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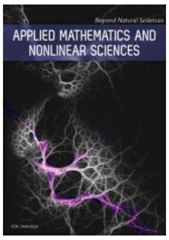

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Portfolio Optimization Strategy Based on Risk Diffusion Model in Emerging Industry Development

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Abstract

In this paper, we first sort out the formula of the premium principle and the algorithm of the diffusion model and then study the strategy problem about optimal investment consumption and insurance purchase when investors invest in new developing industries under the risk diffusion model. In real financial markets, there are two types of uncertainty regarding asset prices: normal fluctuations and abnormal shocks. The risk diffusion model is used to plan the optimal investment strategy based on this basis. In the end, three tests are executed, including two numerical simulations and one investment analysis that determines the investor's age. The computational results show that the optimal strategy in the first set of simulations is the 56% increase in investment volume $A(x)$ at the parameter $\sigma = 0.1$. The standard deviation of the investor's objective in the second set of simulations is 9.287%, and the investor's assets invested in risky securities should be 1.071. In the third set of tests, as the investor's age increases, the value of the investor's investment in risky assets continues to decline from 2.0 after 30 years, and by the time it reaches 40 years, it is already close to 0.25, and there is a continued decline, converging to 0. Investors can invest in providing effective reference data by investing in the portfolio optimization strategy in this paper, which predicts stock market volatility and vibration.

Keywords: Emerging Industries; Risk Diffusion Model; Optimal Portfolio; Optimization Strategy.
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1 Introduction

Slow or even stagnant economic development has been experienced by various countries and regions around the world due to the financial crisis. The experience of generational economic development shows that accelerating the cultivation and development of strategic emerging industries is the fundamental driving force that helps a country's economy to come out of the downturn and promotes the progress of the financial market. In the course of overcoming major economic crises, wave after wave of emerging industries sprouted and grew up, revitalized and became new economic growth points [1-2]. The innovation and invention of new technologies and the birth of new industries indicate that the world economy is about to enter a new growth cycle, and whether it can realize a breakthrough in the field of high technology will, to a large extent, rewrite the future of the country's power contrast [3-4]. The concept of a low-carbon economy has emerged in response to the financial crisis and economic downturn. Some countries such as the United States, the European Union and Japan are committed to promoting the technological revolution of strategic emerging industries with new energy as the main line in order to revitalize the economy and promote economic recovery by emerging industries. China also takes the development of strategic emerging industries as a breakthrough and strives to seize the high ground of science and technology, stimulate economic development and enhance the comprehensive strength of the country [5-6].

The financial market is faced with many uncertainties due to the highly volatile market environment in China's general economic environment. The importance of risk metrics for emerging industry markets has increased. The work of measuring financial market risk is particularly difficult due to the interdependence of markets in transmitting and spreading [7-8].

In terms of equity investment, each fund focuses on equity investment strategy, relatively downplaying macro indicators, analyzing different characteristics of industries and listed companies from the meso level, identifying industries with good development prospects and relatively stable competitive structure in the process of economic development, and selecting high-quality enterprises with excellent business models and strong competitiveness [9-10]. China's emerging industries are the focus of this group of stocks, which reflect its future economic development and have high representative and research value. China's new industries are concentrated in the manufacturing and information industries, which are also where industrial development is heading. In the present layout of China's emerging industries (11-12), health and social work, scientific research, and technology services are the areas where its remaining assets are invested [11-12].

The development of strategic emerging industries has not yet reached the standards of mature markets, so it is more likely to be affected by the impact of changes in the market environment, which, in turn, may not be able to meet the hedging needs of investors. Literature [13] points out that investment institutions and private investors in developed economies focus on emerging stocks in emerging markets, with little attention paid to stocks in frontier markets, where significant development has taken place, with significant improvements in poverty and literacy rates. Investors should be aware that the most inefficient areas of the market may have the best opportunities, according to Larry Speidell. The current state of risk estimation using high-moment models in the process of portfolio diversification in international emerging and developed markets is explored in the literature [14]. By studying a sample of 33 global stock exchange market indices from 2001 to 2012, the data processing in the analysis found that the high-moment model outperforms the traditional mean-variance model in this period. Investors can choose the optimal portfolio for their investment by relying on this research finding, which provides an effective reference value. Literature [15] draws on option investment theory to study the portfolio characteristics of OFDI and how the characteristics affect the investment performance of OFDI and finds that when the overall quality of the invested country's institutions is higher, or the diversity of the strategic market is higher, the OFDI of enterprises in emerging economies contributes more to the short-term performance, and when the portfolio has a higher strategic factor market and the institutional environment has the When the portfolio has higher strategic factor markets and the institutional environment is characterized by diversity, OFDI by firms from emerging economies contributes more to long-term performance. This serves as a reference for comprehending the factors that contribute to OFDI performance in developing countries. Literature [16] in order to study the impact of structural ruptures on the optimal weights, hedging ratios and hedging effectiveness indices of risk-minimizing portfolios consisting of the S&P index and the emerging market indices of Eastern Europe, South America and Asia. A binary DCC+ARCH model is used, and the study shows no evidence that structural breaks have a significant effect on the above three, verifying that there is no room for arbitrage for international investors. Literature [17] states that it is important for investors to differentiate between emerging and developed markets, using hierarchical clustering to objectively identify countries or regions from an investment perspective. This important geographic footprint extends beyond the basic classification of the economy and the classification of financial returns at the country level. Active and index investors can benefit greatly from this significant investment reference.

