www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

A PID BASED ANFIS & FUZZY CONTROL OF INVERTED PENDULUM ON INCLINED PLANE (IPIP)

Ashwani Kharola
Department of Mechanical Engineering
Graphic Era University, Dehradun, India
Email: ashwanidaa@gmail.com

Submitted: Dec. 28, 2015 *Accepted: Mar. 8, 2016* *Published: June 1, 2016*

Abstract- The objective of this study is to present an offline control of highly non-linear inverted pendulum system moving on a plane inclined at an angle of 10° from horizontal. The stabilisation was achieved using three different soft-computing control techniques i.e. Proportional-integral-derivative (PID), Fuzzy logic and Adaptive neuro fuzzy inference system (ANFIS). A Matlab-Simulink model of the proposed system was initially developed which was further simulated using PID controllers based on trial and error method. The ANFIS controller were trained using data sets generated from simulation results of PID controller. The ANFIS controllers were designed using only three membership functions. A fuzzy logic control of the proposed system is also shown using nine membership functions. The study compares the three techniques in terms of settling time, maximum overshoot and steady state error. The simulation results are shown with the help of graphs and tables which validates the effectiveness of proposed techniques.

Index terms: PID, ANFIS, fuzzy logic, inverted pendulum, Matlab, simulink.

I. INTRODUCTION

Inverted Pendulum are non-linear systems which are objects of theoretical investigation and experimentation in the area of control engineering [1]. It comprises of a pole pivoted on a cart which can be moved horizontally. It has inherently two equilibrium states i.e. a stable equilibrium state and a unstable equilibrium state. In stable equilibrium the pendulum is oriented vertically downwards whereas in unstable equilibrium, pendulum is oriented vertically upwards and hence a counter force is required to keep it align to this position [2]. The control objective is to bring the pole to upper unstable equilibrium position. The standard non-linear control techniques were ineffective to control the Inverted pendulum system [3]. Various intelligent control techniques have been developed for the control of these non-linear systems [4, 5, 6]. The recent development in areas of soft-computing techniques like ANFIS [7], fuzzy logic [8], Neural networks (NNs) [9], Genetic algorithm (GA) [10], Particle swarm optimisation (PSO) [11], Linear quadratic regulator (LQR) [12], Proportional-integral-derivative (PID) [13] etc have given novel solution to control of IP systems. The PID control provides the simplest and efficient control of IP systems. It controls both transient and steady-state response of non-linear systems. Past literature showed that researchers have shown keen interest in the control of IP system using PID control. Yusuf and Magaji [14] considered PID controller optimized with GA for control of inverted pendulum angle. The PID gains were tuned manually for obtaining an optimum response. The results obtained showed the superiority of GA-PID controller over conventional PID controller. Benrejeb and Boubaker [15] proposed a real time control of swinging pendulum using Labview software. The control was achieved using Astrom-Furuta energy control strategy. The validation test of the proposed system was achieved using Proteus software using virtual simulation models.

Dastranj et al. [16] used combination of PSO algorithm and PID for control of non-linear inverted pendulum. The simulation results showed the acceptability of proposed approach. Omatu et al. [17] proposed a self-tuning neuro PID controller for stabilisation of an inverted pendulum system. A single input and multi-output system is considered for control of single stage and double stage inverted pendulums. The PID gains were tuned using two types of neural networks. The simulation results proved the effectiveness of proposed approach. Prasad et al. [18] presented an optimal control of non-linear inverted pendulum-cart dynamic system using PID and LQR controller. The non-linear states are fed to LQR from state-space response. Simulation results justify the advantage of LQR control. Dong et al. [19] discussed about the problem of balancing first-order inverted pendulum using PID and fuzzy controllers. The simulation results showed satisfying performance and different characteristics of proposed schemes. Wang et al. [20] proposed a double loop PID control scheme for controlling unstable inverted pendulum system with single-input and double-outputs. The simulation results proved the superiority of double-loop PD-PD control scheme over other kinds of double-loop PID control methods. Wang [21] applied PID controllers for tracking control and stabilisation of three types of inverted pendulum. The paper illustrated step by step designing of PID controllers. The Lagrange's and state equation models were used for analysing relationship between three types of inverted pendulum. Chakraborty et al. [22] presented various types of mathematical modelling for inverted pendulum system. The PID controller was designed for the stabilisation and errors were optimised using GA. Mousa et al. [23] used PID controller with a feed forward gain and Reduced linear quadratic regulator (RLQR) for stabilising cart position and swinging up of pendulum angle. The tuning of controller gains were achieved using PSO. The system was subjected to disturbance with different amplitudes, the results proved the robustness of the controller.

Mishra and Chandra [24] used fractional calculus to design a robust fractional-order PID controller for stabilisation of Inverted pendulum system. PSO based tuning was used to design two PID controllers. Jia et al. [25] designed an improved ANFIS controller based on fuzzy controller for stabilisation of inverted pendulum on inclined rail. A neuro-fuzzy hybrid approach was used for designing the fuzzy rule base based on sugeno model. Almeshal [26] developed a hybrid fuzzy control strategy for two-wheeled robotic vehicle having a movable payload. The system was designed to move in different environments and terrains. The Euler-Lagrange approach was used for deriving model of the system. Furuta et al. [27] proposed a digital control of double inverted pendulum on cart, where the cart is placed on an inclined plane. The controller design was based on linear servo control theory which eliminates the effect of rail incline. Almeshal et al. [28] developed a two-wheeled robotic vehicle with five degree of freedom (DOF). The authors derived a mathematical model for incorporating a friction model and plane inclination angle to provide vehicle stability in irregular and inclined terrains. Nasrallah et al. [29] proposed a novel control scheme for an anti-tilting mobile robot moving on an inclined plane. The paper highlights an internal feedback control structure, composed of two imbricated loops and based on an intrinsic dynamical property of the system. Kausar et al. [30] applied numerical methods for estimating the stability region of two-wheeled mobile robot. The authors investigated the effects of inclined surface on the performance and stability region of the robot. A Full state feedback (FSFB) controller based on optimal gains was designed to stabilise the proposed system. This paper presents a PID based ANFIS control of inverted pendulum on an inclined plane (IPIP). The mathematical equations of motion for the system were developed using Newton's second law. The governing equations were further used to develop a Matlab-simulink model of the proposed system. The study also considers a fuzzy logic approach for the control of IPIP system. The three approaches were further compared in terms of settling time, steady state error and maximum overshoot.

II. MATHEMATICAL MODELING OF IPIP

This study considers a variant of Inverted pendulum on cart by restricting the motion of cart on an inclined plane. The system comprises of a pendulum of mass (m) and length (l) pivoted on a cart of mass (M) and makes an angle (θ) with the vertical axis. The cart is free to move on an inclined plane, inclined at an angle (γ) from the horizontal axis. The Simulink of IPIP was built from governing equations derived using Newton's law of motion. A 3D view of inverted pendulum on inclined plane is shown in Figure 1, where, x and y denotes pendulum position with reference to horizontal and vertical axis respectively. let, x_1 be the horizontal distance from reference point to the ground and x_2 be the horizontal distance from ground to pendulum axle. Therefore,

$$x = x_1 + x_2 \quad (1)$$

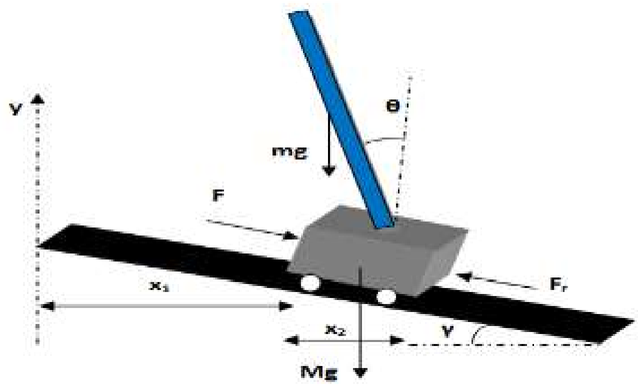

Figure 1. 3D view of IPIP

The complete system was further divided into two sub-systems i.e. cart and pendulum for studying the complete dynamics of the system. The free body diagrams (FBD) for pendulum and cart system are shown in Figure 2 and Figure 3 respectively. Let 't' be the thickness and 'l_c' be the length of the cart in meter. Applying Newton's second law following equations of motion were obtained.

