity of the strategies created and developed by playing Pentago game numbers of times with different players can be checked in various ways.

Step 6: *Evaluating the game and the process.* This step can be taken when it is ensured that the steps presented above have been passed properly and the strategies developed are considered sufficient. First, the players may be asked to evaluate the Pentago game, whether they find the rules clear, whether the game is fun, whether there is anything they see as missing in the game. Issues such as what they would do if they wanted to change the Pentago game, how the game rules should be on a larger board size can be discussed. After the evaluations on the game, it is possible to talk about the evaluation of the analyzes made during the process of examining the Pentago game, where they found the most interesting and enjoyable part of the steps passed, and at which step they got bored. Comments can be allowed such as, “I develop to create a four, the name of the game by making Tetrago.”; “I release the game sections’ rotation rule, the player should be able to rotate any section in any direction they want. This makes the game more difficult.”; “I would renew the rule by preventing the player from placing marbles again in the section where they placed marbles in the previous turn.”; “I would have made the colors of the game board in black and white.” and the comments made can be discussed. In this process, if desired, students can replay and evaluate the game with the rules they have determined.

Conclusion and Suggestions

The environments playing board games are social environments, and social and emotional interactions (joking, competition, solidarity, sportsmanship, etc.) can often be observed in these environments. However, the cognitive processes experienced by the players in these environments are mostly individual and unless there is a valid reason, the players do not share these experiences and do not reflect them in their non-game interactions. Cognitive experiences must be systematically reflected in social interactions by transforming social and affective environments into learning environments. In this study, the cognitive processes based on strategy development in board games were modeled. How these processes could be reflected in the game environment and thus how the game environments

could gain an instructional dimension was examined. In this context, a framework for game playing process was developed. The actions of the students were defined at each step determined in this framework, and what the teacher could do to make these actions happen was discussed. Thus, a guide was presented to teachers on how they can support their students' social and affective skills and their cognitive skills while playing board games. Teachers can use this guide in different games and thus improve their practices and related skills related to "playing board games". In this way, board games will have meaning beyond entertainment and having a good time for both teachers and students, and it will be possible to support the basic cognitive skills of the curriculum, especially reasoning, critical thinking, and communication, with board games. Certainly, it should be taken into consideration that a general framework is presented here, and the teacher can change and develop this framework according to the game in question, according to the social, affective, and cognitive skill levels of her/his students, and according to the skill she/he wants to support.

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