Robust controller design
for a heat exchanger using $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$,
and μ-synthesis approaches

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Abstract: Possibilities of using robust controllers for a shell-and-tube heat exchanger control were studied, tested and compared by simulations and obtained results are presented in this paper. The heat exchanger was used to pre-heat petroleum by hot water; the controlled output was the measured output temperature of the heated fluid — petroleum, and the control input was the volumetric flow rate of the heating fluid — water. Robust controllers were designed using $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ strategies and μ-synthesis. A comparison with the classical PID control demonstrated the superiority of the proposed robust control especially in case when the controlled process is affected by disturbances.

Keywords: heat exchanger, PID control, $\mathcal{H}_2$ control, $\mathcal{H}_\infty$ control, μ-synthesis

Introduction

Heat exchangers are nonlinear processes (Janna, 2009) and their control is a complicated problem due to their non-linear behaviour and complexity caused by many phenomena such as leakage, friction, temperature-dependent flow properties, contact resistance, unknown fluid properties, etc. (Serth and Lestina, 2014). Many factors enter into the design of heat exchangers, including thermal analysis, weight, size, structural strength, pressure drop and cost. The heat exchanger modelling framework has been described and demonstrated previously (Skaugen et al., 2013). The main purpose of Daróczy et al. (2014) was to illustrate and analyse a heat exchanger arrangement problem in its most general form. In Manassaldi et al. (2014), a mathematical model for optimal design of an air cooled heat exchanger is described. Owing to the wide utilisation of heat exchangers in industrial processes, their cost minimisation is an important task for both, designers and users (Pan et al., 2011). Cost evaluation is obviously an optimisation process dependent upon other design parameters. A method capable of utilising the maximum allowable stream pressure drops has been described by Panjeshahi et al. (2010). This approach can result in minimum surface area requirements. Economics plays a key role in the design and selection of heat exchanger equipment. The weight and size of heat exchangers are significant parameters in the overall application and thus may be considered as economic variables (Holman, 2009). Effective control of heat exchangers requires the application of some advanced methods, e.g. robust control (Gerhard et al., 2004;

Tlacuahuac et al., 2005). Robust control has emerged as one of the most important areas in the modern control design since Doyle (1981), Zames (1983), and many others. In Veselý (2013), a survey of robust control design procedures is given. $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control theories have been the area of active research for years and they have been successfully implemented in many engineering applications. While the $\mathcal{H}_2$ norm represents the mean energy of the system, the $\mathcal{H}_\infty$ norm represents the maximum energy. If there are uncertainties in the system model, some quantity combining the $\mathcal{H}_2$ norm and the $\mathcal{H}_\infty$ norm can be a desirable measure of a system's robust performance (Bansal and Sharma, 2013). Thus, the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance criterion provides an interesting measure for the controller evaluation. Theoretic motivation for mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control has been discussed in literature (Doyle, 1984; Kwakernaak, 2002). The goal of Zarabadipour et al. (2011) was to design a reduced order robust controller based on the balanced realisation technique. The simulation showed that the reduced order controller design based on the $\mathcal{H}_2/\mathcal{H}_\infty$ approach has good results in the frequency and time domains. In Ganji et al. (2013), different conventional and intelligent controllers were implemented with a clear objective to control the outlet fluid temperature of a shell-and-tube heat exchanger system. For the dynamic system with time varying characteristics and parametric uncertainties, a sliding mode controller was developed and an optimal $\mathcal{H}_\infty$ controller was designed based on μ-synthesis with the *DK*-iteration algorithm by Moradi et al. (2012). In De Souza et al. (2014), the problems of robust stability analysis and robust control of linear

discrete-time periodic systems was investigated. In Ahmad et al. (2013), the design of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control for a twin rotor system is described. Dlapa (2015) presented an application of algebraic μ-synthesis to the air-heating system, where the temperature of bulb was controlled by its voltage. Zhu and Yang (2016) solved a simultaneous $\mathcal{H}_2/\mathcal{H}_\infty$ stabilisation problem for a chemical reaction system that can be modelled as a finite collection of subsystems. Mahitthimahawong et al. (2016) developed a control system for a split range control in a heat exchanger network. Vasičkaninová and Bakošová (2013) dealt with the design and application of robust $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers for a heat exchanger.

In this study, robust controllers are designed using $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis approaches. The $\mathcal{H}_2$ control technique is based on the minimisation of the quadratic norm of the transfer function between the input disturbance signal and the plant's output signal. The disadvantage is that the $\mathcal{H}_2$ control does not show guaranteed robustness a priori, when unstructured uncertainties are present. The $\mathcal{H}_\infty$ control has the basic objective to minimise the effect of disturbances on the plant output, but, on the other hand, it presents limitations of the system's performance. The $\mathcal{H}_2/\mathcal{H}_\infty$ mixed control joins the ability of the $\mathcal{H}_\infty$ control to minimise the effect of the input disturbances on the plant output employing the $\mathcal{H}_2$ control. In the μ-synthesis design, a perturbation matrix Δ is chosen and through an optimisation procedure, a stabilising controller for the worst perturbation is obtained. As all these approaches have been implemented in the Robust Control Toolbox of Matlab, the robust controllers have been designed using this toolbox. The controllers were tested in set point tracking and disturbance rejection of the shell-and-tube heat exchanger and compared with conventional PID controllers. The presented simulation results confirm that robust controllers are able to provide better results especially in case of disturbance rejection.