a. Equation of motion for Pendulum

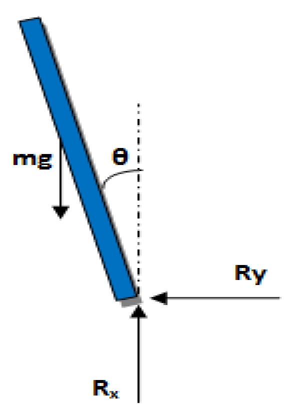

Figure 2. FBD for Pendulum system

Summation of forces along X-axis

$$-\sum F_x = m\ddot{x} \quad (2)$$

$$-R_x = m\frac{d^2}{dt^2}(x - l_c\cos\theta) \quad (3)$$

$$R_x = -m\ddot{x} + ml_c\ddot{\theta}\cos\theta - ml_c\dot{\theta}^2\sin\theta \quad (4)$$

Summation of forces along Y-axis

$$-\sum F_y = m\ddot{y} \quad (5)$$

$$-R_y - mg = m\frac{d^2}{dt^2}(y + l_c\cos\theta) \quad (6)$$

$$R_y + mg + m\ddot{y} = ml_c\ddot{\theta}\sin\theta + ml_c\dot{\theta}^2\cos\theta \quad (7)$$

Summation of torque acting on Pendulum

$$\sum\tau_p = I\ddot{\theta} \quad (8)$$

$$-R_y\, l_c\sin\theta - R_x\, l_c\cos\theta = I\ddot{\theta} \quad (9)$$

b. Equation of motion for Cart

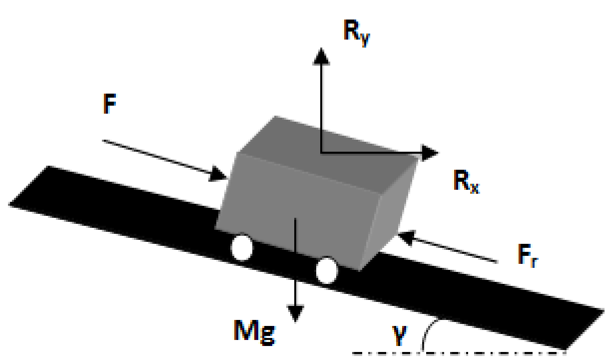

Figure 3. FBD for Cart system

Summation of forces along X-axis

$$-\sum F_x = M\ddot{x} \quad (10)$$

$$F\cos\gamma - F_r\cos\gamma + R_x\, M\frac{d^2}{dt^2}(x - \frac{t}{2}\sin\gamma) \quad (11)$$

$$F\cos\gamma - b\dot{x}\cos\gamma + R_x = M\ddot{x} \quad (12)$$

Summation of forces along Y-axis

$$-\sum F_y = M\ddot{y} \tag{13}$$

$$-F\sin\gamma + b\dot{x}\sin\gamma + R_y = M\ddot{y} \tag{14}$$

Summation of torque acting on Cart

$$\sum \tau_c = I\ddot{\theta} \tag{15}$$

$$b\dot{x} - R_y\sin\gamma + R_x\cos\gamma = 0 \tag{16}$$

Variable x and y depends mutually because the pendulum is always in contact with ground surface. Thus deleting one of the two variable in order to work with a simpler system.

$$x_2 = \frac{t}{\sin\gamma} \tag{17}$$

from equation nos. 1 we obtained,

$$x = x_1 + x_2 = x_1 + \frac{t}{\sin\gamma} \tag{18}$$

$$\tan\gamma = \frac{y}{x_1} \tag{19}$$

$$\dot{y} = \dot{x}\tan\gamma \tag{20}$$

$$\ddot{y} = \ddot{x}\tan\gamma \tag{21}$$

applying equation nos. 21 to equation nos. 7 and 14 we get,

$$R_y + mg + m\ddot{x}\tan\gamma = ml_c\ddot{\theta}\sin\theta + ml_c\dot{\theta}^2\cos\theta \tag{22}$$

$$-F\sin\gamma + b\dot{x}\sin\gamma + R_y = M\ddot{x}\tan\gamma \tag{23}$$

Substituting eq^n. 4 and eq^n.22 in eq^n. 9 and solving we obtained,

$$\ddot{\theta}(I + ml^2) - mgl_c\sin\theta = \ddot{x}ml_c(\cos\theta + \tan\gamma\sin\theta) \tag{24}$$

Substituting eq^n. 4 in eq^n. 12 we obtained,

$$F\cos\gamma = (M+m)\ddot{x} + b\dot{x}\cos\gamma + ml_c\dot{\theta}^2\sin\gamma - ml_c\ddot{\theta}\cos\theta \tag{25}$$

eq^n. 24 and eq^n. 25 were further used for building Matlab-Simulink model of IPIP subsystem as shown in Figure 4.

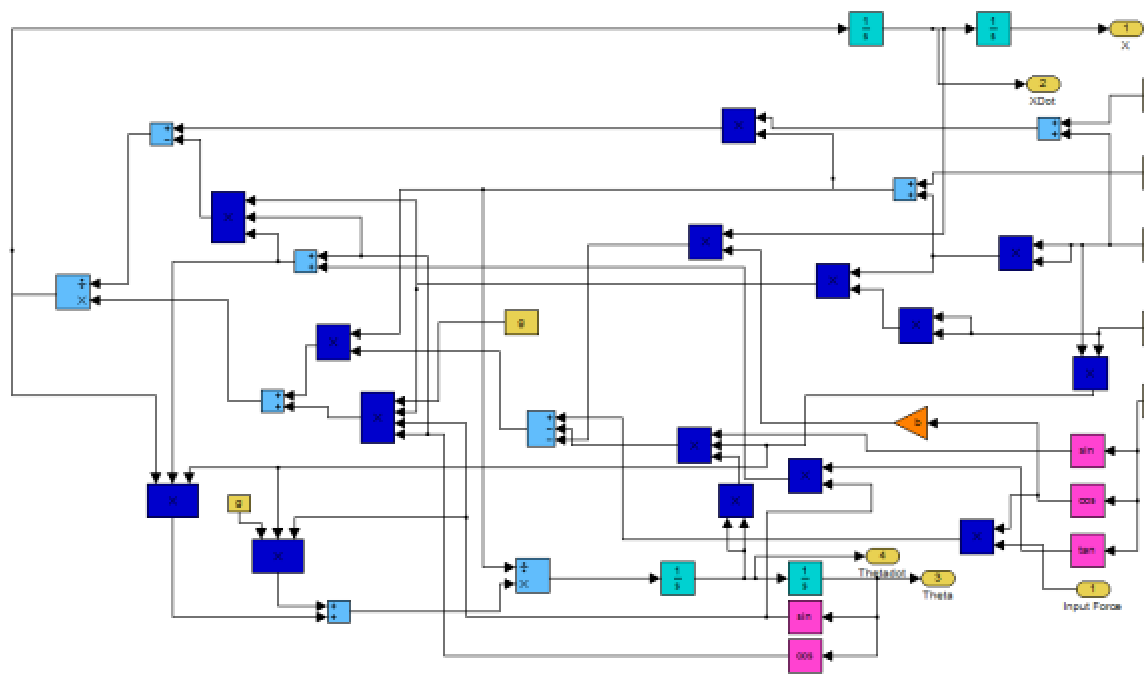

Figure 4. Matlab-Simulink model of IPIP subsystem

The values of input parameters considered for simulation are given in table.

Table 1: Values of input parameters

Parameter	Value
Mass of cart (M)	0.2 Kg
Mass of Pendulum (m)	0.1 Kg
Acceleration due to gravity (g)	$9.81 m/s^2$
Length of the pendulum (L)	0.1 m
Angle of inclination (γ)	10°
Moment of inertia (I)	$0.006 Kgm^2$

III. PID CONTROL OF IPIP

The PID is the most common form of feedback controllers used in control engineering. Most of the practical feedback loops are based on PID control. The control signal in PID is sum of P-term (which is proportional to the error), the I-term (which is proportional to the integral of the error) and the D-term (which is proportional to derivative of the error). The general representation of PID controller is given in equation 26 [14, 31].

$$u(t)= k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de}{dt} \qquad (26)$$

where u is the control signal and e is the control error (e=r-y). The controller parameters are proportional gain K_p, integral gain K_i and derivative gain K_d. In this study two different PID controllers were separately designed for controlling cart position and pendulum angle. The gains for PID controllers were obtained using trial and error method which makes the complete process very tedious. The gains considered for cart and pendulum controllers are given in Table 2.

Table 2: Values of PID gains for different controllers

Controller	K_p	K_i	K_d
Cart	1	0	-1
Pendulum	-40	-40	-20

The Matlab-Simulink model for IPIP using PID controllers is shown in Figure 5. The time considered for simulation was set to 10sec.

