

# CELESTIAL NAVIGATION FIX BASED ON PARTICLE SWARM OPTIMIZATION

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## ABSTRACT

*A technique for solving celestial fix problems is proposed in this study. This method is based on Particle Swarm Optimization from the field of swarm intelligence, utilizing its superior optimization and searching abilities to obtain the most probable astronomical vessel position. In addition to being applicable to two-body fix, multi-body fix, and high-altitude observation problems, it is also less reliant on the initial dead reckoning position. Moreover, by introducing spatial data processing and display functions in a Geographical Information System, calculation results and chart work used in Circle of Position graphical positioning can both be integrated. As a result, in addition to avoiding tedious and complicated computational and graphical procedures, this work has more flexibility and is more robust when compared to other analytical approaches.*

**Keywords:** Particle Swarm Optimization; Celestial navigation; Intercept method

## INTRODUCTION

Even today, when navigation is dominated by GPS, a traditional celestial fix still serves as a valuable backup measure. Nevertheless, as we enter the 21st century, traditional methods for computing a fix using celestial navigation can no longer meet the requirements of modern vessels in terms of calculation speed and precision. The need arises for further improvements that utilize information technology. The 2010 amendment to the STCW Code placed a continuing emphasis on celestial navigation-related education and training. It also encouraged the usage of an electronic nautical almanac and celestial navigation calculation software. In response, many researchers have resorted to computer programs to deal with celestial navigation positioning. Problems that could not be solved previously using an inspection table can now be solved with the Spherical Triangle Method or the Vector-Matrix method, which give vessel positions directly. With these efforts, great advancements have been made in celestial navigation technology.

Because the independence of celestial navigation can complement other navigation methods, research into how to apply information technology in a celestial navigation approach proves especially relevant. This study uses particle swarm optimization (PSO) from the field of swarm intelligence, which mimics natural swarm optimization behaviours, due to its superior search ability. This technique is combined with a geographical information system (GIS) and the principle of using celestial circles of equal altitude for a fix to give a fast and accurate calculation of the Most Probable Position (MPP). The proposed method and framework can potentially be integrated into an Electronic Chart Display and Information System (ECDIS).

## PRINCIPLES OF A CELESTIAL NAVIGATION FIX

The purpose of celestial navigation, as traditionally practiced, is to determine the latitude and longitude of a vessel at a specific time, through observations of the altitudes of celestial bodies, which are used to determine the observed circle of position (COP). When more than two sets of data are obtained, the vessel's position can be calculated through graphical, combined graphical and computational, or direct computational procedures using sight reduction methods such as High-Altitude Observation and the Intercept Method (IM), or computational methods. The basic principles of a celestial fix remain unchanged today, and are the basis for a number of methods. The following is a review of several celestial navigation fix methods with their respective advantages and shortcomings.

### CIRCLE OF POSITION FIX PRINCIPLE

According to the relationship between celestial and Earth coordinates, in which they are each other's projections, an observer's COP is the projection of the circle of zenith distance onto the surface of the Earth. The centre of the COP is the Geographic Position (GP) of the celestial body. The radius of the circle is the zenith distance (co-altitude) of the celestial body (Figure 1). In order to estimate the vessel's position, one must observe at least two celestial bodies from the same location, thus producing two celestial COPs and two points of intersection. The point of intersection closest to the estimated vessel position is the observed vessel position. The principle of the COP fix is quite simple and can theoretically be carried out as long as one can plot the COP directly onto a chart.

However, it is not feasible in practice for the following reasons. Firstly, the radii of most circles of equal altitude are too large to be plotted on a chart. Secondly, graphical distortion at high latitudes is apparent on the commonly used Mercator chart, and the distortion increases with the latitude of the GP. Therefore, a graphical fix by directly plotting the COP is limited only to high-altitude observations. However, a high-altitude observation is only suitable for an observed altitude of greater than  $87^\circ$  (Chen et al., 2003). It is difficult to use a sextant for high-altitude sighting, and the probability of a bright star being near the zenith at any given time and place is small. Thus, there is little desire for high-altitude observations in ordinary navigation.