Robust control

The basic feature of robust control is that it guarantees closed-loop stability and performance even though the controlled process has uncertain parameters or is affected by disturbances or measurement noise.

$\mathcal{H}_2$, $\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_\infty$ control

Consider a plant, $P(s)$, that has to be controlled (Bosgra et al., 2007). The goal of the $\mathcal{H}_2$, $\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_\infty$ control strategies is to design an internally stabilising linear time-invariant (LTI) dynamic feedback controller, $K(s)$, that minimises the $\mathcal{H}_2$ and the $\mathcal{H}_\infty$ norm

of the performance transfer function, T_{zw}, of the closed-loop system (see Fig. 1).
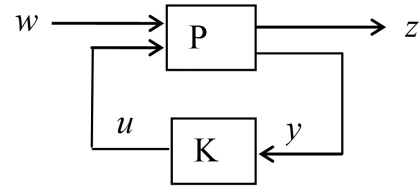

Fig. 1. Standard feedback control loop.

T_{zw} is described as follows:

$$T_{zw} = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \qquad (1)$$

where $F_l(P, K)$ is the lower linear fractional transformation of P and K (Zhou et al., 1998). The transfer matrix, $P(s)$, and the signals in the feedback control loop (see Fig. 1) can be expressed as:

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \qquad (2)$$

Here, z are performance outputs, w the exogenous inputs, y the measurements, and u are the control inputs:

$z = P_{11}w + P_{12}u,\ \ y = P_{21}w + P_{22}u,\ \ u = Ky$.

State-space realisation of the controlled process is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{12}u(t) \qquad (3) \\ y(t) &= C_2x(t) + D_{21}w(t) \end{aligned}$$

and $P(s) = C(s\,I - A)^{-1}\,B + D$.

Assumptions A1 to A8 are typically made for $\mathcal{H}_2$ or $\mathcal{H}_\infty$ or $\mathcal{H}_2/\mathcal{H}_\infty$ control design problems (Skogestad and Postlethwaite, 2005):

A1. system (A, B_2, C_2) is stabilisable and detectable,

A2. matrices D_{12} and D_{21} have full rank,

A3. matrix $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for

all ω, where $\omega \in [0, \infty)$ is the frequency,

A4. matrix $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all

values of ω,

A5. $D_{11} = 0$, $D_{22} = 0$.

A1 is required for the existence of a stabilising controller, K, and A2 is required to achieve proper value of K. A3 and A4 ensure that the controller does not cancel the poles or zeros on the imaginary axis, which leads to instability. A5 is required for the $\mathcal{H}_2$ control. Condition $D_{11} = 0$ results in strictly proper P_{11} which is required because $\mathcal{H}_2$ is a set of

proper transfer matrices. Condition $D_{22} = 0$ provides a strictly proper P_{22} and simplifies the $\mathcal{H}_2$ design problem. In the $\mathcal{H}_\infty$ control, none of the assumptions A1–A5 is required but if met, they simplify the problem. Sometimes, additional assumptions can be made to simplify the solution significantly (Skogestad and Postlethwaite, 2005):

A6. $D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & I \end{bmatrix}$,

A7. $D_{12}^T C_1 = 0$, $B_1 D_{21}^T = 0$,

A8. (A, B_1) is stabilisable and (A, C_1) is detectable.

Assumption A6 is required for the $\mathcal{H}_2$ problem and it means that there is no cross term in the cost function. A7 means common LQG control with no cross-coupling. If A7 is true, A8 replaces A3 and A4.

For a plant, $P(s)$, satisfying assumptions A1–A8 and the closed-loop configuration according to Fig. 1, the $\mathcal{H}_2$-optimal controller, $K(s)$, is found to minimise the $\mathcal{H}_2$ norm

$$\gamma_{2min} = \min_K \|T_{zw}\|_2 = \min_K \|F_l(P,K)\|_2 \qquad (4)$$

The suboptimal $\mathcal{H}_2$ design problem is defined as: find all internally stabilising controllers that assure

$$\|T_{zw}\|_2 = \|F_l(P,K)\|_2 < \gamma \qquad (5)$$

where $\gamma > \gamma_{2min}$.

A unique optimal controller is obtained as follows. Find positive definite

$X_2 = X_2^T \geq 0$ and $Y_2 = Y_2^T \geq 0$

that are the solutions of

$$A^T X_2 + X_2 A + C_1^T C_1 - X_2 B_2 B_2^T X_2 = 0 \qquad (6)$$

$$AY_2 + Y_2 A^T + B_1 B_1^T - Y_2 C_2^T C_2 Y_2 = 0 \qquad (7)$$

such that

$$\Re \left\{ \lambda_i \left[A - B_2 B_2^T X_2 \right] \right\} < 0 \ \forall i$$

and

$$\Re \left\{ \lambda_i \left[A - Y_2 C_2^T C_2 \right] \right\} < 0 \ \forall i$$

(stabilising solutions), where $\Re(.)$ is the real part of the argument.