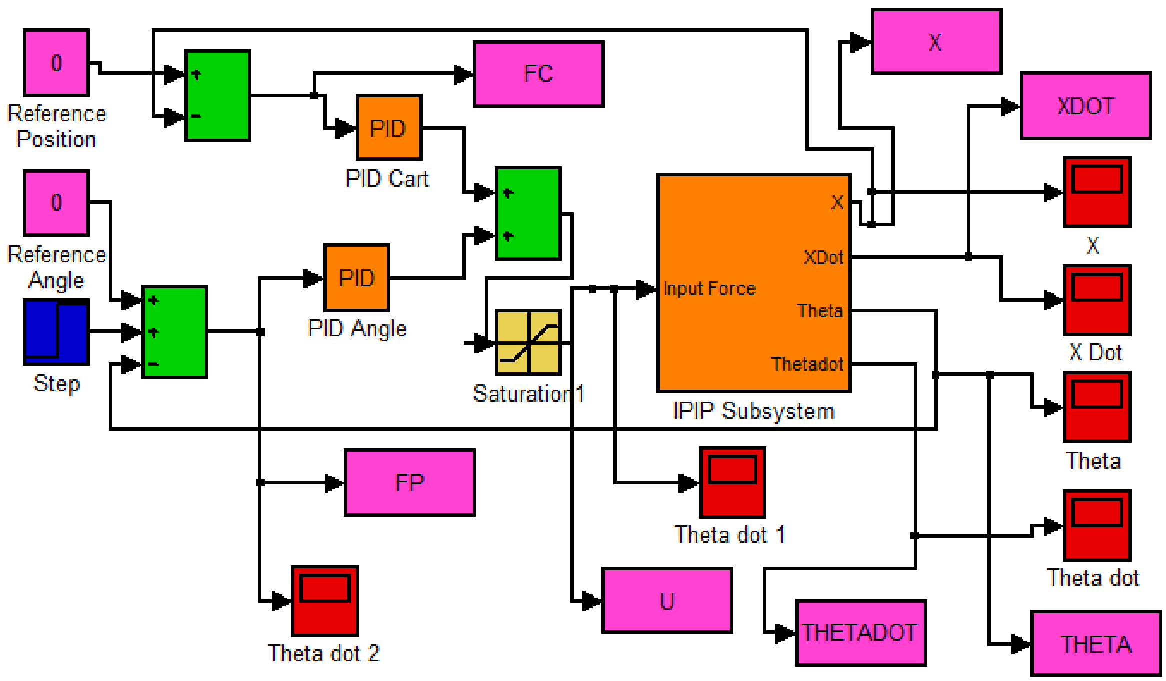

Figure 5. Simulink of IPIP with PID controllers

The Simulation results using PID controllers are shown with the help of graphs and tables below.

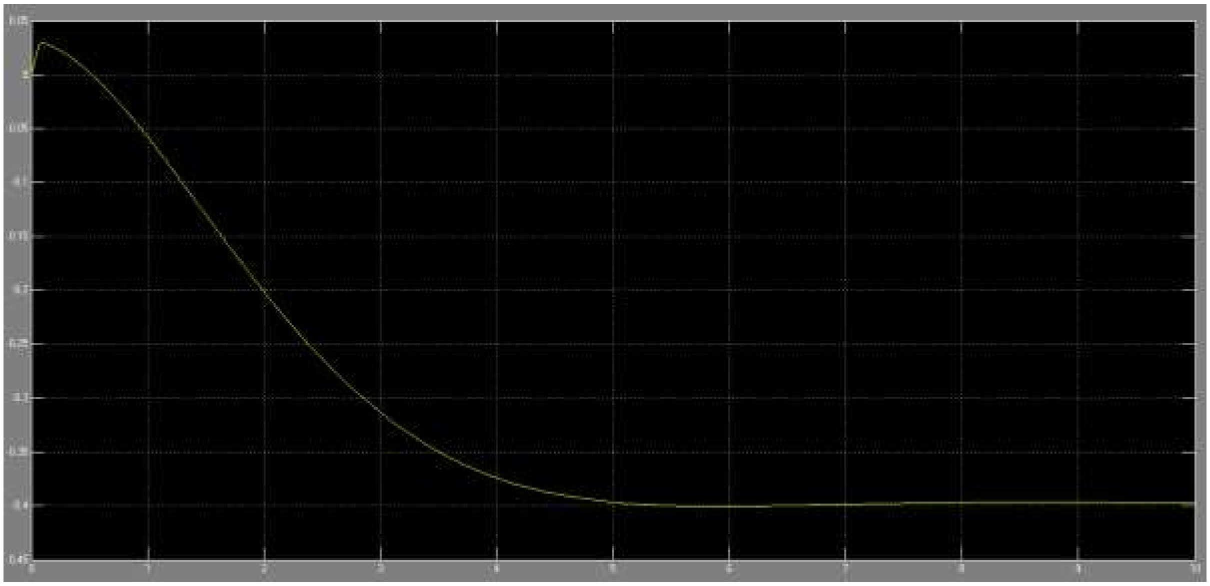

Figure 6. Simulation response for Cart velocity

Table 3: Simulation results for Cart velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
5.2 sec	0.03° to -0.4°	0