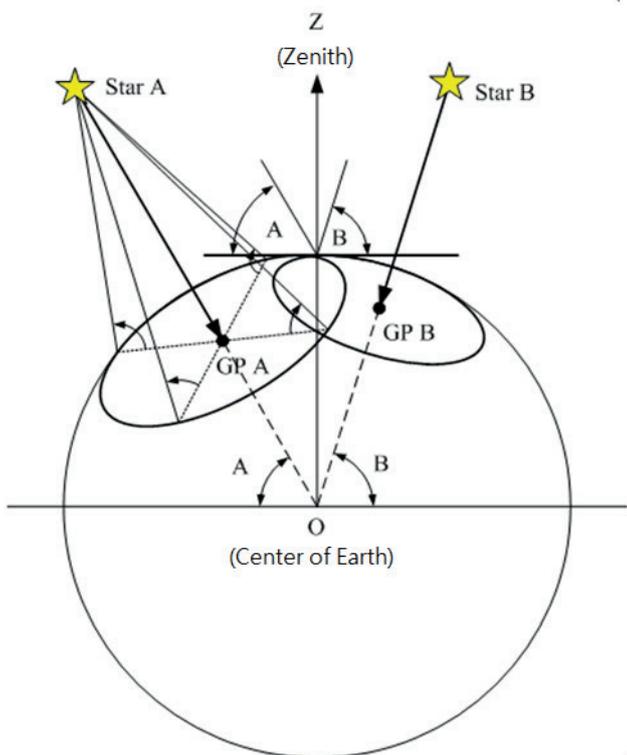


Fig. 1. The COP fix principle

### LINE OF POSITION FIX PRINCIPLE

Because the altitudes of most celestial bodies are less than  $87^\circ$ , Marcq de St Hilaire first introduced the Assumed Position (AP) to form the altitude difference, or the Intercept Method (IM), to overcome the limitation of high-altitude observations (Peacock, 2011), which has become the basis of virtually all present-day celestial navigation methods. The basic concept of the IM is to choose an AP at the nearest probable position and take it as the reference position to calculate the altitude and azimuth. By comparing the computed altitude ( $H_c$ ) and observed altitude ( $H_o$ ), the difference between the two altitudes (called the intercept or altitude difference) can be obtained. Therefore, once the AP, the computed azimuth of the body, and the intercept are all determined, the Line of Position (LOP) can be plotted. The COP can then be converted to the line of position (LOP).

However, the entire process consists of observing, calculating, and plotting. Even when performed by professional seafarers, one astronomical positioning will take around 20 minutes. Moreover, the accuracy of finding astronomical vessel positions by the IM is subject to the following two restrictions:

1. The distance between the AP and vessel's actual position largely affects the accuracy of the result. Therefore, this distance should not exceed 30 nautical miles (NM).
2. When the altitude of the observed celestial body exceeds  $70^\circ$  or the vessel is navigating in waters at high latitude, the resulting error of curvature will increase when using the LOP in lieu of the COP on the Mercator Chart.

Because of these restrictions limiting the accuracy, the IM has inherent drawbacks. Thus, a direct computation method is required, and the concept of the circle of equal altitude is reconsidered.

### DIRECT COMPUTATION METHOD FOR CELESTIAL NAVIGATION FIX

With recent advances in information technology, the problems of the celestial navigation fix can be solved using the Spherical Triangle Method, the Vector-Matrix Method, or other computational methods with the aid of computer programs. There are three classes of solutions:

1. Exact solutions to a two-body fix, such as those by Chiesa and Chiesa (1990), Spencer (1990), Gibson (1994), Chen et al. (2003), Hsu et al. (2005) and González (2008). These methods are based on full 3-D geometry, vector solution, or spherical trigonometry, and are not dependent on approximations or an assumed position.
2. Methods which are based on straight lines of position (each of which is a small arc of a circle of position) on a flat Earth near an estimated or assumed position. These methods can be applied to any number of observations  $\geq 2$ . Traditional hand-calculation and chart-based methods fall into this category, as does the least-squares method by DeWitt (1974), which was independently derived by Yallop and Hohenkerk (1990) and described in the Nautical Almanac and used in HMNAO's NavPac software.
3. Other least-squares techniques, including those of Watkins and Janiczek (1978), Severance (1989), Metcalf and Metcalf (1991), Kaplan (1995) and Wu (1991), are not based on circles or lines of position at all. Although they minimize the sum of the observational variances (the variance being the square of the altitude intercept) they do not rely on any geometric approximations. Some of them require an estimated (a priori) position, and in the case of Kaplan's algorithm, an estimated course and speed. Tsou (2012) employs genetic algorithm, similar to this study, to solve celestial navigation fix problems. This method can prevent from converging toward a local optima, but require a longer calculation time.