Then, with

$F_2 = -B_2^T X_2$, $L_2 = -Y_2 C_2^T$, $\hat{A}_2 = A + B_2 F_2 + L_2 C_2$,

the central controller is

$$K_{2c}(s) = \left[\begin{array}{c|cc} \hat{A}_2 & -L_2 & B_2 \\ \hline F_2 & 0 & I \\ -C_2 & I & 0 \end{array} \right] \qquad (8)$$

and the family of all stabilising controllers such that

$\|T_{zw}\| \leq \gamma$ holds, is obtained from

$$K_2(s) = F_l(K_{2c}, Q) \qquad (9)$$

where $Q(s)$ is stable, strictly proper, and meets

$$\|Q\|_2^2 \leq \gamma^2 - \gamma_{2min}^2.$$

The optimal controller is obtained from Eq. (9) by setting $Q(s) = 0$.

$\mathcal{H}_\infty$ suboptimal controller design

Consider a plant, $P(s)$, with configuration as in Fig. 2, which satisfies assumptions A1–A8. All stabilising controllers, $K(s)$, have to be found that satisfy

$$\|F_l(P,K)\| < \gamma \qquad (10)$$

for a suboptimal bound $\gamma > \gamma_{min}$.

They can be obtained as follows (Glover and Doyle, 1988; Doyle et al., 1989): search for a positive definite

$X_\infty = X_\infty^T \geq 0$ and $Y_\infty = Y_\infty^T \geq 0$ solving

$$\begin{array}{c} A^T X_\infty + X_\infty A + C_1^T C_1 + \\ + X_\infty \left(\gamma^{-2} B_1 B_1^T - B_2 B_2^T \right) X_\infty = 0 \end{array} \qquad (11)$$

$$\begin{array}{c} AY_\infty + Y_\infty A^T + B_1 B_1^T + \\ + Y_\infty \left(\gamma^{-2} C_1^T C_1 - C_2^T C_2 \right) Y_\infty = 0 \end{array} \qquad (12)$$

such that

$$\Re \left\{ \lambda_i \left[A + \left(\gamma^{-2} B_1 B_1^T - B_2 B_2^T \right) X_\infty \right] \right\} < 0 \ \forall i$$

(stabilising solution of (11)),

$$\Re \left\{ \lambda_i \left[A + Y_\infty \left(\gamma^{-2} C_1^T C_1 - C_2^T C_2 \right) \right] \right\} < 0 \ \forall i$$

(stabilising solution of (12)) and $\rho(X_\infty Y_\infty) < \gamma^2$.

All controllers, $K(s)$, are given by $K = F_l(K_c, Q)$

where the central controller, K_c, is of the same order as P and is given by

$$K_c = \left[\begin{array}{c|cc} A_\infty & -Z_\infty L_\infty & Z_\infty B_2 \\ \hline F_\infty & 0 & I \\ -C_2 & I & 0 \end{array} \right] \qquad (13)$$

where

$A_\infty = A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$,

$Z_\infty = \left(I - \gamma^{-2} Y_\infty X_\infty \right)^{-1}$, $L_\infty = -Y_\infty C_2^T$, $F_\infty = -B_2^T X_\infty$

and $Q(s)$ are stable proper transfer functions for which $\|Q\|_\infty < \gamma$ holds.

The central controller can be, similarly as the LQG controller, separated into the estimator

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$$\dot{\hat{x}} = A\hat{x} + B_1^T \gamma^{-2} B_1^T X_\infty \hat{x} + B_2 u + Z_\infty L_\infty (C_2 \hat{x} - y) \quad (14)$$

and the state feedback

$$u = F_\infty \hat{x} \quad (15)$$

If the stated conditions in the Riccati equations listed above are not met, the tested value of γ is too low and hence infeasible. This is used to formulate a bisection algorithm, called γ-iteration, to search for a feasible value $\gamma \geq \gamma_{min}$ close to the optimum within tolerance γ such that $\gamma - \gamma_{min} < \varepsilon$.

Mixed sensitivity control

Mixed sensitivity stands for the transfer function shaping problems in which the sensitivity function $S = (I + GK)^{-1}$ is shaped along with one or more other closed-loop transfer functions such as KS or the complementary sensitivity function $T = I - S$ in a typical one degree-of-freedom configuration, where G denotes the plant and K the (sub-)optimal controller to be found (Skogestad and Postlethwaite, 2005). Shaping of multivariable transfer functions is based on the idea that a satisfactory definition of gain (range of gains) for a matrix transfer function is given by its singular values, σ. Hence, the classical loop-shaping ideas of feedback design can be generalised to multivariable systems. In addition to the requirement that K stabilises G, the closed-loop objectives are as follows (Skogestad and Postlethwaite, 2005):

- for disturbance rejection: low $\overline{\sigma}(S)$,
- for noise attenuation: low $\overline{\sigma}(T)$,
- for reference tracking: $\overline{\sigma}(T) \approx \underline{\sigma}(T) \approx 1$,
- for input usage (control energy) reduction: low $\overline{\sigma}(KS)$,
- for robust stability in the presence of an additive perturbation, $G_P = G + \Delta$: low $\overline{\sigma}(KS)$,
- for robust stability in the presence of a multiplicative output perturbation, $G_P = (I + \Delta)G$: low $\overline{\sigma}(T)$,

where $\overline{\sigma}(A)$ is the maximum and $\underline{\sigma}(A)$ is the minimum singular value of A. It is known that robust controllers are designed to minimise the $\mathcal{H}_\infty$-norm of the plant. Three weight functions were added to the control system for loop shaping (Bansal and Sharma, 2013). The classical feedback control system structure with weighting is shown in Fig. 2.