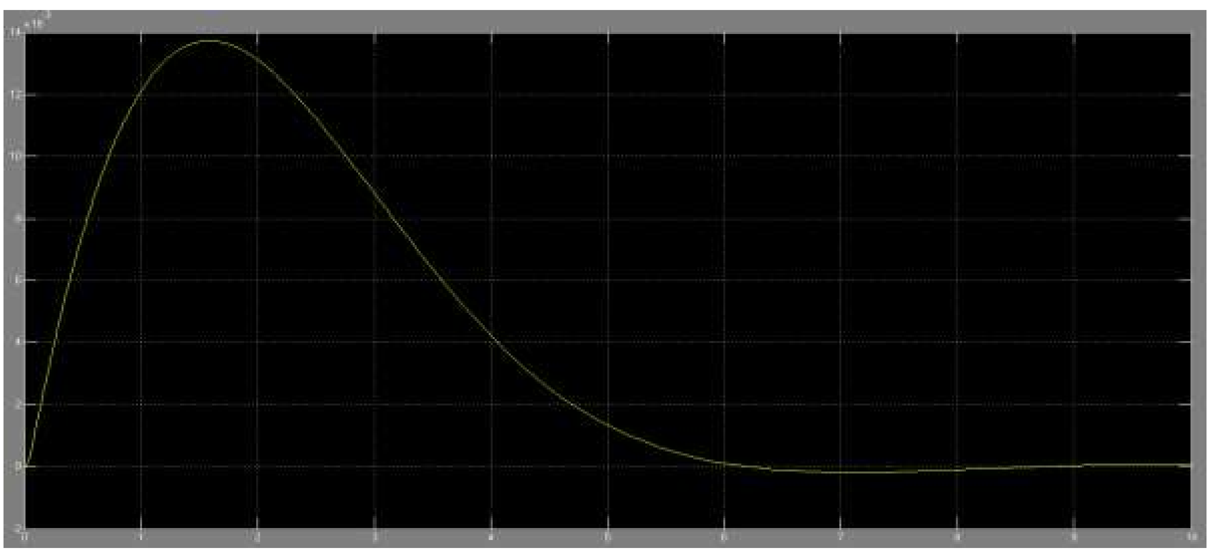

Figure 7. Simulation response for Pendulum angle

Table 4: Simulation results for Pendulum angle

Settling time (sec)	Max. Overshoot (°)	Steady state error
6.5 sec	0.014°	0

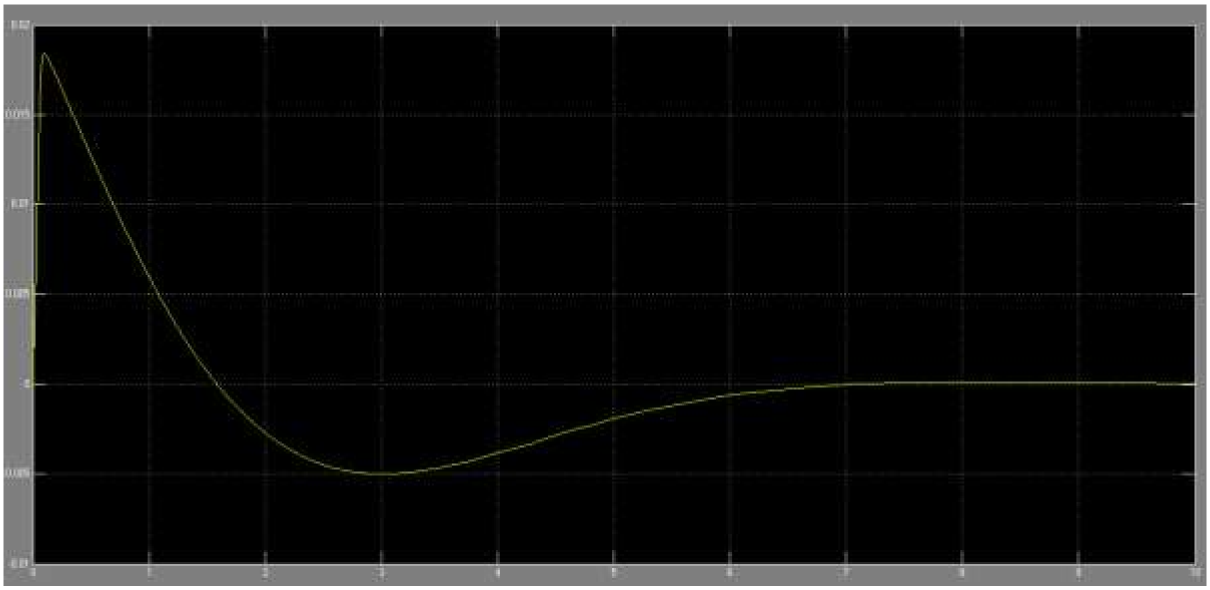

Figure 8. Simulation response for Pendulum angular velocity

Table 5: Simulation results for Pendulum angular velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
7.6 sec	0.018°	0

The simulation results showed that the PID controller smoothly controls the cart velocity with one small overshoot. The settling time and maximum overshoot obtained for cart velocity were 5.2 sec and 0.03° to -0.4° respectively. The pendulum angle stabilises after one overshoot and pendulum angular velocity after one minor overshoot and undershoot respectively. The settling time obtained for pendulum angle and angular velocity were 6.5 sec and 7.6 sec respectively. The steady state responses were negligible using PID controllers.

IV. FUZZY CONTROL OF IPIP

Fuzzy logic theory was introduced by L.A. Zadeh, in 1965 [32] and is widely used when conventional reasoning fails. It is a computational paradigm which is based on application of linguistic variables whose values are words or sentences [33]. Fuzzy logic has found a wide application in designing of certain complex industrial and management systems which cannot be modeled precisely under various assumptions and approximations [34]. This paper considers gbell membership functions (MFs) for designing of fuzzy logic controllers (FLCs) for the proposed system. A total of 7 gbell MFs were considered which gives 49 if-then fuzzy rules. A view of input MFs for cart position and surface viewer for cart controller is shown in Figure 9 and Figure 10 respectively.

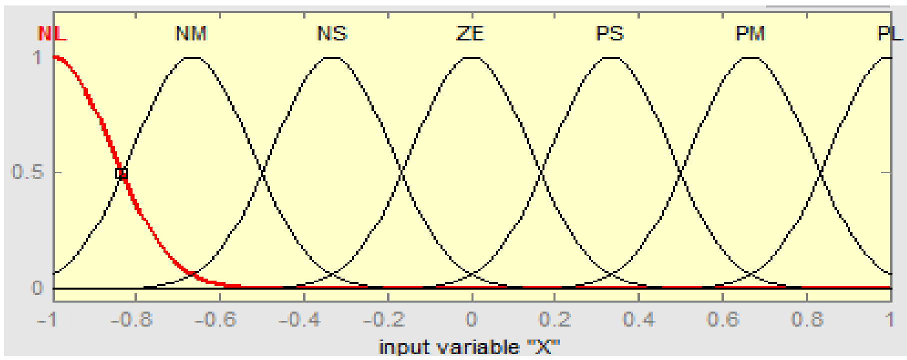

Figure 9. Input MF's for Cart controller

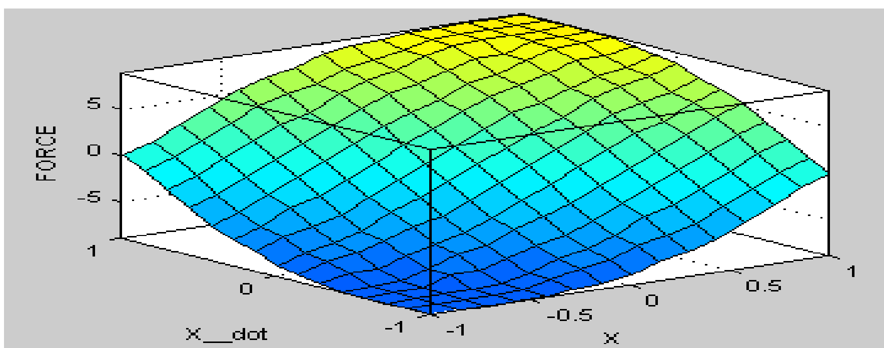

Figure 10. Surface viewer for Cart controller

The Matlab-Simulink model for IPIP using FLC is shown in Figure 11. The time considered for simulation was set to 20sec.

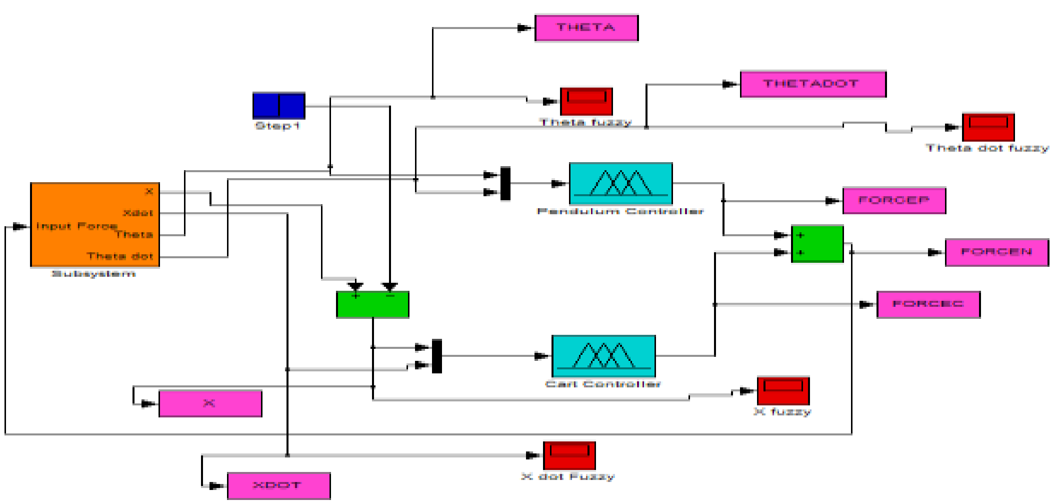

Figure 11. Simulink of IPIP with Fuzzy controllers

The Simulation results using Fuzzy controllers are shown with the help of graphs and tables below.