In recent years, the development of the financial market has soared, and more and more investors are keen on speculative behavior in the financial market and more of it to inject fresh vitality. Investors are now focusing on choosing an effective portfolio as the most important topic. The study of investment portfolios is of particular importance in the new situation. Literature [18] explores the optimal portfolio problem of risk-free assets and two jump-diffusion risk assets based on the stochastic control theory and the corresponding extended Hamilton-Jacobi-Bellman equations, solves the optimal equilibrium strategy and the closed expression of the value function, and arrives at the optimal strategy of investing the amount of risky assets, i.e., the risky investment is directly proportional to the current wealth, and through the numerical arithmetic example illustrate the effect of model parameters on the optimization results. A stochastic decision model for renewable energy investments is proposed in the literature [19] that allows for maximum risk avoidance and expected returns under current risk control. Conditional value-at-risk is used to assess uncertainty, and the results of the study show that diversification of firms' assets and complementarity of power generation sources reduces the risk of investors' portfolios, the decision maker's risk control ability affects the firm's market positioning, and the results of the study confirm that there is a trade-off relationship between risk aversion and expected return. Literature [20] examined how pension management can avoid excessive risk for investment, even if the quadratic deviation card expectation is minimized between the actual terminal pension fund and the preset terminal fund size. To solve the optimal portfolio control problem and obtain its closed-form solution, stochastic dynamic programming and matching methods are employed. The optimal strategy values are also derived by developing new algorithms and discussing, in-depth, the effects of preset terminal targets, input delay lengths and risk jump intensities on optimal investments. Literature [21] proposes a two-stage dynamic portfolio optimization approach to obtain the optimal asset allocation in order to accurately predict the price of the investment assets, modeling the asset returns through coherent fuzzy numbers. The optimization model is solved through the integration of investor attitudes using the genetic algorithm. After the example to verify the feasibility of the method, mean-variance and other models for comparison to analyze the advantages of the method and have a certain positive impact on the behavior of investor investment decision-making. Literature [22] approximated the utility maximization problem using the semi-harness approximation technique to confirm the convergence speed of the optimal insurance investment strategy and also proposed the dual-control Monte Rocca method to solve the approximate control problem. Literature [23] uses a two-dimensional dependent claims model, a Heston SV model and a reduced form approach to describe the insurance market, the financial market and the default risk to obtain the wealth evolution equations of the insurer, respectively. According to the findings, risk management is the primary factor in insurance companies' wealth management and investment strategies. Literature [24] confirms that the minimum variance optimal weight vector constructed using NERIVE in a portfolio has an upper bound on the maximum risk and an upper bound on the actual risk of order p. The theoretical minimum variance portfolio weights meet the same upper bound on the maximum risk while simultaneously complying with the same upper bound on the maximum risk. Literature [25] points out that ambiguity aversion significantly affects the optimal exposure to stocks and covariances, and ignoring the model uncertainty of the investment, this loss will be greater, which is an important reference for optimizing the portfolio.

In this paper, we choose to use the diffusion risk model to explore the optimal portfolio strategy for investors when investing in emerging industries based on the irregular movements of market prices. The focus is on the optimal investment problem in a diffusion risk environment for an exponential utility function, considering insurance and reinsurance, variable interest rates and investment objects containing options, respectively, and arriving at an optimal solution while satisfying the dual requirements of the mean being as large as possible and the variance being as small as possible. After completing the computational arithmetic, three sets of tests were performed. One group focuses on numerical simulation of investment, while another group focuses on emerging industry stock prices and investment analysis. The results proved that the algorithm of this paper can incorporate various variables and can calculate the effect of the variable on the results in addition to this algorithm in the numerical simulation of investment can be solved for the standard deviation of the investor's expectations and the price that the investor should invest.

2 Risk diffusion models

Insolvency can occur for both investment companies and investors in the event of a catastrophe. By sharing the risk, reinsurance reduces the risk of insolvency. Reinsurance is a form of insurance in which the investor or investment company pays a premium to a reinsurance company to share risks. The investor is charged a premium by the reinsurer that exceeds the expected value of the risk, just like any other insurance contract. The investor's solvency is enhanced by rationalizing surplus planning through investments. Investing companies find it important to make reinsurance investments.

To optimize portfolio optimization and avoid risk while maintaining profitability, the jump-diffusion model can be combined with the investor's reinsurance-investment model, which can calculate the investor's surplus process.

2.1 Premium rationale

The market price can fluctuate between high and low depending on the agreed-upon price, and sometimes, it is not even straightforward to compute. If the random variable Z is used to denote the fraction of the claim amount paid by the company, the reinsurance premium is a function of the distribution function of Z, denoted $\pi(Z)$. In this section, we present the computational principles that pertain to optimal investment and reinsurance.

Expected Value Principle:

$$\pi(Z)=(1+\rho)E(Z) \tag{1}$$

Where parameter $\rho>0$, is called the relative security load. If $\rho=0$, it is the net premium principle.

Index Principle:

$$\pi(Z)=\frac{1}{\beta}\ln E(\exp(\beta Z)) \tag{2}$$

Where the parameter $\beta>0$, called the adjustment factor, the exponential principle occupies a very important place in the study of the principle of premiums, and was studied and applied by a wide range of experts and scholars in the 1970s and 1980s.

Variance principle:

$$\pi(Z)=E(Z)+\beta Var(Z) \tag{3}$$

Where parameter $\beta>0$.

Corrected variance principle:

$$\pi(Z)=E(Z)+\beta_1 D(Z)+\beta_2\frac{Var(Z)}{E(Z)} \tag{4}$$

Where both parameter β_1 and parameter β_2 are greater than zero.

Semi-Variance Principle:

$$\pi(Z)=E(Z)+\beta E(Z-EZ)_+^2 \tag{5}$$

Where parameter $\beta>0$.

Mean value principle:

$$\pi(Z)=\sqrt{E\left(Z^2\right)}=\sqrt{(E(Z))^2+Var(Z)} \tag{6}$$

Standard Deviation Principle:

$$\pi(Z)=E(Z)+\beta D(Z) \tag{7}$$

Where parameter $\beta>0$.

Mixing principle:

$$\pi(Z)=E(Z)+\beta_1 D(Z)+\beta_2 Var(Z) \tag{8}$$

Where both parameter β_1 and parameter β_2 are greater than zero.

Zero Utility Principle:

$$\pi(Z)\text{ meets }U\left(w_0\right)=E\left[U\left(w_0+\pi(Z)-Z\right)\right] \tag{9}$$

Where w_0 denotes the initial wealth of the firm and $U(w)$ is the utility function of the investor satisfying $U'(w)>0$ and $U''(w)\le 0$. When the utility function is taken as an exponential function, the premium formula is $\pi(Z)=\frac{1}{\lambda}\log\frac{Ee^{-\lambda(w_0-Z)}}{Ee^{-\lambda w_0}}$. When Z and w_0 are independent of each other, the classical exponential principle is obtained.