For this problem, the cost function given by Eq. (16) with weighting functions W_1, W_2, and W_3 penalising the error signal, control signal and output signal, respectively, can be used.

$$\|F_{zw}\|_\infty = \|F_l(P, K)\|_\infty = \left\| \begin{matrix} W_1 S \\ W_2 KS \\ W_3 T \end{matrix} \right\|_\infty < \gamma \quad (16)$$
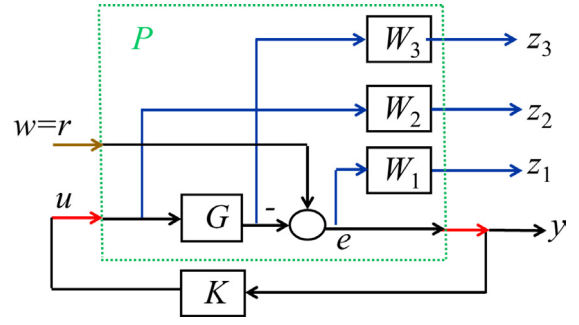

Fig. 2. Mixed-sensitivity configuration.

For this structure it is possible to use the generalised plant, P, as follows

$$P(s) = \begin{bmatrix} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ I & -G \end{bmatrix} \quad (17)$$

The returned values of S, KS, and T satisfy the following loop shaping inequalities:

$$\begin{aligned} \overline{\sigma}(S(j\omega)) &\leq \gamma \underline{\sigma}(W_1^{-1}(j\omega)) \\ \overline{\sigma}(KS(j\omega)) &\leq \gamma \underline{\sigma}(W_2^{-1}(j\omega)), \forall \omega \\ \overline{\sigma}(T(j\omega)) &\leq \gamma \underline{\sigma}(W_3^{-1}(j\omega)) \end{aligned} \quad (18)$$

It is necessary to choose low W_1 inside the desired control bandwidth to achieve good disturbance attenuation (i.e. performance), and to choose low W_3 outside the control bandwidth to ensure good stability margin (i.e. robustness).

μ-synthesis with DK-iteration

The DK iteration controller design method combines the $\mathcal{H}_\infty$ synthesis that provides robustness with the μ-analysis that considers parametric uncertainties and yields a good performance of the robust controller. The basic idea is to find a controller that minimises the maximum value of the upper bound on μ over the whole range of frequency.

At present, no analytical method to calculate a μ-optimal controller is available. However, a numerical method for complex perturbations known as DK-iteration can be used (Balas et al., 1998). The generalised open-loop representation of the configuration in Fig. 3 is given by Eq. (19) (Griffin and Fleming, 2003). The input and output vectors for this configuration contain inputs and outputs related to the input perturbations u_Δ and y_Δ, respectively. Uncertainty at the plant input and performance requirements at the system output are described by weights W_u and W_p, respectively, K is the controller, and G_d is the disturbance model:

$$\begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 & W_u \\ W_p G & W_p G_d & W_p G \\ -G & -G_d & -G \end{bmatrix} \begin{bmatrix} u_\Delta \\ d \\ u \end{bmatrix} = P \begin{bmatrix} u_\Delta \\ d \\ u \end{bmatrix} \quad (19)$$

The closed-loop interconnection structure N which includes P and K is given as:

$$N = \begin{bmatrix} -W_u KGS & -W_u KG_d S \\ W_p GS & W_p G_d S \end{bmatrix} \quad (20)$$

μ-synthesis with DK-iteration is based on the minimisation of the upper bound on μ specified in terms of the scaled singular value

$$\mu(N) \le \min_{D \in \mathcal{D}} \overline{\sigma}\left(DND^{-1}\right) \quad (21)$$

A scaling matrix, D, was chosen so as to commute with the plant perturbation Δ, i.e. $D\Delta = \Delta D$. Then, the synthesis problem is to find controller K that minimises the peak value of this upper bound over the given frequency range, namely:

$$\min_{K} \left(\min_{D \in \mathcal{D}} \left\| DN(K)D^{-1} \right\|_\infty \right) \quad (22)$$

by alternating the minimisation with respect to K and D (keeping the other ones fixed). The iteration proceeds as follows (Skogestad and Postlethwaite, 2005):

1. K-step. Synthesise an $\mathcal{H}_\infty$ controller for the scaled problem with fixed $D(s)$.

$$\min_{K} \left(\left\| DN(K)D^{-1} \right\|_\infty \right) \quad (23)$$

2. D-step. For N fixed, find $D(j\omega)$ to minimise

$$\overline{\sigma}\left(D(j\omega)ND^{-1}(j\omega)\right) \quad (24)$$

at each frequency.