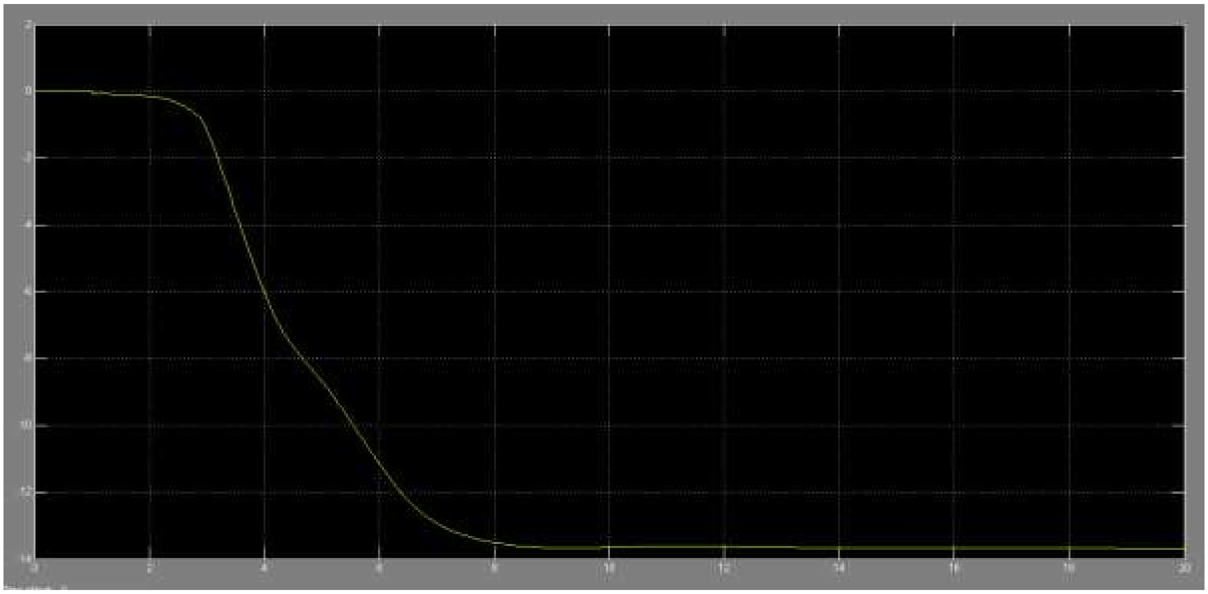

Figure 12. Simulation response for Cart position

Table 6: Simulation results for Cart position

Settling time (sec)	Max. Overshoot (°)	Steady state error
8.2 sec	-13.7°	0

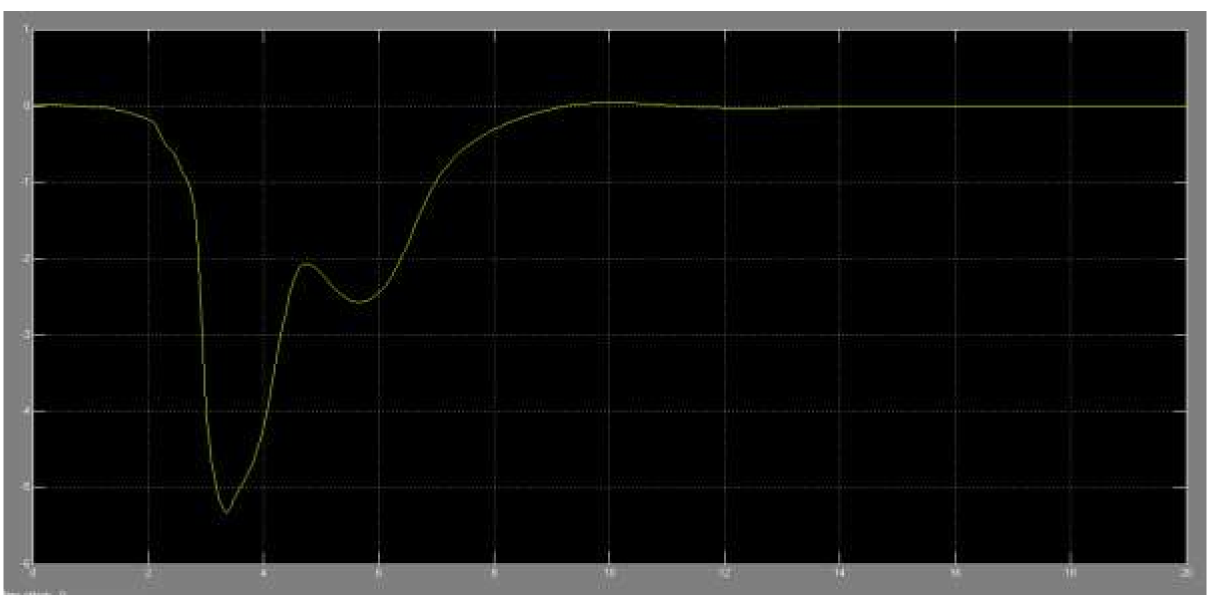

Figure 13. Simulation response for Cart velocity

Table 7: Simulation results for Cart velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
10 sec	-5.4 °	0

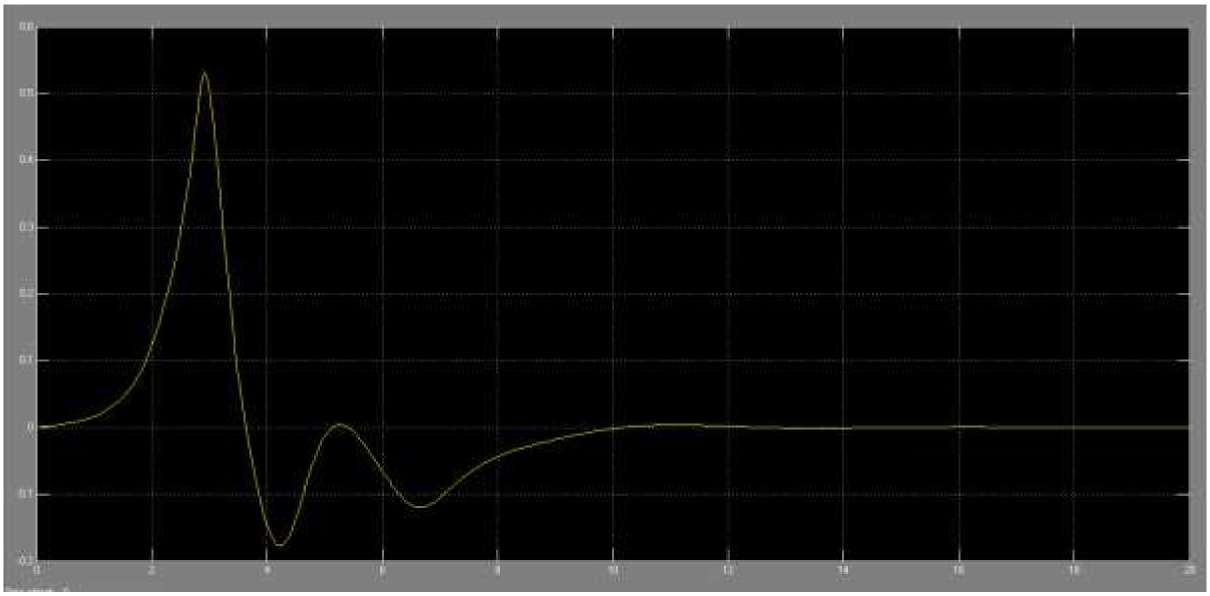

Figure 14. Simulation response for Pendulum angle

Table 8: Simulation results for Pendulum angle

Settling time (sec)	Max. Overshoot (°)	Steady state error
10 sec	0.54° to -0.18°	0

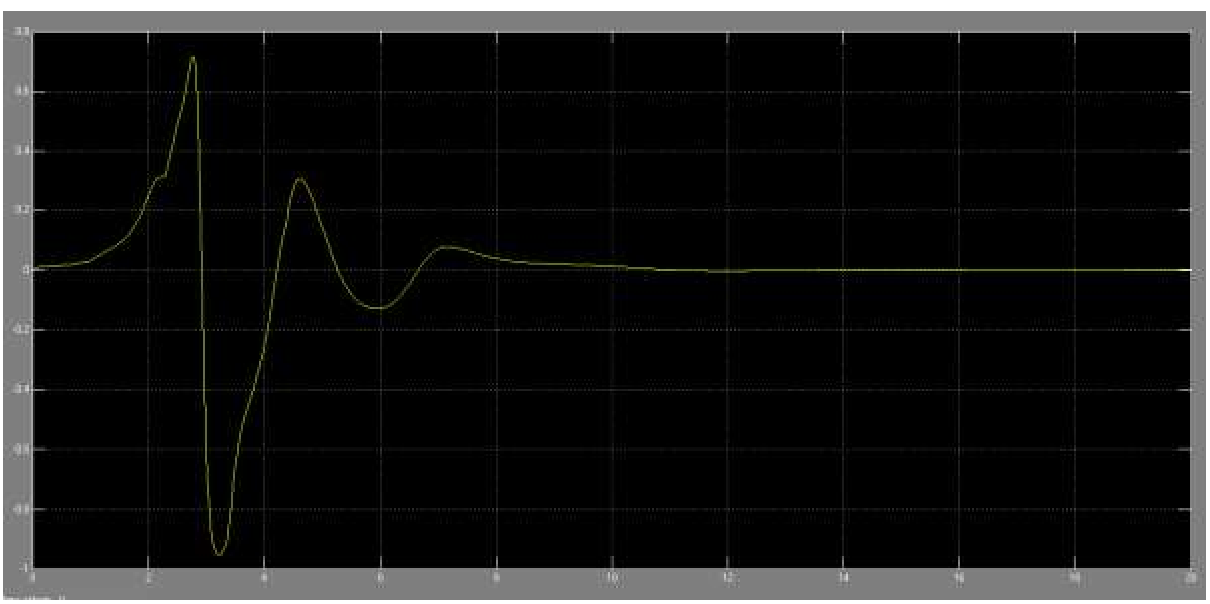

Figure 15. Simulation response for Pendulum angular velocity

Table 9: Simulation results for Pendulum angular velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
8.2 sec	0.72° to -0.95°	0

The simulation results showed that the fuzzy controller stabilises the cart position and velocity in 8.2 sec and 10 sec respectively. The cart velocity was smoothly controlled after two undershoots and one overshoot. The settling time obtained for pendulum angle and angular velocity were 10 sec and 8.2 sec respectively. Both the pendulum angle and angular velocity were smoothly controlled after two overshoots and two undershoots.