If direct computation is adopted, the choice of initial reference positions, such as the DR position or the EP, can be unconstrained. Furthermore, the previously described limitations are eliminated. In a computational procedure, the precision of a solution is no longer a major issue, as no graphical procedures and tables are required. In these circumstances, upon obtaining an accurate celestial Greenwich Hour Angle (GHA) and Declination (Dec), the precision of a celestial fix primarily depends on the accuracy of the observed altitude of the celestial body, provided that calculations are carried out correctly. This study is founded on these principles and improves upon the theory of the COP fix.

## CELESTIAL NAVIGATION FIX BASED ON PARTICLE SWARM OPTIMIZATION

This study utilizes the PSO technique from the field of swarm intelligence to solve celestial fix problems. Modifications are made specifically for its integration and implementation into celestial navigation.

### DIRECT FIX USING THE EQUATION OF COP.

In this study, the central idea behind the celestial circle of equal altitude fixing is to find the best fit to the altitude of a celestial body observed as a function of time. The  $H_c$  of a celestial body is given as a function of the Dec and GHA of the celestial body, and the observer's probable longitude ( $\lambda$ ), and latitude ( $L$ ) by:

$$\sin H_c = \sin L \cdot \sin Dec_i + \cos L \cdot \cos Dec_i \cdot \cos(GHA_i - \lambda) \quad (1)$$

where  $i = 1, \dots, n$  stands for the  $i$ th celestial body and its observation data.

The GHA and the Dec can be found according to time of observation in the Nautical Almanac or its electronic edition. Equation 1 is the equation of the celestial COP. Since the actual vessel position can be seen as a function of the altitude of the celestial body, which is a nonlinear function of  $L$  and  $\lambda$ , we cannot solve Equation 1 directly. Therefore, the latitude and longitude of the actual vessel position may be obtained by comparing the difference between  $H_c$  and  $H_o$  using the PSO technique and an appropriate fitting algorithm. With all the appropriate Dec and GHA values in Equation 1, any combination of altitudes of celestial bodies can be used.

### PARTICLE SWARM OPTIMIZATION

PSO is a computational method that was originally proposed by Eberhart and Kennedy (1995) for simulating behaviours in a bird flock to optimize a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. When using PSO to solve optimization problems, the solution corresponds to the position of a particle in the search space. Each particle has its own position and velocity, which decides the direction and distance of movement. In the process of iteration, every particle's movement is influenced by its local best-known position ( $p_{best}$ ) and is also guided

toward the best-known position ( $g_{best}$ ) in the search-space, which are updated as better positions are found by other particles. This is expected to move the swarm towards the best solutions. PSO does not require that the optimization problem be differentiable as is required by classic optimization methods such as gradient descent and quasi-Newton methods and can search very large spaces of candidate solutions.

Particle information can be represented by an  $n$ -dimensional vector. Its position can be expressed by  $X = (x_1, x_2, \dots, x_n)$ , and its velocity by  $V = (v_1, v_2, \dots, v_n)$ . The update equations for particle velocity and position are, respectively:

$$v_j^{t+1} = wv_j^t + c_1r_1(pb_{best_j}^t - x_j^t) + c_2r_2(g_{best_j}^t - x_j^t) \quad (2)$$