2.2 Diffusion risk modeling

To better understand the diffusion risk model, the classic Cramer-Lundberg risk model is first given:

$$R(t)=x_0+ct-\sum_{i=1}^{N(t)}Y_i=x_0+ct-S(t) \tag{10}$$

Where $x_0\geq 0$ is the firm's initial surplus, $c>0$ is the firm's premium income per unit of time for $\{N(t),t\geq 0\}$. The chi-squared Possion process with parameter $\lambda>0$ denotes the total number of claims up to moment t for $\{Y_i,i=1,2,\cdots\}$ is an independent and identically distributed (strictly) positive random variable, and Y_i denotes the size of the i th claim. Furthermore, assume that $\{N(t),t\geq 0\}$ and $\{Y_i,i=1,2,\cdots\}$ are independent of each other.

Approximate this with the following diffusion risk model:

$$X(t)=x_0+\mu t+\sigma W(t) \tag{11}$$

Where $\mu=c-\lambda E[Y_i]>0,\sigma^2=\lambda E\left[Y_i^2\right]>0,W(t)$ is the standard Brownian motion.

Please assume that the investment firm controls its level of risk by purchasing insurance, and for each claim, assume that the firm purchases a proportional reinsurance level of $1-a$, where a is called the risk exposure, i.e., it is the firm's proportion of retained risk. At the same time, the investment company pays a portion of the premium to the reinsurance company, assuming η is the premium income per unit of time required by the reinsurance company, the reinsurance premium is $(1-a)\eta$, and $\eta\geq\mu$. The surplus process of the company at the company's retention ratio of a is:

$$X(t,a)=x_0+[\mu-(1-a)\eta]t+a\sigma W(t) \tag{12}$$

If retention a satisfies $0\leq a\leq 1$, then a is said to be a feasible reinsurance strategy. The set of all feasible reinsurance strategies is denoted as Π_1. Assuming that the reinsurer itself is not involved in any other insurance activities and that its source of income consists entirely of reinsurance premiums, the surplus process of the reinsurer after underwriting is given:

$$Y(t,1-a)=y_0+(1-a)\eta t+(1-a)\sigma W(t) \tag{13}$$

Where $y_0\geq 0$ is the initial surplus of the reinsurer and $(1-a)$ is the reinsurer's distribution ratio. If the distribution ratio $1-a$ satisfies $0\leq 1-a\leq 1$, then $1-a$ is said to be a feasible distribution strategy. The set of all feasible diversion strategies is denoted as Π_2.

2.3 Risk asset model

Investing is necessary for investment firms to achieve profitability, and the following models are frequently employed to depict the cost of risky assets in actual financial markets.

GBM model:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{14}$$

CEV modeling:

$$dS(t) = \mu_s S(t)dt + \sigma_s S(t)^{\beta+1} dW(t) \tag{15}$$

Where μ_s is the average return and $\sigma_s S(t)^{\beta+1}$ is the instantaneous volatility. $\beta \le 0$ is the elasticity factor.

Most of the literature provides three main criteria for measuring optimal strategies when it comes to investment strategies. The expected utility of terminal wealth can be maximized by minimizing the criteria in the first category. The second category is the bankruptcy probability minimization criterion. The third category is the mean-variance criterion. Furthermore, the first type of criterion to use the utility function $U(x)$, that is, consider $\max E[U(x)]$ where the risk aversion index is $r(x) = -\frac{U''(x)}{U'(x)}, U(x)$ satisfies $U'(x) > 0$ and $U''(x) \le 0$.

The exponential utility function formula is:

$$U(x) = -\frac{1}{m} e^{-mx} \tag{16}$$

Where $m > 0$ denotes the risk aversion coefficient.

The power utility function is given by:

$$U(x) = -\frac{1}{m} x^m, x > 0 \tag{17}$$

Where the constant $0 < m < 1$.

In summary, in this paper, we will consider the optimal investment problem in a diffuse risk environment for the exponential utility function with variable interest rates on both sides of the reinsurance and investment objects containing options, respectively.

3 Optimal Portfolio Problem of Emerging Industries Based on Diffusion Risk Modeling

Emerging industries greatly impact the national economy of a country. Their development is not smooth, but varies, fluctuates and even jumps frequently. Excessive or frequent price changes will adversely affect emerging industries, leading to an imbalance in resource allocation, an imbalance in industrial structure and business difficulties, affecting the government's macro-control of the economy and even destroying the normal operation of the national economy as a whole. Based on the intermittent and sudden change in the macroeconomic environment and market, this chapter studies the mechanism and influencing factors of price change of emerging industries establishes the risk diffusion model of emerging industries' investment, demonstrates the price characteristics and applicability of the model, and defines the meanings of the model parameters. The statistical characteristics of the model are the subject of discussion. The further empirical analysis adopts the market price index of emerging industry markets in different places, compares the jump degree of the price index of the emerging industry in different levels and types of different geographic regions horizontally from the macro level, and compares the strength of the transaction price jump in each period of a specific region vertically from the micro level, and finds out the specific influencing factors of the jump, and analyzes the effectiveness of each type of policy, and the specific formulas are as follows:

$$V(t,x)=\lambda_0-\frac{\gamma}{m}e^{-mxe^{r_0(T-t)}+h(T-t)} \tag{18}$$

Where h is a definite function with:

$$V_t=\left[V(t,x)-\lambda_0\right]\left[mxr_0e^{r_0(T-t)}-h'(T-t)\right] \tag{19}$$

$$V_x=\left[V(t,x)-\lambda_0\right]\left[-me^{r_0(T-t)}\right] \tag{20}$$

$$V_{xx}=\left[V(t,x)-\lambda_0\right]\left[me^{r_0(T-t)}\right]^2 \tag{21}$$

Substituting the above equation into the equation gives:

$$\begin{aligned}&\inf_{\pi\in\Pi}-h'(T-t)+\left[c-(1-a)c_1+\left(r_1-r_0\right)bx\right]\left(-me^{r_0(T-t)}\right)\\&+\frac{1}{2}\left(b^2\sigma^2x^2+\beta^2\right)\left(me^{r_0(T-t)}\right)^2+\lambda\int_0^\infty\left[e^{amye^{r_0(T-t)}}-1\right]G(dy)\Bigg\}=0\end{aligned} \tag{22}$$

So, we need to find the function that makes the following:

$$\begin{aligned}\tilde{f}(a,b)=&-h'(T-t)+\left[c-(1-a)c_1+\left(r_1-r_0\right)bx\right]\left(-me^{r_0(T-t)}\right)\\&+\frac{1}{2}\left(b^2\sigma^2x^2+\beta^2\right)\left(me^{r_0(T-t)}\right)^2+\lambda\int_0^\infty\left[e^{amye^{r_0(T-t)}}-1\right]G(dy)\end{aligned} \tag{23}$$