V. ANFIS CONTROL OF IPIP

ANFIS are artificial neural network based on Takagi-Sugeno fuzzy inference system [35,36] which combines the features of both FLC and Neural networks [37]. The data sets generated after simulation results of PID controllers were used to train ANFIS controllers. A total of 455 data sets were collected and 3 gbell MFs were used for training. The training was done using Hybrid learning algorithm [38] which is a combination of Back-propagation [39] and least square method [40]. The Grid partition method was further used for building an initial FIS structure. The values of error tolerance and epochs were set to 0 and 10 respectively. The error obtained after training was 0.0052838 which is satisfactory. A view of loading, training and ANFIS architecture is shown in Figure 16, Figure 17 and Figure 18 respectively.

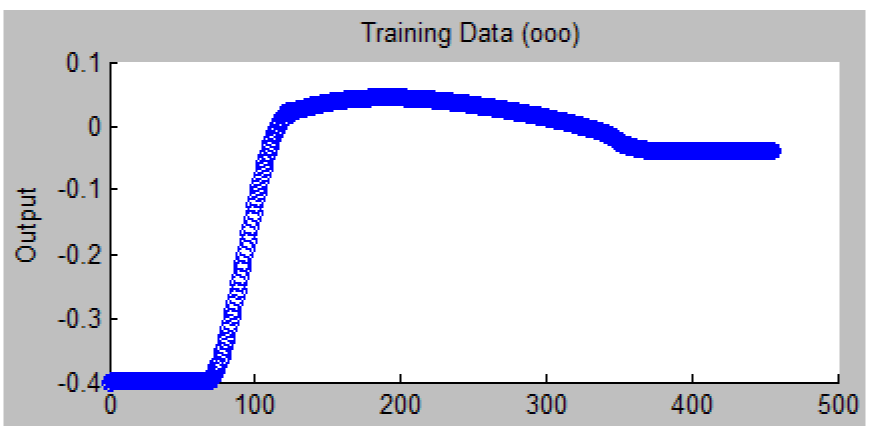

Figure 16. Loading of data in ANFIS

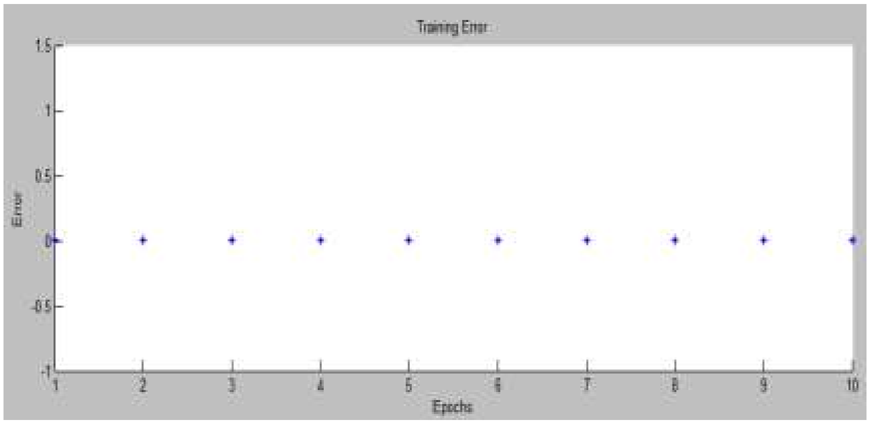

Figure 17. Training of data in ANFIS

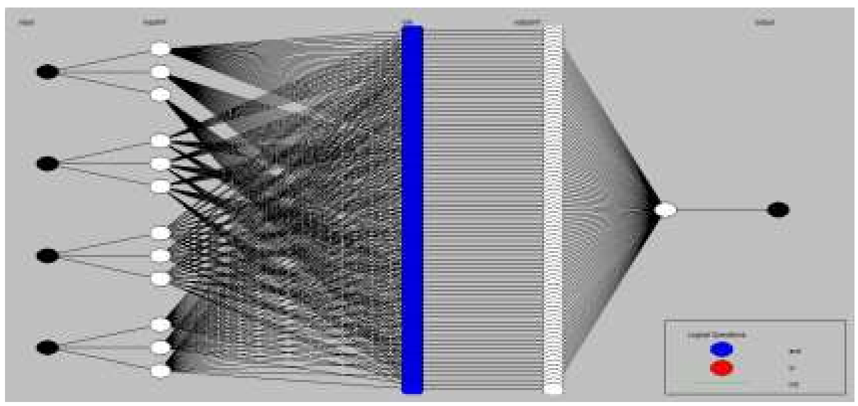

Figure 18. ANFIS architecture

The Matlab-Simulink model for IPIP using ANFIS controller is shown in Figure 19. The time considered for simulation was set to 15sec.

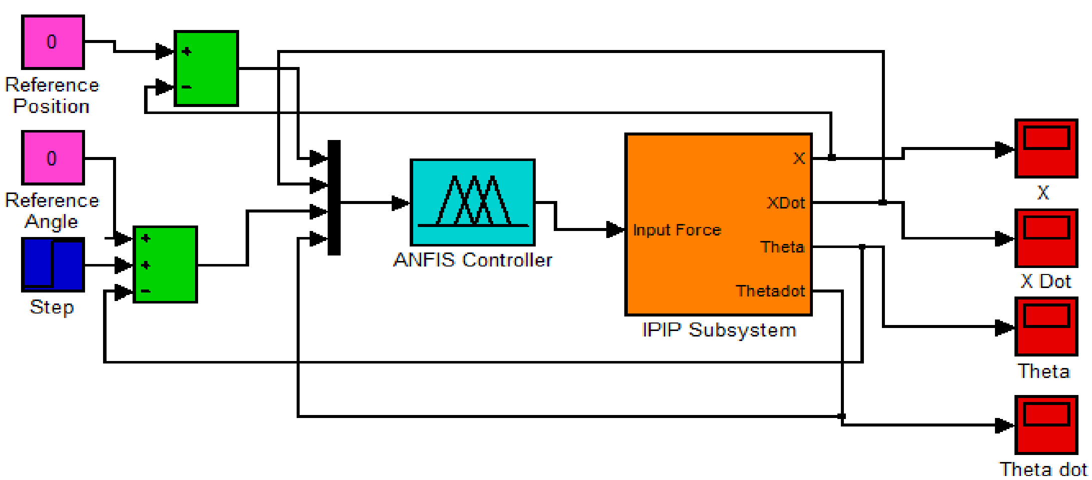

Figure 19. Simulink of IPIP with ANFIS controller

The Simulation results using ANFIS controllers are shown with the help of graphs and tables below.

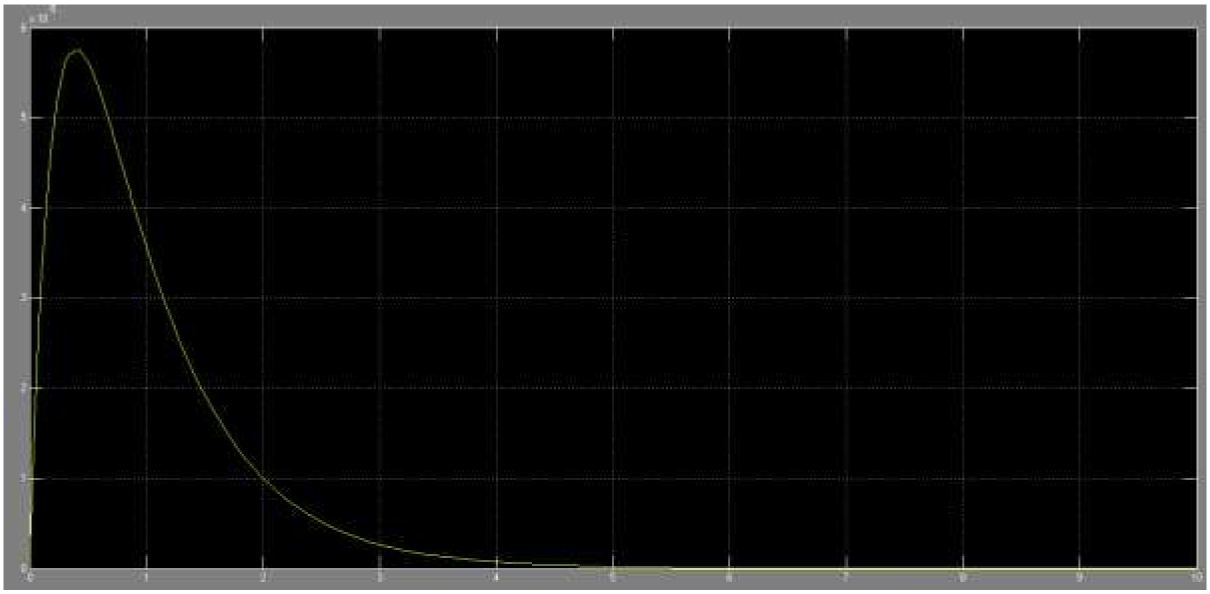

Figure 20. Simulation response for Cart velocity

Table 10: Simulation results for Cart velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
5 sec	5.7×10^{-5}°	0