and

$$x_j^{t+1} = x_j^t + v_j^t \quad (3)$$

where  $w$  is the inertial coefficient, the main purpose of which is to generate a disturbance in order to prevent the calculation from becoming trapped in a local optimum. The maximum value of  $w$  is 0.9, and it decreases linearly with the evolutionary process to 0.4. A larger  $w$  means a stronger global optimization capacity and a weaker local optimization capacity, and vice versa. The terms  $c_1$  and  $c_2$  are acceleration coefficients (or learning factors). Usually,  $c_1 = c_2$  and both are random numbers in the interval  $[0, 2]$ . The coefficient  $c_1$  adjusts the flying velocity towards the best-known solution of the particle, while  $c_2$  adjusts the flying velocity towards the best-known solution of the entire swarm. Overly small values of  $c_1$  and  $c_2$  may cause particles to move further away from the target area and, conversely, overlarge values of  $c_1$  and  $c_2$  can result in them overshooting the target area. The values  $r_1$  and  $r_2$  are random numbers in the interval  $[0, 1]$ . The velocity  $v$  is usually limited to a certain range, i.e., it lies in the interval  $[-v_{max}, v_{max}]$ .

This study also includes the concept of a constriction factor that was later proposed by Kennedy and Eberhart (1997). It further improves on Equation (3) with:

$$x_j^{t+1} = x_j^t + nv_j^t \quad (4)$$

and

$$n = \frac{2}{|2 - f - \sqrt{f^2 - 4f}|} \quad (5)$$

where  $f = c_1 + c_2$  and  $n$  is the constriction factor. Similar to  $v_{max}$ , it is used to control and constrain the particle velocity. It also enhances the local search ability of the algorithm and improves the overall convergence.

The underlying principles and mechanisms of PSO are relatively simple, and the algorithm is easy to realize. At present, multi-body fixes in celestial navigation positioning or computer programs for the IM generally use the least squares mean method. However, a least squares mean method that proceeds from some initial value by decreasing the gradient of the goodness-of-fit parameter can converge to a local

minimum that is not the best solution when high-altitude observations are used (Metcalf and Metcalf, 1991). It was found in this study that the PSO method cannot only avoid this problem, but also possesses some advantages over other computational methods. Ten particles were used in this study. Each particle X represents a possible vessel position. It has two dimensions: longitude and latitude. The MPP is searched for using the optimization mechanism of the PSO method. More observation data will produce better estimation results.

## DESIGN OF THE OBJECTIVE FUNCTION

In the past, the most optimal vessel position in the majority of multi-body celestial fixing problems was solved for by data fitting. In this study, the variance of the altitude residuals is minimized through PSO. The altitude residual is defined as the difference between  $H_o$  and the  $H_c$  computed from Equation 1:

$$F(L, \lambda) = \min \frac{\sum_{i=1}^n \sqrt{[(\text{Sin}L \cdot \text{Sin}Dec_i + \text{Cos}L \cdot \text{Cos}Dec_i \cdot \text{Cos}(GHA_i - \lambda)) - H_{o_i}]^2}}{n} \quad (6)$$

The aim of Equation 6 is to find the most appropriate location (of longitude and latitude) in the solution space from the observational data of n celestial bodies so that F is minimized. When the result converges to meet the acceptance criteria, the MPP is found.

### CONSTRAINT CONDITIONS

Although PSO is not sensitive to initial conditions, the position of each particle X can be determined according to whether a reference position exists. If there is one, the particle position is chosen randomly based on this reference position and limited to the search range. Otherwise, the particle position can be any random location on the Earth. The reference position here is essentially different from the AP or the DR position in the IM. The reference position in this study is used to limit the search range and provide the effects of a heuristic search. Therefore, although the reference position can be set to be the same as the DR position, there is no limitation imposed on the DR position in the IM that the distance to the actual vessel position cannot exceed 30 NM. It is only an approximate reference position, and it can be hundreds or even thousands of NMs away.

However, a reference position closer to the DR position will result in a faster convergence. Therefore, if a DR position is available, it should be used as the reference position to accelerate the search. A reference position should be set in two-body celestial fix problems in order to determine which one of the two points of intersection from the COPs of the two celestial bodies is the actual vessel position. Multi-body celestial fixing does not require an initial guess for the position in this study, which means that a reference position does not need to be set. However, the number of iterations can be reduced by setting one, thus speeding up the search. The velocity of each particle must be within the search range.