3. Fit the magnitude of each element of $D(j\omega)$ to a stable and minimum-phase transfer function, $D(s)$. Go to step 1.

Continue the iteration until $\left\| DND^{-1} \right\|_\infty < 1$, or until the norm no longer decreases.

Simulations and results

Process description
Consider a shell-and-tube heat exchanger (Vasičkaninová and Bakošová, 2015), where petroleum is heated by hot water passed through a copper tube (Fig. 4).
Among the input variables, the hot water flow rate, $q_3(t)$, was selected as the control input. The controlled variable was the outlet petroleum
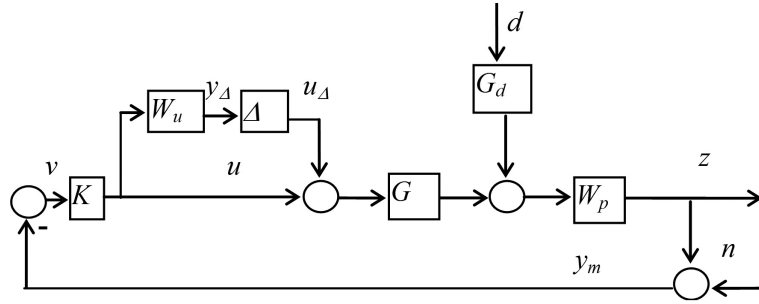

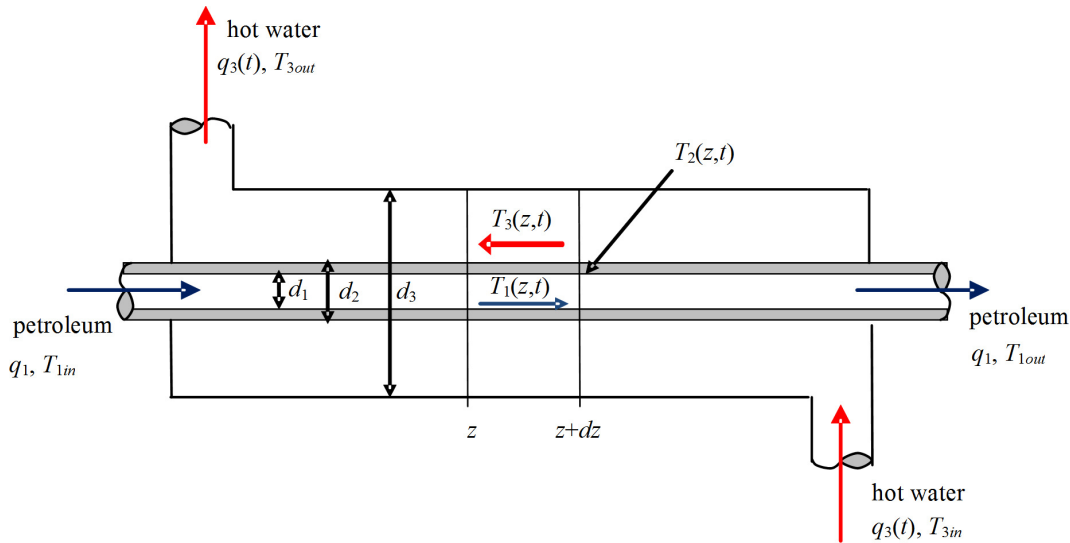**Fig. 3.** Closed-loop system with uncertainty and performance weighting.

Fig. 4. Scheme of the counter-current heat exchanger.

temperature, T_{1out}. The mathematical model of the heat exchanger was derived under several simplifying assumptions. Coordinate z measures the distance of a modelled section from the inlet. The fluids move in a plug velocity profile and the petroleum, tube and water temperatures: $T_1(z,t)$, $T_2(z,t)$ and $T_3(z,t)$, are functions of the axial coordinate, z, and time, t. The petroleum, water and tube material densities, ρ_i, as well as the specific heat capacities, c_{pi}, $i = 1, 2, 3$, were assumed to be constant.

The simplified nonlinear dynamic mathematical model of the counter-current heat exchanger is described by three partial differential equations:

$$-T_1(z,t) + T_2(z,t) =$$
$$= \frac{d_1 \rho_1 c_{p1}}{4h_1} \frac{\partial T_1(z,t)}{\partial t} + \frac{d_1 \rho_1 c_{p1}}{h_1} \frac{q_1}{\pi d_1^2} \frac{\partial T_1(z,t)}{\partial z} \quad (25)$$

$$\frac{d_1 h_1}{d_1 h_1 + d_2 h_2} T_1(z,t) - T_2(z,t) + \frac{d_2 h_2}{d_1 h_1 + d_2 h_2} T_3(z,t) =$$
$$= \frac{(d_2^2 - d_1^2)\rho_2 c_{p2}}{4(d_1 h_1 + d_2 h_2)} \frac{\partial T_2(z,t)}{\partial t} \quad (26)$$

$$T_2(z,t) - T_3(z,t) =$$
$$= \frac{(d_3^2 - d_2^2)\rho_3 c_{p3}}{4 d_2 h_2} \frac{\partial T_3(z,t)}{\partial t} -$$
$$- \frac{(d_3^2 - d_2^2)\rho_3 c_{p3}}{d_2 h_2} \frac{q_3(t)}{\pi(d_3^2 - d_2^2)} \frac{\partial T_3(z,t)}{\partial z} \quad (27)$$