Taking the minimum value of a^*,b^*. Taking the partial derivative of a,b and making it equal to 0, respectively, yields:

$$\frac{\partial\tilde{f}}{\partial a}=-c_1me^{r_0(T-t)}+\lambda me^{r_0(T-t)}\int_0^\infty ye^{amye^{r_0(T-t)}}G(dy)=0 \tag{24}$$

$$\frac{\partial\tilde{f}}{\partial b}=\left(r_1-r_0\right)\left(-me^{r_0(T-t)}\right)x+b\sigma^2x^2\left(me^{r_0(T-t)}\right)^2=0 \tag{25}$$

Can be obtained directly from the above equation:

$$b^* = \frac{r_1 - r_0}{\sigma^2 mx} e^{-r_0(T-t)} \tag{26}$$

From the above equation:

$$-c_1 + \lambda \int_0^\infty y e^{amye^{r_0(T-t)}} G(dy) = 0 \tag{27}$$

Since the proportional reinsurance retention $a \le 1$, we also need to give the existence condition for the optimal value a^* of the above equation.

Lemma, when $c_1 \le \lambda \int_0^\infty y e^{mye^{r0(T-t)}} G(dy)$, the above equation has a unique positive root a_1 and this root also satisfies $0 < a_1 \le 1$. When $c_1 > \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$, there is a unique positive root a_2 and $a_2 > 1$.

Prove the setting:

$$f(a) = -c_1 + \lambda \int_0^\infty y e^{amye^{r_0(T-t)}} G(dy) \tag{28}$$

Then:

$$\begin{aligned} f(0) &= -c_1 + \lambda\mu < 0 \\ f(1) &= -c_1 + \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy) \\ f'(a) &= \lambda m e^{r_0(T-t)} \int_0^\infty y^2 e^{amye^{r_0(T-t)}} G(dy) > 0 \\ f''(a) &= \lambda \left(m e^{r_0(T-t)} \right)^2 \int_0^\infty y^3 e^{amye^{r_0(T-t)}} G(dy) > 0 \end{aligned} \tag{29}$$

Therefore, $f(a)$ is a monotonically increasing convex function, so when $c_1 \le \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$, $f(a)$ has a unique positive root a_1 on the interval $(0,1]$. When $c_1 > \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$, $f(a)$ has a unique positive root a_2 and $a_2 > 1$, thus leading to the proof.

Therefore, when $c_1 \le \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$, $a^* = a_1$. when $c_1 > \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$ $a^* = 1$, i.e., the investment firm does not engage in a reinsurance strategy.

When $c_1 \le \lambda \int_0^\infty y e^{mye^{r_0(T-t)}} G(dy)$, substituting $a^* = a_1, b^* = \frac{r_1 - r_0}{\sigma^2 mx} e^{-r_0(T-t)}$ 5 back into the equation gives:

$$\begin{aligned} h'(T-t) = &-m\left[c - (1-a_1)c_1\right] e^{r_0(T-t)} - \frac{(r_1 - r_0)^2}{2\sigma^2} + \frac{1}{2}\beta^2 m^2 e^{2r_0(T-t)} \\ &+ \lambda \int_0^\infty \left[e^{a_1 mye^{r_0(T-t)}} - 1 \right] G(dy) \end{aligned} \tag{30}$$

From the initial condition $h(0)=0$, the expression for the function $h(T-t)$ can be found.

Similarly, when $c_1>\lambda\int_0^\infty ye^{mye^{r_0(T-t)}}G(dy)$, substituting $a^*=1, b^*=\frac{r_1-r_0}{\sigma^2 mx}e^{-r_0(T-t)}$ back into the above equation gives:

$$\begin{aligned} h'(T-t) &= -cme^{r_0(T-t)}-\frac{(r_1-r_0)^2}{2\sigma^2}+\frac{1}{2}\beta^2m^2e^{2r_0(T-t)} \\ &+\lambda\int_0^\infty\left[e^{mye^{r_0(T-t)}}-1\right]G(dy) \end{aligned} \tag{31}$$

From the initial condition $h(0)=0$, the expression for function $h(T-t)$ can be found. Substituting the obtained $h(T-t)$ back into the equation above, we get the corresponding optimal value function.

Thus from the above discussion, we have the following theorem. Let $\tilde{a}(t)$ be the equation:

$$c_1-\lambda\int_0^\infty ye^{amye^{r_0(T-t)}}G(dy)=0 \tag{32}$$

The optimal investment strategy is the unique positive root of:

$$b^*(t)=\frac{r_1-r_0}{\sigma^2 mx}e^{-r_0(T-t)} \tag{33}$$

The optimal reinsurance strategy is:

$$a^*(t)=\tilde{a}(t)\wedge 1 \tag{34}$$

The value function is:

$$V(t,x)=\lambda_0-\frac{\gamma}{m}e^{-mxe^{r_0(T-t)}+h(T-t)} \tag{35}$$

Where $h(T-t)$ satisfies the following conditions:

1) If $a^*(t)<1$, then $h(T-t)$ is determined by the above equation and initial condition $h(0)=0$.

2) If $a^*(t)=1$, then $h(T-t)$ is determined by the above equation and initial condition $h(0)=0$.

It follows from the theorem that the determination of the optimal reinsurance strategy does not affect the determination of the optimal investment strategy.

It follows from the theorem that both the optimal investment strategy and the value function depend on the investment firm's wealth x at moment t. The optimal investment strategy is a subtractive function of the investor's and the investment firm's wealth x at moment t. That is, the larger x is, the smaller the surplus from investing in risky assets. The simultaneous value function is an increasing function of the wealth x of the investment firm at moment t, which is consistent with the actual situation. The optimal investment strategy obtained does not depend on the wealth x of the insurance company at moment t, and the reason for this is because the risk-free rate is 0 in the middle.