### CORRECTION OF THE OBSERVED ALTITUDE IN A RUNNING FIX.

When performing the multi-body celestial fix at sea, there is some time difference between observations. It is therefore necessary to correct the zenith of the COP in the last observation to the same zenith position of the

following observation. When the distance travelled between two consecutive observations does not exceed 30 NM, the following formula is used to calculate the correction of the altitude ( $\Delta h$ ) in the running fix:

$$\Delta h = \frac{V}{60} \text{Cos} \Delta A \cdot \Delta T^m \quad (7)$$

in which V is the speed (in knots),  $\Delta A$  is the angle between the observed azimuthal direction of the celestial body and the true course of the vessel, and  $\Delta T^m$  is the time interval (in minutes) between the two observations.

### PLOTTING THE COP

Due to the large radius of the COP and graphic distortion at high latitudes when conducting graphical positioning on the Mercator chart, the application of direct graphical fix method was restricted to high-altitude observations. If the functionality of the GIS can be further modified, then the previous limitations on positioning with manual chart work will not apply on an electronic chart under a GIS environment.

This study proposes to include a function that constructs a COP, based on vector analysis (González, 2011), in a celestial navigation fix module in GIS. As long as there is a GP of the celestial body and a  $H_o$ , the entire COP can be directly constructed in a GIS environment in any projection. By utilizing the spatial analysis function in a GIS to obtain intersection points of the COP, the vessel's position can be measured, which can then facilitate positioning and serve as an aid to set the initial position, to assess the likely accuracy of the fix.

### RESULTS VALIDATION

The validity of this study was verified by using observation data from both two celestial bodies and multiple celestial bodies, and also by using data at high altitude. Using several important methods of astronomical navigation fixing published in the literature in recent years, three cases were tested and compared. The adopted comparison methods are: the traditional IM; the computerized IM by DeWitt (1974) that was published in the Nautical Almanac, U.S. Naval Observatory (Kaplan, 1995); the vector-matrix method by

Metcalf and Metcalf (1991); the Simultaneous Equal-Altitude Equation Method (SEEM) by Hsu et al., (2005); and the vector analysis method by González (2008). The methods proposed by Hsu et al., and González are only applicable in a two-body celestial fix. The other methods do not have this restriction. Visual Basic.Net 2010 was used as the development tool. In order to plot the COP and to add visual effects, the COM component of the GIS was also used. This was also used for education on celestial navigation and integration with an ECDIS in the future.

### TWO-BODY FIX

The data for this case study was taken from Hsu et al., (2005). It only contains observational data of two celestial bodies. All methods were tested using two different DR positions. Table 1 contains the relevant observation data and the results are presented in Table 2.

Tab. 1. Extract of relevant information from Hsu et al. (2005) for two-body observation

Body	ZT	Ho	GHA	Dec
Capella	20-03-58	15° 19.3'	131° 24.8'	45° 58.4' N
Alkaid	20-02-56	77° 34.9'	003° 14.2'	49° 25.7' N

Tab. 2. Two-body fix positions

Method	DR 1: $L = 41^\circ \text{N}$ , $\lambda = 017^\circ \text{W}$		DR 2: $L = 44^\circ \text{N}$ , $\lambda = 019^\circ \text{W}$	
	$L_1$	$\lambda_1$	$L_2$	$\lambda_2$
IM	41° 38.6' N	017° 08.1' W	41° 26.5' N	017° 28.1' W
DeWitt (5 iterations)	41° 39.1' N	017° 07.3' W	41° 39.1' N	017° 07.3' W
Metcalf (virtual star)	41° 39.1' N	017° 07.3' W	41° 39.3' N	017° 07.5' W
Hsu	$L = 41^\circ 39.1' \text{N}$ , $\lambda = 017^\circ 07.3' \text{W}$ (DR for reference)			
González	$L = 41^\circ 39.1' \text{N}$ , $\lambda = 017^\circ 07.3' \text{W}$ (DR for reference)			
PSO	$L = 41^\circ 39.1' \text{N}$ , $\lambda = 017^\circ 07.3' \text{W}$ (DR for reference)			