Here, l is the length of the tube, d is the tube diameter, ρ the density, c_p the specific heat capacity, h the heat transfer coefficient, q the volumetric flow rate, 1 is petroleum, 2 is copper, and 3 is hot water. Parameters and steady-state inputs of the heat exchanger are listed in Table 1, where the subscript in denotes the inlet, and the superscript s denotes the steady state.

Tab. 1. Heat exchanger parameters and inputs.

Variable	Unit	Value
d_1	m	0.025
d_2	m	0.028
d_3	m	0.050
h_1	W m^{-2} K^{-1}	750
h_2	W m^{-2} K^{-1}	1480
ρ_1	kg m^{-3}	810
ρ_2	kg m^{-3}	8960
c_{p1}	J kg^{-1} K^{-1}	2100
c_{p2}	J kg^{-1} K^{-1}	418
c_{p3}	J kg^{-1} K^{-1}	4186
l	m	10
q_1	m^3 s^{-1}	3.7723×10^{-4}
q_{3in}^s	m^3 s^{-1}	1.1111×10^{-4}
T_{1in}^s	K	309.6
T_{2in}^s	K	317.76
T_{3in}^s	K	324.82

For identification, $\pm 10\%$, $\pm 30\%$, $\pm 50\%$ step changes of the inlet volumetric flow rate of heating water were generated. Step responses of the outlet temperature are shown in Fig. 5. Here, the response to the input change of $\pm 10\%$ is represented by a blue solid line, $\pm 30\%$ by a red dashed line, $\pm 50\%$ by a brown dotted line.

According to the step responses, the heat exchanger is a nonlinear system with asymmetric dynamics. The nominal model was identified using the Strejc method (Mikleš and Fikar, 2007) from the step responses in form of the nth order transfer function in Eq. (28). As for various step responses, intervals of the values of gain, K, time constant, τ, and time-delay, D, were obtained and the heat exchanger is represented as a system with interval parametric uncertainty, where $K \in [37000, 70000]$, $\tau \in [20, 30]$,
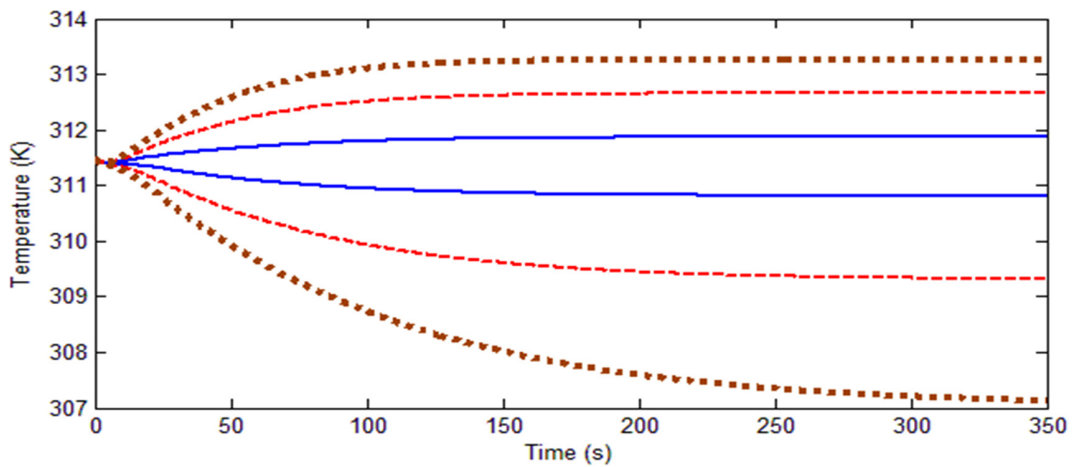

Fig. 5. Step responses of the outlet temperature to the step changes of the volumetric flow-rate of heating water.

$D \in [0.2, 2.8]$, $n = 2$. The nominal values of the parameters are the mean values of the identified parameter intervals.

$$G = \frac{K}{(\tau s + 1)^n} e^{-Ds} = \frac{53500}{(25s+1)^2} e^{-1.5s} \qquad (28)$$

Control of the heat exchanger

PID control
Conventional PID controllers described by the transfer function

$$C = k_p + \frac{k_i}{s} + k_d s \qquad (29)$$

were tuned using the Ziegler-Nichols and the Rivera-Morari methods (Bequette, 1991).
The PID controller parameters tuned using the Ziegler-Nichols formulas for the nominal values of parameters are: $k_p = 2.92 \times 10^{-4}$, $k_i = 9.72 \times 10^{-5}$, $k_d = 2.19 \times 10^{-4}$, and those obtained using the Rivera-Morari formulas are: $k_p = 7.27 \times 10^{-5}$, $k_i = 1.02 \times 10^{-6}$, $k_d = 2.44 \times 10^{-4}$.