4 Numerical testing of optimal investments

4.1 Numerical simulation of investments

This simulated investment test is conducted. Setup $a = 0.5, \lambda = 3, \delta = 0.06, r = 0.04, k = 1, c = 3.6, \theta = 0.6, \rho = -0.1, \beta = 0.1, h = 0.01$. Figure 1 shows the effect of parameter σ on investment strategy $A(x)$ at time $\mu = 0.1$. Figure 2 shows the effect of parameter μ on investment strategy $A(x)$ at time $\sigma = 0.1$. From Fig. 1, it can be seen that $A(x)$ gradually decreases as σ increases. At $\sigma = 0.1$, Z grows to 56% at $A(x)$ in the interval of 0 to 0.25, and then decreases rapidly to maintain a steady state. Because σ indicates the fluctuation of risky assets, the larger the σ indicates, the higher the risk, so the investment in risky assets should be less, which is consistent with the actual. At $\sigma = 0.5$, the growth rate is only 13%, and $A(x)$ stops growing at a Z of 0.3 or less and enters a plateau.

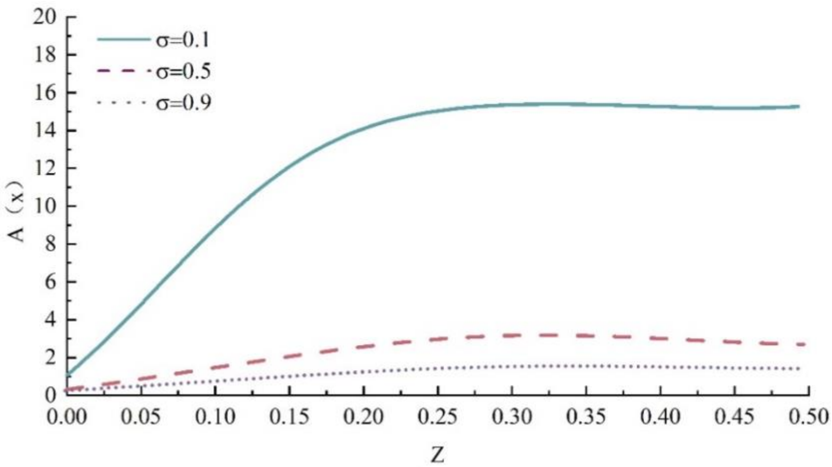

Figure 1. The impact of σ on investment with μ =0.1

μ represents the interest rate on the risky asset, and a larger μ indicates that the investor's expected rate of return is greater, so the amount invested in the risky asset is greater, which is consistent with reality. It can be seen that $A(x)$ gradually increases as μ increases. At $\mu = 0.1$, $A(x)$ grows most slowly as Z increases. The growth rate decreases substantially at Z = 0.2. During the period when Z increases from 0.2 to 0.5, the value of $A(x)$ only changes from 12 to 14. At $\mu = 0.5$ the stagnation period changes from 0.2 to 0.25. At $\mu = 0.1$ the stagnation period is around 0.3.

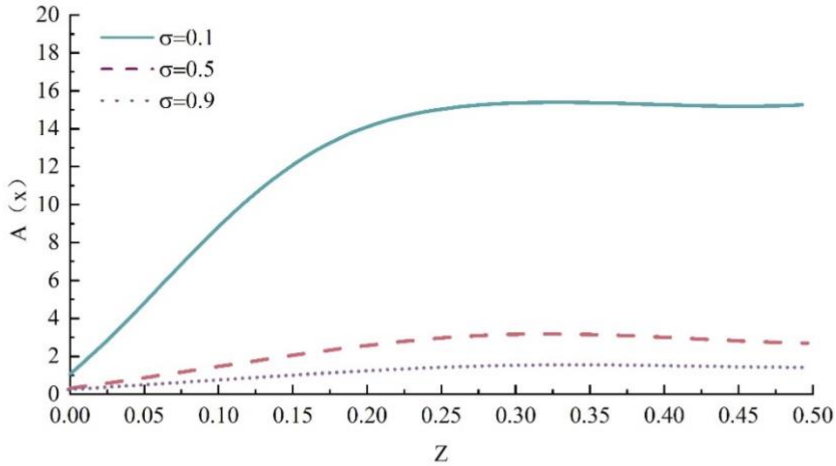

Figure 2. The impact of μ on investment with σ=0.1

4.2 Risk-free portfolio investment simulation

Suppose that there exists risk-free security in the market for the new development industry with an average annual return of $r=5\%$, and another stock with an average annual return of $b=15\%$ and a standard volatility of $\sigma=16\%$. Assume that each jump has a magnitude of 1, i.e., $\varphi=1$, and that the jump occurs with an intensity of 1, i.e., $\lambda=1$. Letting $T=1$ years and $\rho=\dfrac{(b-r+1)^2}{\sigma^2+\varphi^2}=1.07$ yields the market line for the security:

$$E\overline{x}(1)=x_0e^{\int_0^1-(r)dt}+\sqrt{\frac{1-e^{-\int_0^1\rho(t)dt}}{e^{-\int_0^1\rho(1)dt}}}\sigma_{\overline{x}(1)}=x_0e^{0.05}+1.386\sigma_{\overline{x}(1)} \tag{36}$$

Now suppose that an investor has an initial asset of $x_0=200$ and a desired rate of return of 20% after one year, calculate the amount of risk this investor will have to take at this level of desired return.

From the securities market line, when $x_0=200, E\overline{x}(1)=236$ is available:

$$\sigma_{\overline{x}(1)}=\frac{E\overline{x}(1)-x_0e^{0.05}}{1.386}=\frac{236-200\cdot e^{0.05}}{1.386}=18.574 \tag{37}$$

The results indicate that the standard deviation of this investor's objective is 18.574%.

Next the portfolio of this investor is calculated:

$$\gamma=\frac{E\overline{x}(T)-\alpha x_0}{\beta}=\frac{236-200e^{-1.022}}{1-e^{-1.072}}=\frac{164.024}{0.657}=249.4 \tag{38}$$

Therefore, this investor's assets invested in risky securities should be:

$$\begin{aligned}\bar{u}(t,x) &= \left[\sigma(t)\sigma(t)^T + \varphi(t)\varphi(t)^T\right]^{-1}(B(t)+\varphi(t)\lambda(t))^T\left(\gamma e^{-\int_t^T r(s)ds} - x\right) \\ &= 1.071\left(145.01e^{0.05(t-1)} - x\right)\end{aligned} \tag{39}$$

At initial moment $t=0, \bar{u}\left(0,x_0\right)=237.23$, state that he should borrow 23.23 yuan along with the initial principal of 200 yuan to purchase the risky security.