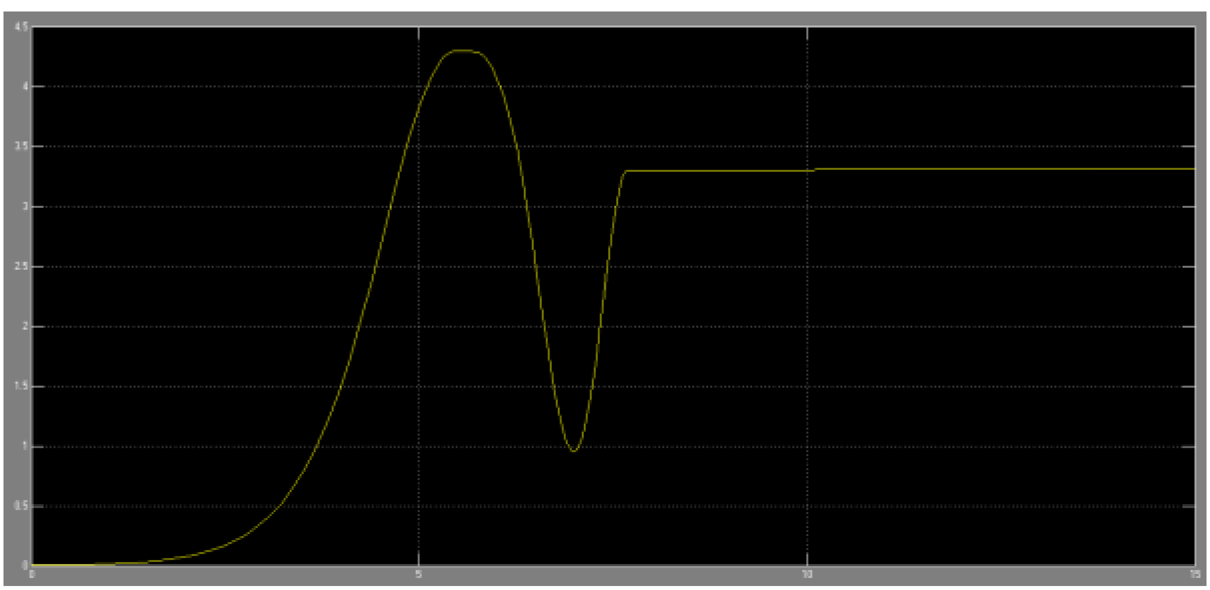

Figure 21. Simulation response for Pendulum angle

Table 11: Simulation results for Pendulum angle

Settling time (sec)	Max. Overshoot (°)	Steady state error
7 sec	4.3 °	0

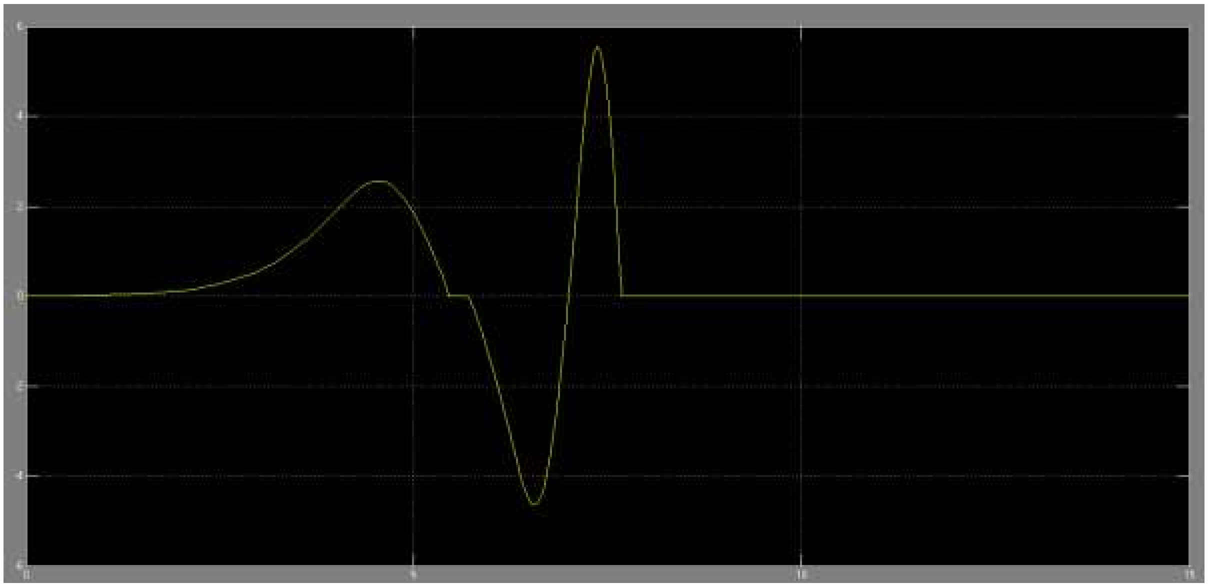

Figure 22. Simulation response for Pendulum angular velocity

Table 12: Simulation results for Pendulum angular velocity

Settling time (sec)	Max. Overshoot (°)	Steady state error
7.5 sec	5.5° to -4.8°	0

The simulation results showed that the ANFIS controller smoothly stabilises the cart velocity after one minor overshoot in 5 sec. The settling time obtained for pendulum angle and angular velocity were 7 sec and 7.7 sec respectively. It is also observed that pendulum angle stabilises after one overshoot and undershoot whereas the pendulum angular velocity is controlled after two overshoots and one undershoot. The responses for steady state error is excellent using ANFIS controller.

VI. CONCLUSION

This paper presented the control and stabilisation of inverted pendulum moving on a plane inclined at an angle of 10° from the horizontal. An offline control of the proposed system has been illustrated. The control strategies considered for stabilisation were PID, fuzzy logic and ANFIS. The three techniques were compared in terms of settling time, steady state error and maximum overshoot. Initially a PID controller was developed using trial and error method. The PID controller was further used to train an ANFIS controller. The results showed better performance of ANFIS controller over other two controllers. The error obtained after training of ANFIS controller was 0.0052838 which is satisfactory. The number of MF's considered for training were only three. The simulation results showed that the settling time and maximum overshoot obtained using ANFIS approach were much better as compared to PID and fuzzy controllers. It is also observed that all the controllers showed an excellent steady state response. The fuzzy, PID and ANFIS controllers stabilises the IPIP system within 7.6 sec, 10 sec and 7.5 sec respectively. As an extension to future work a neural network controller based on PID and fuzzy controllers is also being incorporated for control of non-linear IPIP systems.

ACKNOWLEDGEMENT

This research was supported by Department of Mechanical Engineering, Graphic Era University, Dehradun and Institute of Technology Management (ITM), Defence Research & Development Organisation (DRDO), Mussoorie.

REFERENCES

[1] Xu C and Yu X, "Mathematical modeling of elastic inverted pendulum control system," Journal of Control theory and applications, 2(3): 281-282, doi: 10.1007/s11768-004-0010-1. (2004).

[2] Tripathi SK, Panday H, Gaur P, "Robust control of Inverted Pendulum using fuzzy logic controller," In Proc. of IEEE Students Conference on Engineering and Systems (SCES), 12-14 April, 2013, Allahabad, pp 1-6, doi: 10.1109/SCES.2013.6547577.(2013).

[3] Lozano R and Fantoni I, "Passivity based control of the Inverted Pendulum," In a Book title Perspectives in Control, pp. 83-95, doi: 10.1007/978-1-4471-1276-1_7. (1998)

[4] Astrom KJ and McAvoy TJ, "Intelligent control," Journal of Process control, 2(3):115-127, doi: 10.1016/0959-1524(92)85001-D. (1992).