Robust controllers
The transfer function describing the nominal system was assumed in the form of Eq. (28).
The $\mathcal{H}_2$ controller was found in the form:

$$C = \frac{0.20s^2 + 0.016s + 0.046}{s^3 + 6.77s^2 + 22.91s + 0.05} \qquad (30)$$

the $\mathcal{H}_\infty$ controller was found in the form:

$$C = \frac{0.04s^2 + 0.0035s + 0.00007}{s^3 + 3.38s^2 + 5.15s + 0.05} \qquad (31)$$

the $\mathcal{H}_2/\mathcal{H}_\infty$ controller was found in the form:

$$C(s) = \frac{0.50s^2 + 0.05s + 0.013}{s^3 + 7.2s^2 + 25s + 0.32} \qquad (32)$$

and finally, the μ-synthesis controller was found in the form in Eq. (33).
In Figs. 6 and 7, the reference trajectory, w, and the responses of the outlet petroleum temperature are shown as obtained using the designed PID and $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis controllers. The simulations were performed employing the nonlinear model characterised by Eqs. (25)–(27). Fig. 6 shows the simulation results of set-point tracking, while Fig. 7 presents the simulation results of set-point tracking and disturbance rejection. Changes of the inlet petroleum temperature from 303.15 K to 308.15 K at 200 s, from 308.15 K to 304.15 K at 600 s and to 307.15 K at 1000 s represent the disturbances.

The proposed controllers were compared using the IAE integral performance index described as follows:

$$\text{IAE} = \int_0^\infty |e| \, dt \qquad (34)$$

The obtained IAE values are given in Table 2. The lowest IAE values were obtained using the μ-synthesis controller, which attenuates disturbances in the shortest time and the overshoots caused by the disturbances are also minimal. However, the best results were obtained applying the most complicated structure of the controller. Among the robust controllers with a simpler structure, the $\mathcal{H}_2/\mathcal{H}_\infty$ controller has been shown to be the best. All robust controllers provided better results in terms of IAE than the conventional PID controllers; again, a more complicated structure of the controllers and a more complicated design were needed.
Stability is often investigated by analysing the Nyquist curve. To achieve stability, the Nyquist curve must be sufficiently far away from the critical point at –1. The sensitivity function, S, expresses how the closed-loop properties are influenced by small variations in the process and disturbances. The complementary sensitivity measures the influence of feedback on the measurement noise. The maximum magnitude of sensitivity and complementary sensitivity,

$$M_S = \max_\omega |S(j\omega)|, \quad M_T = \max_\omega |T(j\omega)|, \qquad (35)$$

are also used as robustness measures. Variable $1/M_S$ can be interpreted as the shortest distance between the Nyquist curve and the critical point at –1. Recommended values for M_S are typically within the range of 1.4–2.0 (Åström and Hägglund, 2006). A good compromise is $M_S = 1.7$ (Toivonen, 1998). The use of the maximum sensitivity as a robustness measure has the advantage that lower bounds on the gain, A_m, and phase, φ_m, margins (Åström and Hägglund, 2006) can be assured according to:

$$A_m > \frac{M_S}{M_S - 1} \qquad (36)$$

$$\varphi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right) \qquad (37)$$

Therefore, $M_S = 2$ provides the minimum robustness requirement, i.e. $A_m > 2$ and $\varphi_m > 29°$. For $M_S = 1.4$, $A_m > 3.5$ and $\varphi_m > 41°$. M_T is the largest magnitude of the complementary sensitivity. Typi-

$$C = \frac{7.35s^7 + 95.4s^6 + 247.6s^5 + 196.3s^4 + 33.47s^3 + 2.37s^2 + 0.07s + 0.001}{s^8 + 32.45s^7 + 426s^6 + 2479s^5 + 5014s^4 + 3313s^3 + 339s^2 + 11s + 0.11} \qquad (33)$$

Tab. 2. Values of IAE, M_S, M_T.