If the occurrence of jumps is not taken into account, i.e., a general diffusion model is used to describe the market price fluctuations, again assuming that there is a risk-free security in the market with an average annual return of $r=5\%$, and there is also a stock with an average annual return of $b=15\%$ and a standardized volatility of $\sigma=16\%$, let $T=1$, we have:

$$\rho_1(t)=\frac{(b(t)-r(t))^2}{\sigma(t)^2}=\frac{(0.15-0.05)^2}{0.16^2}=\frac{0.01}{0.026}=0.391 \tag{40}$$

$$\begin{aligned}E\bar{x}_1(1) &= x_0 e^{\int_0^1 r(t)dt} + \sqrt{\frac{1-e^{-\int_0^1 \rho_1(t)dt}}{e^{-\int_0^1 \rho_1(t)dt}}}\sigma_{\bar{x}_1(1)} \\ &= x_0 e^{0.05} + \sqrt{\frac{1-e^{-0.3906}}{e^{-0.3906}}}\sigma_{\bar{x}_1(1)} \\ &= x_0 e^{0.05} + 0.691\sigma_{\bar{x}_1(1)}\end{aligned} \tag{41}$$

Suppose an investor has an initial asset $x_0=200$ and a desired rate of return of 20% after one year to calculate how much risk he will take at this desired level.

Knowing $x_0=200, E\bar{x}_1(1)=(1+0.18)200=236$, from $E\bar{x}_1(1)=x_0e^{0.05}+0.6912\sigma_{\bar{x}_1(1)}$, we can obtain $\sigma_{\bar{x}_1(1)}=\frac{E\bar{x}_1(1)-x_0e^{0.05}}{0.6912}=\frac{236-200e^{0.05}}{0.6912}=37.24$.

That is, the standard deviation of this investor's expectation is 37.24%. His portfolio choices are as follows:

$$\gamma=\frac{E\bar{x}_1(T)-x_0e^{\int_0^T (r(t)-\rho_1(t))dt}}{1-e^{-\int_0^T \rho_1(t)dt}}=\frac{236-200e^{0.05-0.3906}}{1-e^{-0.3906}}=\frac{93.732}{0.323}=289.92 \tag{42}$$

From the above equation:

$$u_1(t,x)=\frac{(b(t)-r(t))^2\left(\gamma e^{-\int_t^T r(s)ds}-x\right)}{\sigma_1(t)^2}=\frac{0.01\left(289.92e^{-0.05}-x\right)}{0.16^2}=0.782\left(289.92e^{0.05(t-1)}-x\right) \tag{43}$$

$$\bar{u}\left(0,x_0\right)=0.391\left(289.92e^{-0.05}-100\right)=30.20$$

This result indicates that the investor will have to borrow 30.20 yuan at the beginning of the investment period, plus his own initial principal of 200 yuan to invest in risky securities. The investor will take a little more risk when asset market price fluctuations include jumps in the same expected return.

4.3 Emerging Industries Stock Price and Investment Analysis

Considering the jump-diffusion risk fractional stochastic differential equation satisfied by the price of zero-coupon bonds of emerging industries with credit risk and the diffusion fractional stochastic differential equation satisfied by the stock price, for the change of each parameter of the model, the algorithm in Section III of this paper is used for the calculation and then Matlab simulation run to generate the values of all the parameters used in the model are shown in Table 1. It can be seen that the stock model and bond model scores 3.1 in the case of r. The stock price increases to 3.4 when the parameter is increased to q. With the increase in the σ parameter, the value of both increases from 3.4 to 4.36. In the case of the bond price model for the emerging industry, the stock price increases with a monotonic increase in the parameter rt.

Table 1. Stock bond change

Model type	$r(\%)$	$q(\%)$	$\sigma_1(\%)$	$S_T(\Theta_T)$	H	a	λ
Bond model	3.1	-	4.36	100	0.76	0.71	2.71
Stock model	3.1	3.4	4.36	100	0.76	-	2.71
Model type	$\sigma_1(\%)$	$\sigma_2(\%)$	$\sigma_3(\%)$	t	T	b	
Bond model	0.03	2.21	0.072	20	0	0.02	
Stock model		2.21	0.072	20	80		

After adding different parameters, the relationship between stocks and bonds changes as shown in Figure 3. , the stock price model and the bond price model show a negative relationship. As the parameter increases from 0.5 to 1.0, the stock price increases from 0.5 to 8, while the debt price decreases from 8.5 to about 0.5. In addition to the above findings, in the emerging industry bond price model, stock price decreases with monotonically increasing parameters H, λ, and σ_r. In the emerging industry stock price model, stock prices increase monotonically with parameter rt, H and decrease monotonically with parameter λ.

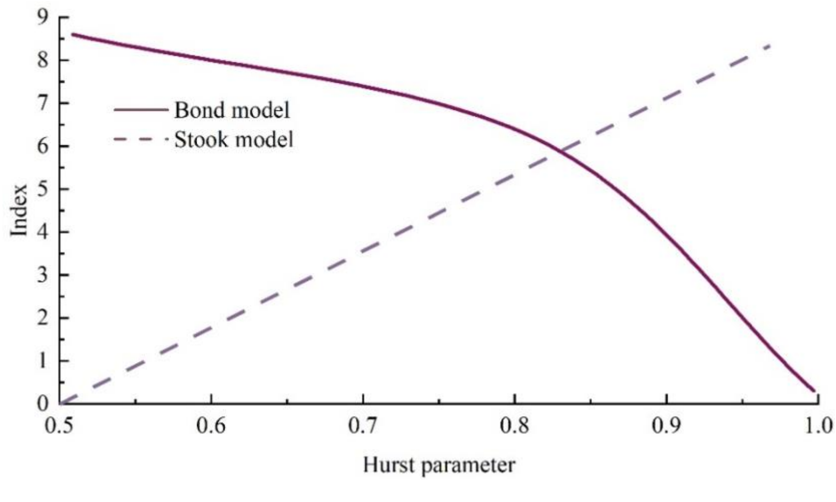

Figure 3. The change trend of the stock ticket and the debt of the H parameter

Figure 4 shows the trend of the stock and debt models after adding the λ parameter. It can be seen that both decrease gradually as the value of λ increases monotonically. A change in λ from 1 to 5 decreases the stock price and debt price from 8.89 to 3 and 0, respectively. The stock price changes even more, with an increase in λ by 1 decreasing the stock price by 20%.