[5] Liu TI, Ko EJ and Lee J, "Intelligent control of Dynamic systems," Journal of the Franklin Institute, 330(3): 491-503. doi: 10.1006/0016-0032(93)90095-C. (1993).

[6] P. Vadakkepat and D. Goswami, "Biped Locomotion: Stability, Analysis and Control", International Journal on Smart sensing and intelligent systems, Vol.1, No.11, pp. 187-207, 2008.

[7] Jang JSR, "ANFIS: Adaptive-network-based fuzzy inference system," IEEE Transactions on Systems, Man and Cybernetics, 23(3): 665-685, 1993.

[8] Tao CW, Taur JS, Wang CM and Chen US, " Fuzzy hierarchical swing-up and sliding position controller for the inverted pendulum-cart system," Elsevier Journal: Fuzzy Sets and Systems, 159, 2763-2784, 2008.

[9] Becerikli Y, Konar AF and Samad T, "Intelligent optimal control with dynamic neural networks," Elsevier Journals of Neural networks, 16, 251-259, 2003.

[10] Man KF, Tang KS and Kwong S, " Genetic Algorithms: Concepts and Applications," IEEE Transactions on Industrial Electronics, 43(5): 519-534, 1996.

[11] Abido MA, "Optimal design of Power-system stabilisers using Particle swarm optimisation," IEEE Transactions on Energy Conversion, 17(3): 406-413 (2002).

[12] Fisher J and Bhattacharya R, "Linear quadratic regulation of systems with stochastic parameter uncertainties," Elsevier Journal of Applied Soft Computing, 45: 2831-2841, 2009.
[13] Johnson MA and Moradi MH, PID Controllers: New identification and Design methods, Springer, 2005.
[14] Yusuf LA and Magaji N, "GA-PID controller for position control of inverted pendulum," In Proc. of IEEE 6th International Conference on Adaptive Science & Technology, 29-31 Oct. 2014, Ota, pp. 1-5, doi: 10.1109/ICASTECH.2014.7068099.
[15] W. Benrejeb and O. Boubaker, "FPGA modelling and real time embedded control design via Labview Software: Application for Swinging up a Pendulum", International Journal on smart sensing and intelligent systems, Vol.5, No. 3, pp. 576-591, 2012.
[16] Dastranj MR, Moghaddas M, Afghu SS and Rouhani M, "PID control of inverted pendulum using particle swarm optimization (PSO) algorithm," In Proc. of 3rd IEEE International Conference on Communication software and networks (ICCSN), 27-29 May 2011, Xian, pp. 575-578, doi: 10.1109/ICCSN.2011.6013972.
[17] Omatu S, Fujinaka T and Yoshioka M, "Neuro-PID control of inverted single and double pendulums," In Proc. of IEEE International Conference on Systems, Man and Cybernetics, 8-11 Oct. 2000, Nashrille, pp. 2685-2690, doi: 10.1109/ICSMC.2000.884401.
[18] Prasad LB, Tyagi B and Gupta HO, "Optimal control of nonlinear inverted pendulum system using PID controller and LQR: Performance analysis without and with disturbance unit," International Journal of Automation and Computing, 11(6), 661-670, doi: 10.1007/s1163-014-0818-1.
[19] Dong Z, Song L and Chen H, "The modeling and simulation of first-order Inverted pendulum control system," In Book title Advances in Electronic Commerce, Web application and Communication, 2, 221-225, doi: 10.1007/978-3-642-28658-2_34.
[20] Wang X, Sun Z and Zai S, "Application of double-loop PID controller in the Inversed pendulum real-time control system," In Book title Green Communications and Networks, pp. 619-626, doi: 10.1007/978-94-007-2169-2_73.
[21] Wang JJ, "Simulation studies of inverted pendulum based on PID controllers,"Simulation Modelling Practice and Theory, 19, 440-449, doi: 10.1016/j.simpat.2010.08.003. 2010.
[22] Chakraborty K, Mukherjee RR and Mukherjee S, "Tuning of PID controller of Inverted pendulum using genetic algorithm," International Journal of Soft Computing and Engineering (IJSCE), 3(1): 21-24, 2013.
[23] Mousa ME, Ebhrahim MA and Hassan MAM, "Stabilising and Swinging-up the Inverted Pendulum using PI and PID controllers based on Reduced linear quadratic regulator tuned by PSO," International Journal of System Dynamics Applications, 4(4): 52-69, 2015.
[24] Mishra SK and Chandra D, "Stabilisation and tracking control of Inverted pendulum using fractional order PID controller," Journal of Engineering, 2014, 1-9, doi: 10.1155/2014/752918.
[25] Jia X, Dai Y and Memon ZA, "Adaptive neuro-fuzzy inference system design of inverted pendulum system on an inclined rail," In Proc. of 2nd WRI Global Congress on Intelligent systems, 16-17 Dec 2010, 137-141, doi: 10.1109/GCIS.2010.67.

[26] Almeshal AM, "Development and control of a novel-structure two-wheeled robotic vehicle maneuverable in different terrains," A PhD Thesis submitted to Dept. of Automobile Control and System Engineering, University of Sheffield, UK (2013).

[27] Furuta K, Kajiwara H, Kosuge K, "Digital control of a double inverted pendulum on an inclined rail," International Journal of Control 32(5): 907-924. doi: 10.1080/00207178008922898, 1980.

[28] Almeshal AM, Goher KM, Tokhi MO, Agouri SA, "A new configuration of a two-wheeled double inverted pendulum like robotic vehicle with movable payload on an inclined plane," In Proc. of 1st IEEE International Conference on Innovative Engineering systems (ICIES), 7-9 Dec 2012, Alexandria, 97-102. doi: 10.1109/ICIES.2012.6530852.

[29] Nasarallah DS, Angeles J, Michalska, "Velocity and orientation control of an anti-tilting mobile robot moving on an inclined plane," In Proc. of IEEE International Conference on Robotics and Automation (ICRA), May 2006, Orlando, Florida, pp 3717-3723. doi: 10.1109/ROBOT.2006.1642270.

[30] Kausar Z, Stol K, Patel N The effect of terrain inclination on performance and the stability region of two-wheeled mobile robots. International Journal of Advanced Robotics systems, Vol. 9, pp 1-11. doi: 10.5772/52894. (2012)

[31] Hanwate SD and Hote YV, "Design of PID controller for inverted pendulum using stability boundary locus," In Proc. of IEEE Indian Conference (INDICON), 11-13 Dec. 2014, Pune, pp. 1-6, doi: 10.1109/INDICON.2014.7030563.

[32] Zadeh LH (1965) Fuzzy Sets, Information and Control 8(3): 338-353.

[33] Zadeh LA (1975) The concept of a Linguistic variable and its application to approximate reasoning-I. Information Sciences 8: 199-249.

[34] Tzafestas SG, Venetsanopoulos AN, Terzakis S (1994) Fuzzy Sets and fuzzy reasoning : An Introduction. In Book title Fuzzy reasoning in Information, decision and control systems, pp 3-29, doi: 10.1007/978-0-585-34652-6_1.

[35] Dalecky S, Zboril FV (2015) An approach to ANFIS Performance. A book title Mendel 2015 and series title Advances in Intelligent Systems and Computing, Vol. 378, pp. 195-206, doi: 10.1007/978-3-319-19824-8_16.

[36] Livingstone DJ (2009) A book title Artificial Neural Networks Methods and Applications. doi: 10.1007/978-1-60327-101-1, ISBN (print): 978-1-58829-718-1, ISSN: 1064-3745.

[37] Cherkassky V (1998) Fuzzy inference systems: A critical review. A book title Computational Intelligence: Soft computing and fuzzy neuro integration with applications and series title NATO ASI series, pp. 177-197. doi: 10.1007/978-3-642-58930-0_10.

[38] Pfister M, Rojas R (1994) Hybrid learning algorithm for feedforward neural networks. A book title Fuzzy logik and series title Informatik aktuell, pp. 61-68. doi: 10.1007/978-3-642-79386-8_8.

[39] Li J, Cheng JH, Shi JY, Fei-Juang (2012) Brief Introduction of Back Propagation(BP) neural networks algorithm and its improvement. A book title Advances in Computer Science and

Information Engineering and series title Advances in Intelligent Soft computing, vol 169, pp. 553-558. doi: 10.1007/978-3-642-30223-7_87.
[40] Welberg J (2006) A book title Data analysis using the method of least squares and subtitle Extracting the most information from experiments.