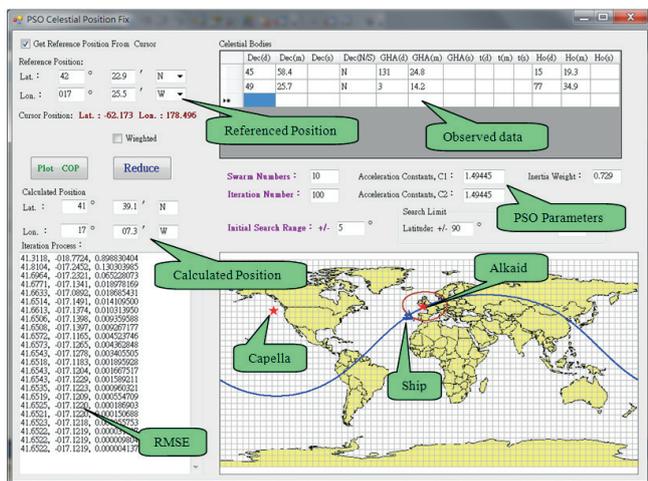


Fig. 2. Results of the two-body fix by PSO

The experimental results of the PSO are shown in Figure 2. Before execution, two COPs may first be plotted and displayed through a GIS module. The vessel position can be measured by mouse control and is used as a reference position. It can be seen from Table 1 that the observed altitude of Alkaid is as high as 77° 34.9', which exceeds the upper limit of 70° specified in the IM. Hsu et al., (2005) found that for high-altitude observation, an LOP drawn using the IM was

shifted due to curvature error, thus resulting in an inaccurate vessel position. It can be seen in Table 2 that all methods but the traditional IM can find the correct vessel position when the first DR position (DR 1) is used; i.e., the distance between the DR position and the actual vessel position is less than 30 NM. This implies that the traditional IM indeed has a limitation on the observed altitude.

It is worth noting that the method proposed by Metcalf was designed for a multi-body celestial fix. When used in a two-body fix, a virtual star must be assumed, and the latitude of the DR position is set to be its Dec. The longitude of the DR position needs to be converted to the GHA of the star. The computation can be performed after setting the observed altitude of the star as 90°. Therefore, the accuracy of positioning when using Metcalf's method in a two-body fix is affected by the selected DR position.

In the case of the second DR position (DR 2), a greater error occurs in the IM due to the larger distance between the DR position and the actual vessel position. DeWitt's method can reach the correct vessel position after 5 iterations. Metcalf's method, meanwhile, has produced a noticeable error. Although other methods, including PSO, can obtain the correct vessel position, some degree of prior DR knowledge is needed to help identify the answer. However, relatively speaking, the requirement on the accuracy of the initial position is lowered.

Some unlikely extreme experiments are designed in this study. One experiment is setting the DR position north of  $L = 60^\circ \text{N}$ ,  $\lambda = 17^\circ \text{W}$ , i.e., with high latitude and far away. Another is to have two COPs intercept at only one tangent point. In these two cases, the IM, DeWitt, and Metcalf methods fail while the other methods can still give the correct vessel position.

### MULTI-BODY FIX.

There are four celestial bodies involved in this case study. Table 3 presents the relevant observation data. In this multi-body fix problem, a correction on the running fix is also applied. González's and Hsu's methods are not applicable in this situation. The experimental results are tabulated in Table 4 and Figure 3. The vessel position obtained using the IM is slightly different due to errors in the graphical procedures. All other methods reach the same vessel position. Metcalf's method and the PSO technique do not require a DR position to find the correct vessel position.

Moreover, in a multi-body celestial fix, to prevent some abnormal observation data from affecting the overall accuracy, a correction on the weights is applied. Those closer to the vessel position are given a larger weight. This point is explained by comparing Figures 4 and 5. The vessel position in Figure 4 is obtained without applying weight correction, and it is at the centre of the big cocked hat. Figure 5 shows the vessel position with corrected weights, in which case it falls into the smaller cocked hat.

Results obtained using PSO show a good agreement with those obtained using other methods. The applicability of PSO on both the two-body fix and multi-body fix is thus verified.