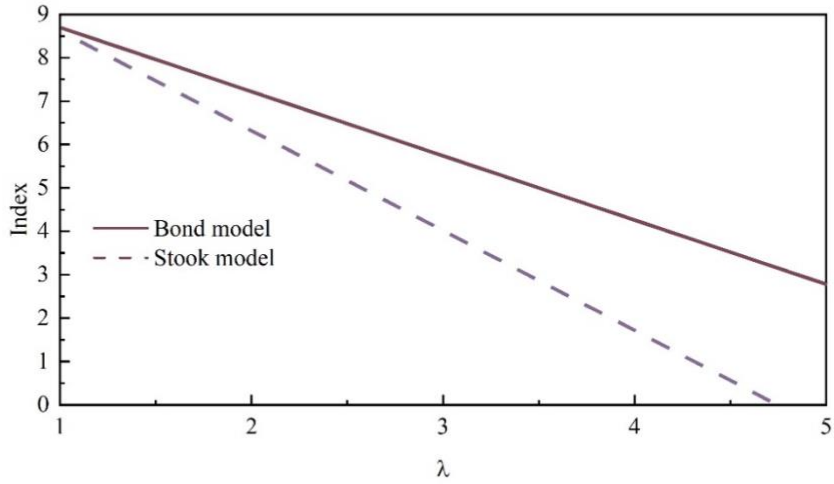

Figure 4. The change trend of the stock ticket and the debt of the λ parameter

Figure 5 shows the trend of the stock model and the debt model after adding the s-parameter. It can be seen that the two and the s-parameter show a positive correlation. The change in debt price is smaller than the stock price. The stock price is at 5.41 and the debt price is at 5.49 when the parameter is 1. As the parameter is increased to 4, the stock price grows to 7.7, and the debt price increases to 6.25, up and down. The growth rate of the former is 42.32%, and the growth rate of the latter is 13.84%.

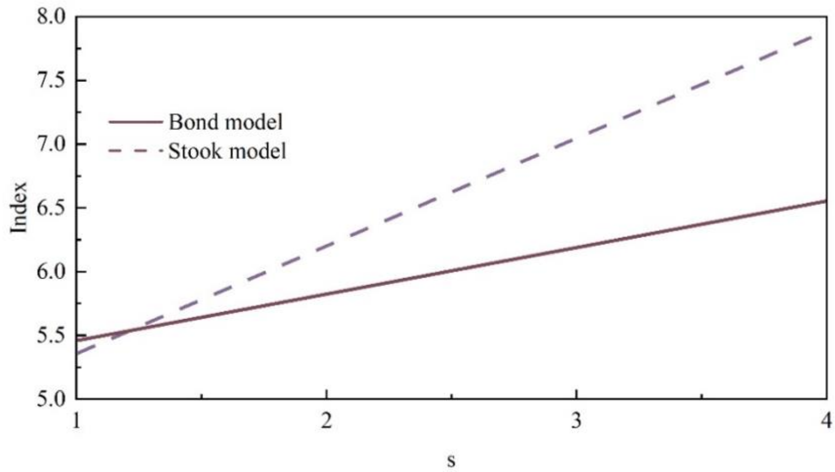

Figure 5. The change trend of the stock ticket and the debt of the S parameter

Figure 6 shows the trend of the stock model and the debt model after adding the π_r parameter. It can be seen that the stock price does not change. The price of debt decreases from 5.543 to 5.51 with the increase of π_r. The value of π_r has a small change from 0.003 to 0.020, but there is a positive correlation between the two.

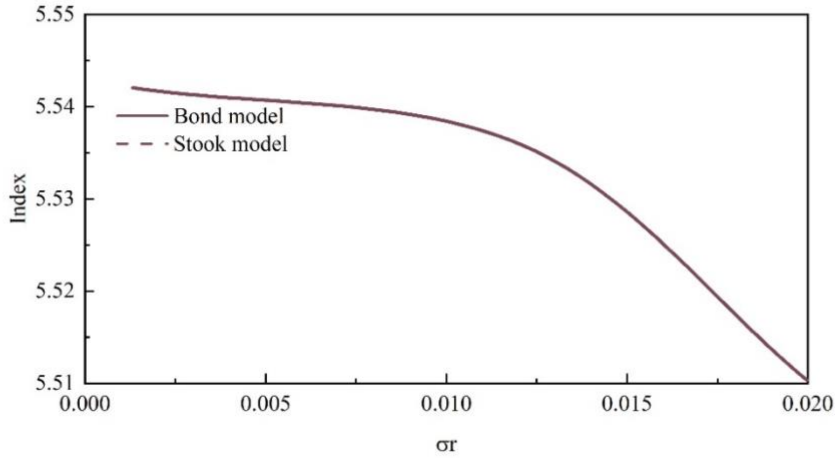

Figure 6. The change trend of the stock ticket and the debt of the π_r parameter

The numerical simulation of the investment strategy considers the evolution of the optimal consumption and investment strategy over the 20 years of the investor's life span from age 30 to 50. The boundary parameters are set in Table 1, but also $\gamma = 0.5, \beta = 1, \sigma_{D_1} = 0.1, \sigma_{D_2} = 0.15, \sigma_{D_3} = 0.05$, $T - t = 30$, $D_0 = 1$, $\int_t^T z_s ds + r_s^2 m_t + n_t = 0.005$, and $X_0 = 1, \lambda_t = \dfrac{120}{120 - t}$.

The relationship between vintage and risky assets is depicted in Figure 7. As the vintage increases from 0 to 40, the price of risky assets, although falling back from 60 to 54 for another 5-10 years, quickly rises back up to an average annual increase of 1 to 5, with an overall upward trend.