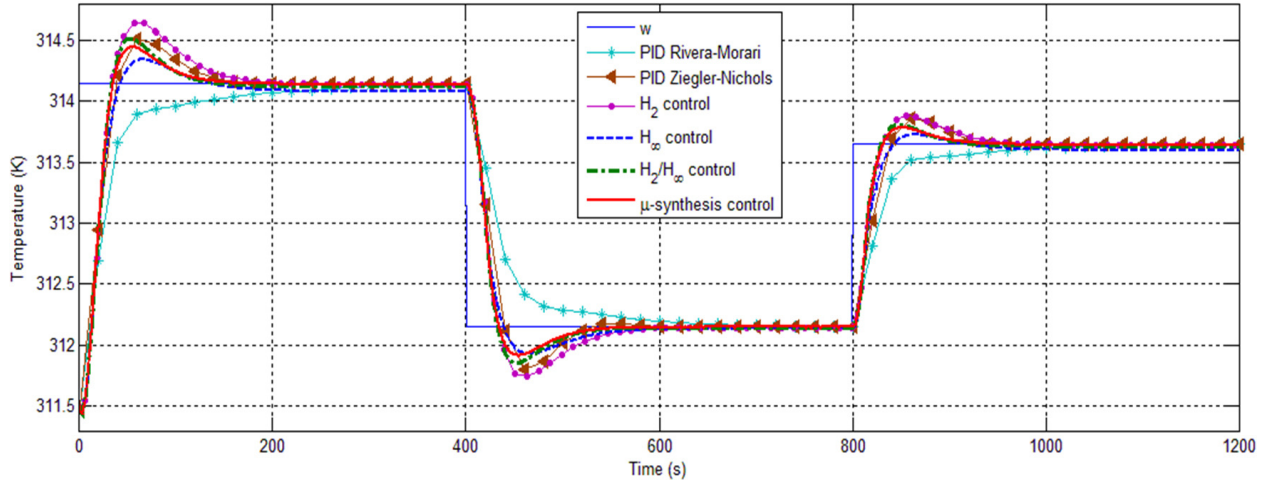| controller	set point tracking	set point tracking and disturbance rejection		
	IAE	IAE	M_S	M_T
PID Ziegler-Nichols	247	381	1.8417	1.7719
PID Rivera-Morari	231	421	1.2290	1.0025
$\mathcal{H}_2$ control	194	299	1.3715	1.0643
$\mathcal{H}_\infty$ control	178	235	1.4593	0.9880
$\mathcal{H}_2/\mathcal{H}_\infty$ control	162	234	1.5605	1.1577
μ-synthesis control	151	220	1.6043	1.0602

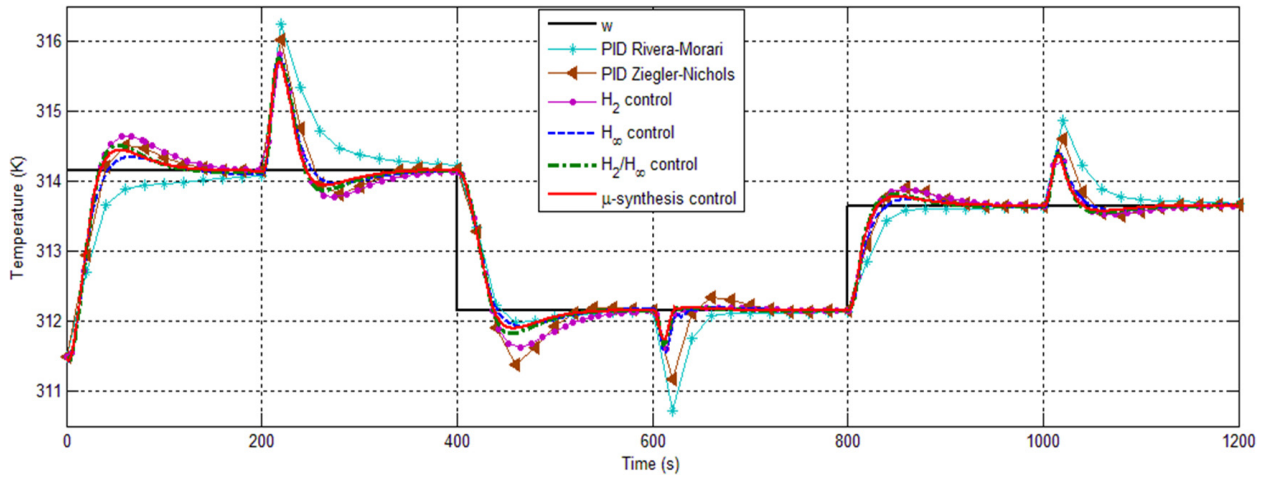**Fig. 6.** Temperature response in set point tracking.

Fig. 7. Temperature response in set point tracking and disturbance rejection.

cally, it is required that M_T is below 1.25 (Skogestad and Postlethwaite, 2005).

In Table 2, values of M_S and M_T are given. M_T is below 1.25 in all cases besides the PID Ziegler-Nichols controller. The condition $M_S \in \langle 1.4; 2 \rangle$ is not met for the PID Rivera-Morari controller and is almost assured by the $\mathcal{H}_2$ controller. According to the values of M_S and M_T it can be stated that all controllers designed using the described advanced control strategies are robust, while the best performance was shown by the μ-synthesis and $\mathcal{H}_2/\mathcal{H}_\infty$ controllers, as they are closest to the recommended value $M_S = 1.7$. Taking into account the controller structure, the $\mathcal{H}_2/\mathcal{H}_\infty$ controller is recommended.

Conclusions

Robust controllers designed using $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis approaches were used to control the heat exchanger. The presented results show satisfactory control responses for the set point tracking as well as for the disturbance attenuation. Design of conventional PID controllers is the most simple one, but these controllers are not sufficiently robust which leads to the worst performance. The $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis controller design requires the choice of weighting functions and of a reference model, but the designed controllers are robust and assure good performance. The μ-synthesis controller design is the most difficult and leads to the most complicated controller. However, such a controller provides the best performance. Taking into account the controller design, controller structure and performance, the $\mathcal{H}_2/\mathcal{H}_\infty$ controller is a reasonable compromise for the control of nonlinear processes with asymmetric dynamics.

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