Tab. 3. Multi-body fix data

Course: 220° Speed: 18 kts				
Body	ZT (1993/9/13)	Ho	GHA	Dec
Altair	18-00-00	37° 53.0'	325° 06.6'	08° 51.4' N
Fomalhaut	18-04-00	27° 54.0'	279° 24.2'	29° 39.1' S
Achernar	18-08-00	17° 46.5'	240° 21.7'	57° 15.8' S
Rasalhague	18-12-00	41° 35.5'	002° 04.8'	12° 34.1' N

Tab. 4. Multi-body fix positions

18-12-00 ZT DR : L = 35° S, λ = 005° E			
Method	L	λ	
IM	35° 19.0' S	005° 26.5' E	
DeWitt (5 iterations)	35° 18.6' S	005° 27.0' E	
Metcalf	L = 35° 18.6' S, λ = 005° 27.0' E (DR not required)		
PSO	L = 35° 18.6' S, λ = 005° 27.0' E (DR not required)		

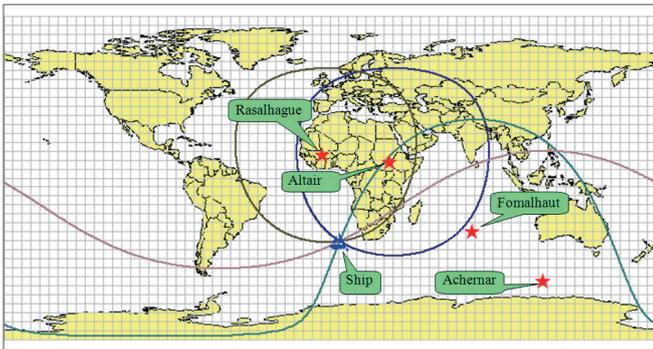


Fig. 3. Results of the multi

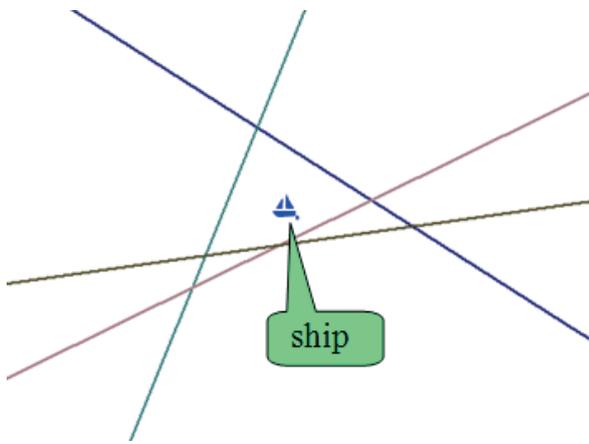


Fig. 4. Vessel position with uncorrected weights

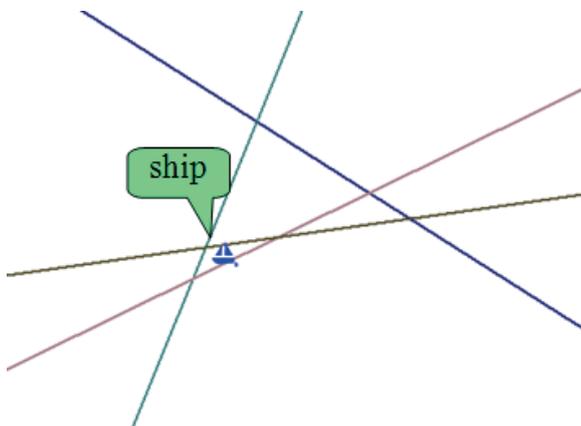


Fig. 5. Vessel position with corrected weights

## HIGH-ALTITUDE OBSERVATION FIX

In this example, a high-altitude observation of the Sun was performed three times before and after its transit. When the Sun is in a low latitude region within a few minutes of the transit, its altitude can reach above 88°, and its azimuth changes remarkably fast. In this situation, the curvature of the LOP makes the IM inappropriate to use. A high-altitude graphical method or a computational method must be used. Therefore, if 2–3 observed altitudes of the Sun before and after the transit time are available, then the vessel position can be obtained from a high-altitude observation of the Sun. Table 5 contains observation data of the Sun's altitude, and experimental results are shown in Table 6.