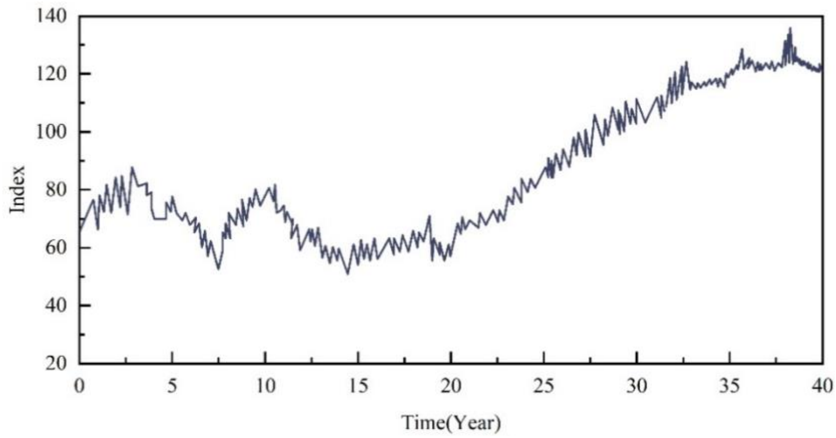

Figure 7. The price change trend of risky assets

Figure 8 demonstrates the correlation between investment strategies for investing in risky assets. Investors are investing in less risky assets. When the year is 0~30, investors are still willing to invest in risky assets, and the value of investment fluctuates up and down from 0.8 to 2.0. However, as the age of the investor increases, the value of the investor's investment in risky assets after 30 years continues to decline, to 40 years has been close to 0.25 and has continued to decline, converging to 0. As time passes, the investor's investment level in risky assets decreases, rather than being able to avoid investment risks more effectively.

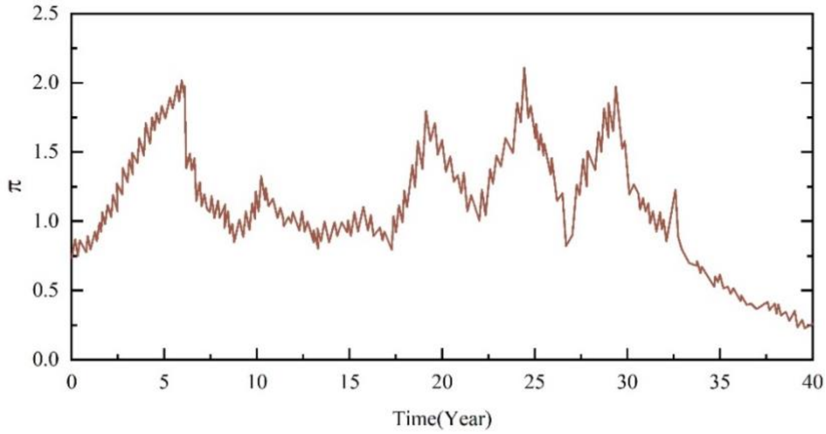

Figure 8. Investment strategies for investing in risky assets

Figure 7 and Figure 8 show that under the diffusion risk model, the trend of the price of risky assets and the trend of the investment strategy of investing in risky assets are opposite. That is, over time, the price of risky assets will become higher and higher, and the volatility of risky asset prices will become larger accordingly. Over time, investors investing in risky assets can instead avoid investment risks more effectively by lowering their investment level.

4.4 Strategies and Recommendations

As the prices of shares and house prices of emerging industries and other aspects are affected by various non-systematic risks, for investors and real estate companies, the first task of risk prevention is to measure the degree of risk. Regulatory policies that are not appropriate do not have the effect of regulation for the government. Thus, the following recommendations are presented in this paper.

1) For home buyers and investors. Generally, we should focus more on the diverse types of emerging industry regulatory policies issued by the state, comprehend the danger of non-systematic risk, and develop logical investment strategies. To prevent sudden changes in emerging industry stock prices that could lead to their losses, it is important to anticipate future regulatory policies and macroeconomic conditions and make the right choice.

2) Suggestions for practitioners of emerging industries. During normal times, pay greater attention to the upstream policy and downstream commodity trading policy and assess the impact of the policy from their perspective. Adjust the strategic positioning in line with future expectations, arrange the progress and sales plan appropriately, and realize the optimal benefit of the enterprise.

3) Suggestions for government policymaking. First of all, we should deeply understand emerging industries and pay attention to the price trend over time. Examine the primary causes of significant fluctuation in the industry’s stock price and rationally formulate regulatory policies. In order to avoid causing more damage when formulating policies, it is necessary to accurately estimate the market jump reaction that the policy may cause. The mainstay of regulation should be market regulation, supplemented by government regulation, as the policy will have an impact on the market.

5 Conclusion

This paper uses the risk diffusion model to explore the optimal portfolio for investors to invest in emerging industries and mainly draws the following conclusions through the construction and empirical analysis:

1) It is reasonable and feasible to follow the risk diffusion model of stock prices for emerging industry prices. By measuring the impact generated by unsystematic risk, the jump parameter in the jump-diffusion model for prices of various aspects of emerging industries can be used to measure the impact generated by unsystematic risk. In the case of $a = 0.5, \lambda = 3, \delta = 0.06, r = 0.04, k = 1, c = 3.6, \theta = 0.6, \rho = -0.1, \beta = 0.1, h = 0.01$, $A(x)$ gradually decreases as σ increases. In the case of $\sigma = 0.1$, when Z is in the range of 0 to 0.25 $A(x)$ the growth is 56%, and the larger σ indicates that the risk is higher, so the amount of investment in risky assets has to be lower, which is consistent with reality. At $\sigma = 0.5$, the growth rate is only 13%.

2) The algorithm of this paper is valid. Assuming that there is risk-free security in the market of the new development industry, with an average annual return of $r = 5\%$, and there is also a stock, with an average annual return of $b = 15\%$, and the standard volatility of $\sigma = 16\%$. Assuming that the magnitude of each jump is 1, i.e., $\varphi = 1$ and the intensity of the jumps occurring is also 1. In this case, the algorithm in this paper can calculate the risk that the investor is going to bear, and the standard deviation of the target is 9.287%. The investor also has to borrow 15.10 yuan at the beginning of the investment period, plus their own initial principal of 100 yuan, all invested in risky securities

3) Age can affect an investor’s decision-making behavior as well as risk. Investment year increases from 0 to 40, the price of risky assets, although then 5-10 years from 60 back down to 54, but quickly back up to 80. over time, the price of risky assets will be higher and higher, and the volatility of risky asset prices become correspondingly larger. When the year is 0~30, investors are still willing to invest in risky assets, and the investment value fluctuates up and down from 0.8 to 2.0. Nevertheless, with the increase in the age of the investor, the investment age of 30 years later, investors invest in risky assets, the value of the value continues to decline, to 40 years of the time has been close to 0.25, and have continued to decline, approaching 0.

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