Tab. 5. High-altitude observations

Course: 290° Speed: 15 kts				
Body	ZT (1996/9/8)	Ho	GHA	Dec
Sun	11-56-13	89° 19.4'	269° 38.9'	05° 34.6' N
Sun	11-58-19	89° 36.2'	270° 10.5'	05° 34.5' N
Sun	12-00-41	89° 19.2'	270° 46.0'	05° 34.5' N

Tab. 6. High-altitude fix positions

12-00-41 ZT DR 1: L = 05° 45' N, λ = 090° E DR 2: L = 05° 33' N, λ = 090° E				
Method	L <sub>1</sub>	λ <sub>1</sub>	L <sub>2</sub>	λ <sub>2</sub>
DeWitt (Iterations)	05° 58.1' N	089° 47.6' E	05° 11.0' N	089° 47.7' W
COP Graphical Method	L = 05° 59.0' N, λ = 089° 47.8' E			
Metcalf	L = 05° 58.1' N, λ = 089° 47.6' E (DR not required)			
PSO	L = 05° 58.1' N, λ = 089° 47.6' E (DR not required)			

The experimental results of using PSO are shown in Figure 6. The COPs from three observations of the Sun give two points of intersection, A and B. The two points are very close, and therefore an inappropriate DR position will lead the algorithm to converge to a non-optimal position. It can be seen from Table 6 that the PSO and Metcalf methods do not require a DR position as reference. As a result, they can find the correct vessel position, or point A.

On the other hand, DeWitt's method is sensitive to the initial DR position. When the DR position is DR 1, the correct vessel position can be obtained. However, when the DR position is DR 2, DeWitt's method finds point B as the vessel position, which is incorrect. Since B and DR 2 are only about 20 NM apart, the algorithm is mistaken in finding the correct vessel position when in fact it has fallen into a local optimum.

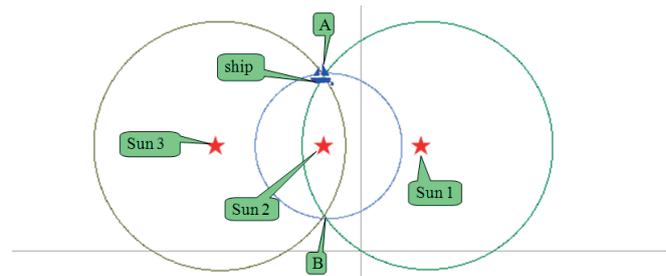


Fig. 6. Results of a high-altitude observation using PSO

## COMPARISON AND DISCUSSION

By comparing the three test cases, it can be determined that, although González's and Hsu's methods are accurate and do not need an AP to solve the problem, they are limited to only a two-body celestial fix. DeWitt's method, Metcalf's method, and the PSO method proposed in this study can perform the fix with any number of celestial bodies and therefore have a wider applicability. Metcalf's method is very convenient, fast, and accurate at solving a multi-body fix and high-altitude observations. However, it becomes inconvenient for solving a two-body fix, in which case a virtual star must be assumed based on the DR position. The accuracy of the positioning is affected by the DR position. It also violates Metcalf's claim that positioning can be performed without needing a DR position as a reference. DeWitt's method has wide applicability and can produce an accurate fix. However, it is still an IM in nature, which means that it has some inherent limitations. The accuracy must be improved by increasing the number of iterations. It is also possible for the algorithm to converge to a non-optimal solution.

The PSO method proposed in this study is applicable in all cases. It is not only accurate and robust, but also relies less on the DR reference position. However, the PSO consumes more computational time than the other methods. Though, a correct vessel position can be obtained within 10 iterations (less than 2 seconds) that can still meet the real time requirement for modern marine navigation.

## CONCLUSIONS

In this work, we utilized Particle Swarm Optimization from the field of swarm intelligence, due to its superior optimization and searching ability, to compute the astronomical vessel position. This technique, used in combination with the GIS and the principle of using circles of position in a celestial navigation fix, allows for a fast, direct, and accurate calculation of a vessel's optimal position. Test results showed that the proposed method not only can be applied in two-body fix, multi-body fix, and high-altitude observation problems, but it is also less reliant on the initial dead reckoning position. In combination with an electronic nautical almanac module, it can be seen as a prototype for integration with an ECDIS. It can serve as an ancillary positioning option and as a useful tool in celestial navigation-related education.

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